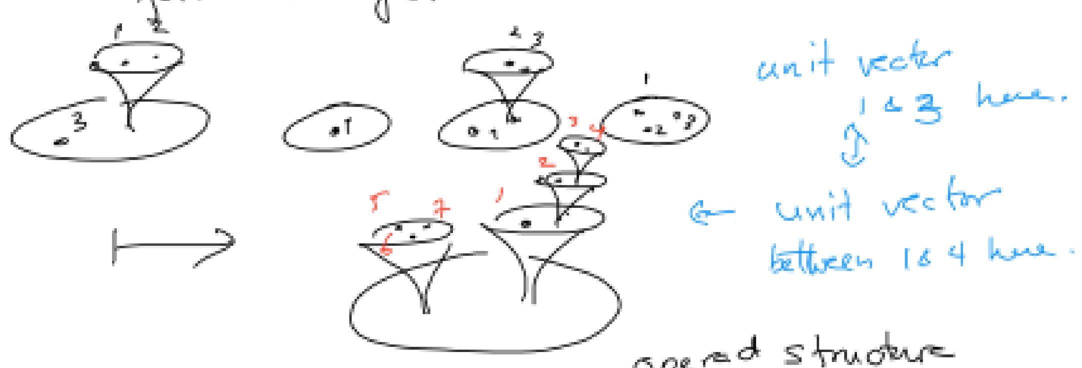
These form an operad.



"Exercise" understand these operad structure maps in coordinates $\left(\prod S^{d-1} \times \prod \mathbb{I}\right)$

In fact looking @ $\prod_{\binom{n}{2}} S^{d-1}$ one sees

that $\mathrm{Binom}(n) = \left(\!\binom{n}{2}\!\right)$ is a co-operad in sets

$\Rightarrow$ $\mathrm{Binom}_X(n) = X^{\left(\!\binom{n}{2}\!\right)}$ forms an operad

(cosimplicial model $\Omega^2 X$)

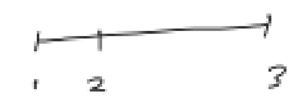
Konsevich operad $\subseteq$ $\mathrm{Binom}_{S^{d-1}}$

? What are algebras over $\mathrm{Binom}_X(\cdot)$?

$d = 1$.

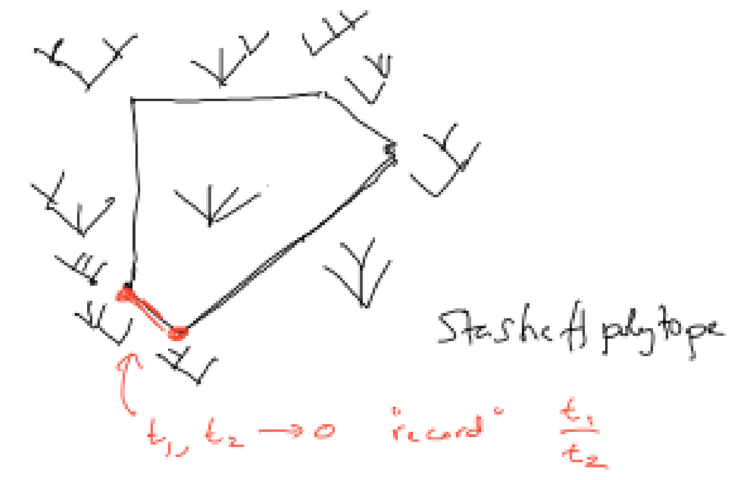
$FM'(2) \cong S_2$ $\cdot_1 \; \cdot_2$ or $\cdot_2 \; \cdot_1$

$FM'(3) \cong \mathbb{I} \times S_3$



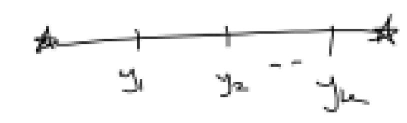
$FM'(4)$ mfld w/ corners. Interior $\mathring{D}^2$

stratification

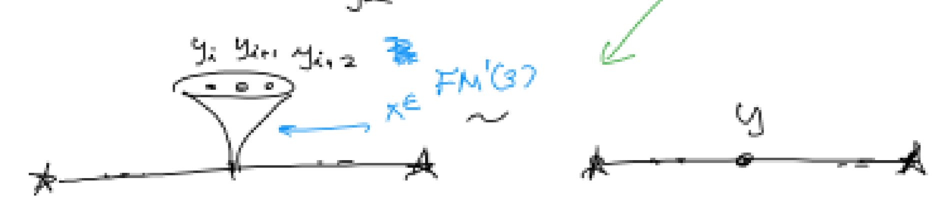


$FM' \curvearrowright Y$ Form $B_{FM} Y = \bigcup_k FM'(k) \times_{S_k} Y^k \big/ \sim$

"generically" $B_{FM} Y$



But eg.

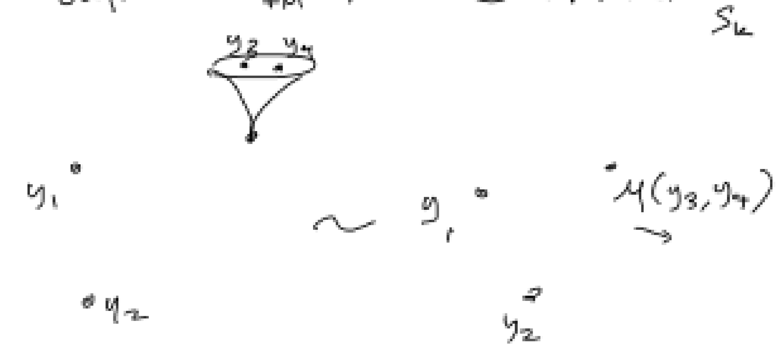


$y = \mu_3(x, y_i, y_{i+1}, y_{i+2})$

Define $E_{FM} Y$ similarly. Need $\pi_0 Y$ gp.

$\simeq *$.

$FM^d \curvearrowright Y$ define $B^d_{FM} Y = \bigcup FM^d(k) \times_{S_k} Y^k \big/ \sim$



Main effort is construction of.

$B^{d-1}_{FM} Y \hookrightarrow E^d_{FM} Y \to B^d_{FM} Y$

If M^d, framed then can form. ($FM^d \curvearrowright Y$)

$\bigcup \mathrm{Conf}_k[M] \times Y^k \big/ \sim \;=:\; \int_M Y$

Recall $\pi_i(\mathbb{Z} Y) \cong H_i(Y)$ Dold-Thom.

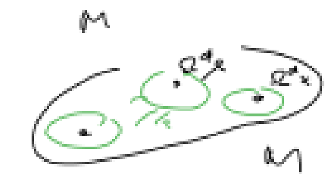
$X \overset{\simeq}{\subseteq} M \subseteq \mathbb{R}^\infty$ E is an "E_∞-alg" (action of FM^∞)

$$\pi_i\left(\int_M E\right) \cong H_i(M; \underline{E}) \quad \left(\int_M E \simeq \Omega^\infty M \wedge \underline{E} \right)$$

Factorization homology: what can we understand of M.

(beyond homotopy type) w/ choices of E.

Salvatore $E = \Omega^d X$ $\int_M \Omega^d X \simeq \mathrm{Map}(M, X)$



Nonabelian Poincaré duality:

$$\int_M E = \int_M \Omega^d(B^d E) \simeq \mathrm{Map}(M, B^d E)$$

$\updownarrow$ $\updownarrow$

$H_i M$ $H^{d-i}(X; \underline{E})$

$B(B^{d-1})$