Choosing a math topic for the museum project

This document contains brief descriptions of some math topics that could be part of the museum exhibit. While I came up with many topics while planning this course during the past few years, several topics listed here were inspired directly by your posts on the discussion board and input on the Week 1 survey.

Assignment for Week 3

This week, we will form teams that will each collaborate on a mathematical topic for the museum project. To assist with this:

1. Read the list of projects below, and decide which ones appeal most to you. (We will also go over these topics in class on Monday, April 13.

2. Post on the message board by 5 pm Tuesday (April 14) with:

   (a) Two skills or things you like to do that could be helpful as your team works on the museum project. For example, do you like writing about math? Are you good at explaining math to others? Have you studied modular arithmetic lots? Do you enjoy looking for patterns? Are you good at figuring out how to make code work? etc. No one knows everything or is able to do everything perfectly. Just share two ways in which you might contribute.

   (b) If there is some skill or trait you hope someone else on your team will have, please share that. For example, do you hope someone else will be good with Mathematica? Do you hope someone else will enjoy writing? This part is optional but could be helpful for forming a team.

   (c) From the list below, name at least three topics (ranked, if you'd like) that you are most interested in working on.

3. The above information might give you ideas about classmates with whom you might be especially interested in working on this project. So you might want to email them to discuss possibly working together on the project.

4. By 5 pm on Friday (April 17), complete the survey on Canvas. In particular, if there are classmates with whom you particularly want to work or there is any additional information you think I should have, please include it there. I will then take all this information into consideration when assigning project teams.

Suggested math topics for the project

For the visualization part of most of these projects, I would provide you with some code to tinker with, similarly to how we began the first "Exercises and Investigations" set, unless someone in your project group particularly wants to do this themself. (The one exception is the final project below, for which I designed an app that outputs intricate pictures in response to basic input but currently only runs on Apple computers.)

1. Families of functions in the complex plane

   In this project, you will explore families of functions of number theoretic interest and their behavior in the complex plane. Building on the ideas from "Exercises and Investigations" set 1, you will also track how circles, lines, and various shapes (and colors) transform under these functions on the complex plane. For lots of inspiration, see the book Visual Complex Functions (linked from our Library Reserve List, or go directly to `https://alliance-primo.hosted.exlibrisgroup.com/permalink/f/3uoa1r/CP71136177190001451`).

2. Algebraic numbers

   In this project, you will explore zeroes of polynomials with rational coefficients. To see some of the spectacular examples of plots of certain families of algebraic numbers, go to:

   `http://math.ucr.edu/home/baez/roots/`

   and

   `https://blogs.ams.org/visualinsight/2013/09/01/algebraic-numbers/`

3. Solving for zeroes of polynomials, Newton's method, and fractals

   It turns out that not only do the zeroes of polynomials form interesting patterns in the complex plane, but so does one approach to finding those zeroes. This project starts with Newton's method, an approach to finding zeroes of polynomials, which leads to *Newton fractals* (which, in turn, sometimes look related to the fractal Azusena posted on the discussion board in Week 1). For an idea of the approach and resulting fractals, see this wikipedia article: `https://en.wikipedia.org/wiki/Newton_fractal`.

4. Arithmetic $\mod n$

   In this project, you will further explore arithmetic $\mod n$, including approaches to visualization of addition and multiplication $\mod n$. For just one example of someone's approach to this, see `https://math.katestange.net/illustration/elementary-number-theory/multiplication-tables-modulo-n/`.

5. Distribution of prime numbers

   In this project, you will explore the distribution of prime numbers. The 3Blue1Brown video showed prime spirals. There are also lots of other ways we might plot and visualize the distribution of primes and how they are divided into different sets (such as congruences classes), though. See, for example, `https://betweenartandscience.com/ulamspiral_words.html`. One might also try to build on the 3-dimensional picture Hunter and Morgan discussed on the discussion board in Week 2.

6. Certain sums of roots of unity

   In this project, you will explore certain sums of roots of unity (i.e. complex numbers z such that $z^n = 1$ for some positive integer n, like we discussed in class), especially sums called *Gauss sums* and *Gaussian periods*. It turns out that these sums are useful in modular arithmetic, for example when determining whether a

given quadratic equation has a solution mod a given integer. As was discovered a few years ago, these sums also have spectacular visual properties. For one example, see the image on the flyer advertising our course: http://nebula.wsimg.com/c1aac287a5783d2bdc3276d7662dccf4?AccessKeyId=49DDC7E543B57EC30D6B&disposition=0&alloworigin=1

For more information, see http://www.elleneischen.com/gaussianperiods.html