Learning and Expectations in Macroeconomics
Problems for Chapter 10

1. Consider the following model which is commonly used to study the implications of monetary policy rules.[1] The first equation, obtained as a linearization of household's Euler equation, relates the output gap x_t to expected output gap and real rate of interest
$$x_t = -\sigma^{-1}(i_t - E_t^*\pi_{t+1}) + E_t^* x_{t+1} + g_t.$$
The second equation is a "new Phillips" curve describing optimal pricing of monopolistically competitive firms
$$\pi_t = \lambda x_t + \zeta E_t^* \pi_{t+1}.$$
π_t is the inflation rate and i_t is the nominal interest rate. $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote private sector expectations of inflation and output gap next period. All the parameters are positive. $0 < \zeta < 1$ is the discount rate of the representative firm and is therefore close to one. g_t is an observable shock following an $AR(1)$ process
$$g_t = \mu g_{t-1} + \hat{g}_t,$$
where $\hat{g}_t$ is iid and $0 < \mu < 1$. The Taylor rule relating the nominal interest rate to output gap and inflation
$$i_t = \phi_x x_t + \phi_\pi \pi_t,$$
where $\phi_x, \phi_\pi > 0$, describes the setting of monetary policy.
(a) Write this model as bivariate linear model in the standard form
$$\begin{aligned} y_t &= \beta E_t^* y_{t+1} + \kappa w_t \\ w_t &= \varphi w_{t-1} + e_t, \end{aligned}$$
that is in the form (10.15) of the book, where the key coefficient matrix is
$$\beta = \frac{1}{\sigma + \phi_x + \lambda \phi_\pi} \begin{pmatrix} \sigma & 1 - \zeta\phi_\pi \\ \lambda\sigma & \lambda + \zeta(\sigma + \phi_x) \end{pmatrix}.$$
(b) Show that the condition for regularity (also called determinacy or saddle-point stability) of the model is
$$\lambda(\phi_\pi - 1) + (1 - \zeta)\phi_x > 0.$$
(Hint: The eigenvalues of a matrix A are inside the unit circle if and only if $|\det(A)| < 1$ and $|\mathrm{tr}(A)| < 1 + \det(A)$.)
(c)Use Proposition 10.3 to verify that the above condition for regularity is also the E-stability condition for the MSV solution.
(Hint: The eigenvalues of a 2×2 matrix A have negative real parts if and only if $\mathrm{tr}(A) < 0$ and $\det(A) > 0$.)

[1] This problem presents one case of the results derived in Bullard, J. and K. Mitra (1999), "Learning Monetary Policy Rules", Working paper, Federal Reserve Bank of St. Louis. For a survey on the literature see Clarida, R., J. Gali and M. Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, vol.37, 1661-1707.

2. We modify the preceding model by adding an observable cost-push shock to the pricing equation, so

$$\pi_t = \lambda x_t + \zeta E_t^* \pi_{t+1} + u_t,$$

where $u_t = \rho u_{t-1} + \hat{u}_t$ with $\hat{u}_t$ *iid* and $0 < \rho < 1$. With a standard quadratic loss function for the policy maker, it can be shown that optimal policy under discretion is described by the interest rate rule

$$\begin{aligned} i_t &= \chi_u u_t + \chi_g g_t, \text{ where} \\ \chi_u &= \frac{\sigma(1-\rho)\lambda + \alpha\rho}{\alpha(1-\rho\zeta)+\lambda^2}, \chi_g = \sigma. \end{aligned}$$

(Here α is the relative weight of the output target in the policy objective function.) Derive the standard form of this bivariate model. Prove that with any interest rate rule which only depends on the observable shocks (like the optimal rule above) the MSV solution is not E-stable.[2]

[2]This problem presents a key result from the the paper Evans, G.W. and S. Honkapohja (2000), "Expectations and the Stability Problem for Optimal Monetary Policies", Working paper, University of Helsinki.