PHYS 391, Homework #5

Key Concepts

- Fourier Series
- Discrete Fourier Transform

Reading: Fourier Handout

Homework Problems

1. Find (by integration) an expression for the real Fourier coefficients a_n and b_n for a 'ramp' function $f(x) = x$ over the range $-\pi$ to π. Write out the first six non-constant (and nonzero) fourier series terms and plot them using python or any program you wish. Hint: save yourself some work by considering whether the function $f(x)$ is even or odd.

2. Find (by integration) the real Fourier coefficients a_n and b_n for a rectified sine function given by

$$f(x) = \begin{cases} -sin(x), & -\pi < x < 0, \\ +sin(x), & 0 < x < +\pi. \end{cases}$$

   What is the lowest non-constant term in the Fourier series? In other words, what is the fundamental frequency of this rectified sine function? Note, this is the hum you hear when you turn on a guitar amplifier...

3. Write a simple python script which will produce a 1D array or list of values sampled at discrete times from the function $A\sin(2\pi\nu t) + B$, where A, B, ν, as well as the sampling frequency ν_s and N (the total number of samples) are input parameters. This is to create "fake data" which can be used in the next problems. Be careful to make sure your discrete time values have a spacing of exactly $\Delta t = 1/\nu_s$. Demonstrate that your function works by producing a plot with $A = 1$, $B = 1$, $\nu = 1$, $\nu_s = 20$ and $N = 100$.

4. Use your script from the last problem to explore aliasing. Generate a curve with $N = 100$ where $\nu = 9.9$ and $\nu_s = 10$. What is the apparent frequency from the plot and how does this compare to the true frequency ν? Explain qualitatively what is going on here. What minimum sampling frequency ν_s would you need here to avoid aliasing?

5. Write a script which will plot the amplitude vs. frequency for a time series of data (like the output of problem 3). In addition to your data, this function will need to know the $\Delta\nu = \nu_s/N$ in order to properly plot amplitude vs. frequency. Check that this works by applying this function to the output of problem 3. Attach the plot of amplitude vs. frequency as well as your code. Hint: You will need to import an FFT package. Scipy has a good one under subpackage fftpack called simply, fft().

6. Take the Fourier transform of the same function with $\nu = 2$ rather than $\nu = 1$. Attach the amplitude vs. frequency plot and explain why you see what you see.

7. Do the same thing for $\nu = 2.05$. What is different here? How can we understand this?

8. Redo the last problem but now use $N = 1000$ rather than $N = 100$. Explain in words what happened. What is the difference in the frequency range sampled by this larger value of N? What is the difference in the frequency resolution?