Learning and Expectations in Macroeconomics
Problems for Chapter 12

1. Consider the nonstochastic model $y_t = F(y_{t+1})^e$. A 2-cycle $(\hat{y}_1, \hat{y}_2)$, with $\hat{y}_1 \neq \hat{y}_2$, is a solution $y_t = \hat{y}_1$ if t is odd and $y_t = \hat{y}_2$ if t is even, where $\hat{y}_2 = F(\hat{y}_1)$ and $\hat{y}_1 = F(\hat{y}_2)$. A steady state $\bar{y} = F(\bar{y})$ can be regarded as a degenerate 2-cycle in which $\hat{y}_1 = \hat{y}_2 = \bar{y}$. Consider PLMs which allow for 2-cycles, i.e. taking the form $y_t = y_1$ if t is odd and $y_t = y_2$ if t is even. Obtain the corresponding E-stability condition for a steady state $\bar{y}$ (this is known as the strong E-stability condition for a steady state).

2. For the model $y_t = F(y_{t+1})^e$, write down the condition for E-stability of a 2-state Markov SSE (y_1^*, y_2^*). Using the fact that the eigenvalues of a matrix are continuous functions of its elements, show that if a 2-cycle $(\hat{y}_1, \hat{y}_2)$ exists then SSEs sufficiently close to the 2-cycle are E-stable if $F'(\hat{y}_1)F'(\hat{y}_2) < 1$.

3. Consider the basic nonstochastic overlapping generations model with temporary equilibrium given by $V'(n_t)n_t = E_t^*(U'(n_{t+1})n_{t+1})$. Under the assumption of point expectations we have

$$V'(n_t)n_t = U'(n_{t+1}^e)n_{t+1}^e,$$

where n_{t+1}^e is the forecast of n_{t+1} made at time t.

(a) Suppose that $U(c) = c^{1-\sigma}/(1-\sigma)$ and $V(n) = n^{1+\varepsilon}/(1+\varepsilon)$,where $\sigma, \varepsilon > 0$. Show that n_t can be solved explicitly as $n_t = F(n_{t+1}^e)$. Show that there is a unique interior steady state at $n_t = 1$.

(b) Show that the steady state is always weakly E-stable and that the condition for strong E-stability, under PLMs which permit 2-cycles, is that $\varepsilon + 2 > \sigma$.

(c) Show that there do not exist 2-cycles if $\sigma > 2 + \varepsilon$, but there do exist sunspot equilibria (SSEs).

4. We continue with the model of Problem 1.

(i) Formulate the $T-$ mapping and the ODE defining E-stability of a two-state Markov SSE (n_1^*, n_2^*) and calculate the E-stability condition analytically. Show that E-stability requires $tr(DT) < 1$ (hint: show that $\det(DT) = 0$).
(ii) Verify numerically that for $\sigma = 4$ and $\varepsilon = 1$ there is a two-state Markov SSE with $\pi_{11} = \pi_{22} = 0.1$. Find the equilibrium values (n_1^*, n_2^*). Check numerically whether it is E-stable.