# EXERCISES RELATED TO THE COHOMOLOGY OF SYMMETRIC GROUPS AND RELATED TOPICS

Everything is over $\mathbb{F}_2$ unless otherwise noted.

(1) Show that $\mathrm{Conf}_n(\mathbb{R}^\infty)$ is contractible. (Hints: start with $n = 2$ and then use the fact that projection maps (forgetting a point) between configuration spaces are fibrations.

(2) Find generators and relations for $\mathbb{F}_2[x_1, y_1, z_1, x_2, y_2, z_2]^{\mathcal{S}_2}$ and $\mathbb{F}_2[x_1, y_1, x_2, y_2, x_3, y_3]^{\mathcal{S}_3}$ where $\mathcal{S}_n$ acts on indices.

(3) Show that the homology of an infinite loop space which represents a ring spectrum is a Hopf ring.

(4) Show directly (geometrically) that $\Delta_{\odot} q_n$ and $\Delta_{\odot} q_{1,1}$ are zero.

(5) Calculate products such as
- $(\gamma_1 \odot 1_2) \cdot \gamma_{1[2]}^2$
- $(\gamma_1 \odot 1_2) \cdot (\gamma_1^2 \odot \gamma_1)$
- $(\gamma_1 \odot 1_2) \cdot \gamma_2$
- $(\gamma_1^3 \odot \gamma_2 \odot 1_2) \cdot (\gamma_{1[2]} \odot 1_4)$.

(6) Find additive basiess for $H^*(B\mathcal{S}_{12})$ and $H_*(B\mathcal{S}_{12})$.

(7) Find generators and relations for $H^*(B\mathcal{S}_8)$.

(8) Fill in the proof that restrictions in group cohomology map to invariants with respect to $W(H)$.

(9) Read the first few pages of Wilkerson's primer on Dickson invariants, at `http://www.math.purdue.edu/~wilker/papers/dickson.pdf` Then verify that $x_1^2 + x_1 x_2 + x_2^2$ and $x_1^2 x_2 + x_1 x_2^2$ generate the second Dickson algebra. Write down generators for the third Dickson algebra, and calculate Steenrod squares on them.

(10) Explicitly give the decomposition $H^*(B\mathcal{S}_m) \cong \bigoplus_{P \in \mathrm{Par}(m)} (\bigotimes_i \wedge_i^s D_i)$ when $m = 12$.

(11) Give additive bases for $B\mathcal{A}_4$ and $B\mathcal{A}_8$ through degree nine.