

Introduction to multivariable functions

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Example 1. *Look at the graphs of the following functions:*

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