- If $Y \simeq \Omega^d X$ then D^d on Y.

  $\Rightarrow H_* Y$ is a graded Poisson alg

Today: if D^d acts on Y & $\pi_0 Y$ gp then $Y \simeq \Omega^d X$.

Important case: $d = \infty$, in which case

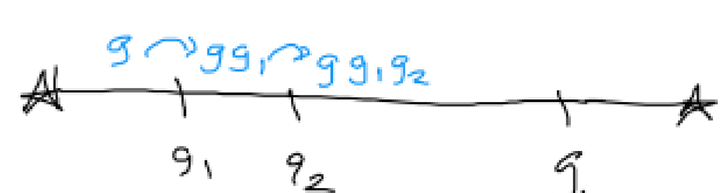
$Y, BY, B^2 Y, \ldots$ forms an Ω-spectrum.

$\Rightarrow$ represents a generalized cohomology

Key ex of 1-fold loop space: G.

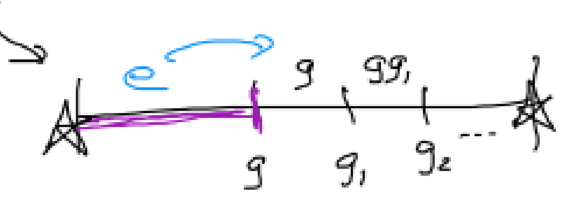
$\Omega BG \simeq G$

pf
$$\begin{tikzcd} \Omega BG \arrow[r] & PBG \arrow[r] & BG \\ G \arrow[u, "\simeq"] \arrow[r] & EG \arrow[u, "\simeq"] \arrow[r] & BG \arrow[u, equal] \end{tikzcd}$$

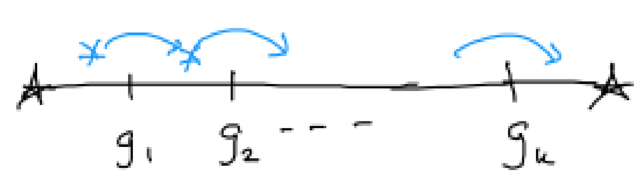
$E_\Delta G$ 

G left mult

id $E_\Delta G \simeq$ const.

s 

Stick (be careful!)

$B_\Delta G$ 

$= |NG|$

lifting

$G \longrightarrow \Omega BG$

$g \longmapsto$ 

After Salvatore

We will replace D^d by FM^d and use the latter to construct $B^d_{FM} Y$ whenever $FM^d \circlearrowleft Y$

Defn $FM^d(k)$ = closure of image of

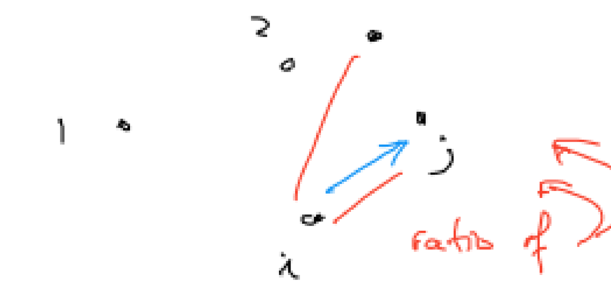
$\widetilde{Conf}_k(\mathbb{R}^d)$ = quotient by scaling & translation.

$\updownarrow$

$Conf_k[\mathbb{R}^d]$

$Conf_n(\mathbb{R}^d) \longrightarrow \prod_{\binom{n}{2}} S^{d-1} \times \prod_{\binom{n}{3}} [0,\infty]$

$(x_1 \ldots x_n) \longmapsto$ $\frac{x_j - x_i}{\|x_j - x_i\|}$ $\frac{\|x_k - x_i\|}{\|x_j - x_i\|}$



Thm $FM^d(k)$ is a manifold w/ corners w/ interior $\cong \widetilde{Conf}_k(\mathbb{R}^d)$, whose stratification poset

$\cong$ trees w/ k leaves and one root

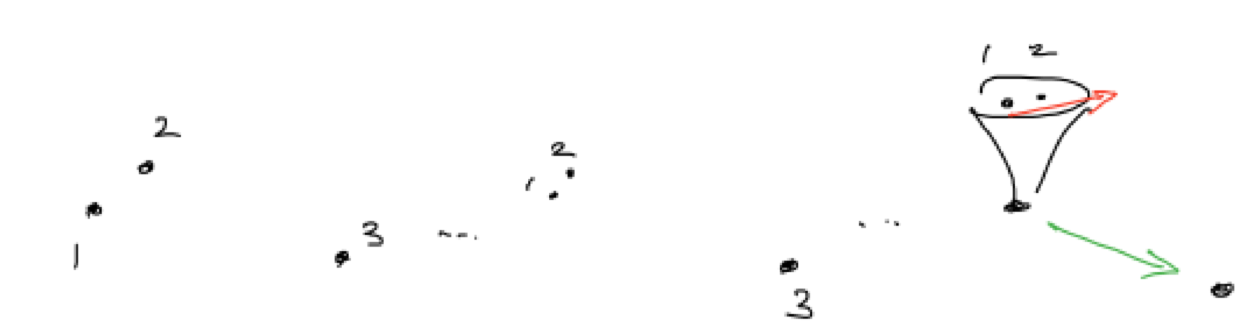
w/ no bivalent vertices (no univ. root)

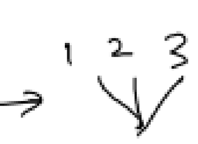
$T \geq T'$ if T obtained from T' by edge contraction

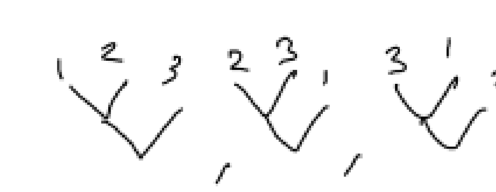
T-stratum, $\widetilde{Conf}_T(\mathbb{R}^d) \cong \prod_{v \in T} \widetilde{Conf}_{\#v}(\mathbb{R}^d)$

"incoming" valence.

Ex $k=2$ $\widetilde{Conf}_2(\mathbb{R}^d) \cong S^{d-1} = FM^d(2)$.



$k=3$ $\widetilde{Conf}_3(\mathbb{R}^d) \longleftrightarrow$ 

$\widetilde{Conf}_2(\mathbb{R}^d) \times \widetilde{Conf}_2(\mathbb{R}^d) \longleftrightarrow$ 

$\cong$

$S^{d-1} \times S^{d-1}$

$FM^2(3) \cong$ "Jacobi manifold"

Exercise? $FM^2(3) \cong$ complement of a trefoil. ?

$\widetilde{Conf}_2 \times \widetilde{Conf}_2 \times \widetilde{Conf}_3 \times \widetilde{Conf}_2 \longleftrightarrow$