Liquidity Traps and Expectation Dynamics: Fiscal Stimulus or Fiscal Austerity?*

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Abstract

We examine global dynamics under infinite-horizon learning in New Keynesian models where the interest-rate rule is subject to the zero lower bound. The intended steady state is locally but not globally stable. Unstable deflationary paths emerge after large pessimistic shocks to expectations. For large expectation shocks that push interest rates to the zero bound, a temporary fiscal stimulus, or in some cases a policy of fiscal austerity, will insulate the economy from deflation traps if the policy is appropriately tailored in magnitude and duration. A fiscal stimulus "switching rule," which automatically kicks in without discretionary fine-tuning, can be equally effective.

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1 Introduction

The recent financial crisis and the subsequent adverse macroeconomic developments in advanced market economies have focused the analysis of macroeconomic outcomes on low rates of inflation and low levels of aggregate economic activity. Worries have emerged that, in the absence of strong policy interventions, the economy might be driven into a deflationary trap and deflationary expectations. Bullard (2010) has stressed the risk of extended periods of deflation, sometimes called a liquidity trap. The Japanese economy seems to have been plagued by such a liquidity trap for nearly two decades.[1] In macroeconomic research these developments have motivated work that has focused on the possibility of multiple equilibria due to the zero interest rate lower bound (ZLB) under standard monetary policy rules, like the Taylor rule. See for example Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe, and Uribe (2001), Benhabib, Schmitt-Grohe, and Uribe (2002), Eggertsson and Woodford (2003), and the more recent work by Werning (2012) and Mertens and Ravn (2014).

This recent literature has explored new types of monetary and/or fiscal policies that can avoid or escape persistent deflationary outcomes. Such policies are based on models that strongly rely on the rational expectations hypothesis: Proposed policies make use of announcements, promises or threats about future policy actions and outcomes, such as policy induced violations of the transversality conditions in order to avoid falling into the liquidity trap. Some of these ideas - especially forward guidance in monetary policy - are now used in actual policy making, e.g. see Bernanke (2012). Others suggest a suitably "irresponsible" fiscal or monetary policies (see e.g. Krugman (1998), Chapter 2.4 of Woodford (2003) and Benhabib, Schmitt-Grohe, and Uribe (2002)), which are controversial as their credibility may be questionable. More recently a number of papers have examined the efficacy of standard fiscal policies, i.e. changes in taxes and/or government spending, when monetary policy is constrained at the zero lower bound. Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011), Eggertsson (2010) and Braun, Korber, and Waki (2012) assume rational expectations and also that the economy is pushed to the zero lower bound as a result of a sustained exogenous negative preference shock, modeled as a two-state Markov process,

[1]See Krugman (1998), Eggertsson and Woodford (2003), and Svensson (2003) for the renewed interest in the liquidity trap sparked by the recent Japanese experience.

with an absorbing value at the normal level, and which therefore disappears in finite time. In our view this perspective does not do justice to the view of expectations as having an independent role in macroeconomic dynamics.

Christiano and Eichenbaum (2012) and Mertens and Ravn (2014) both consider sunspot equilibria, again taking the form of a two-state Markov process, with the normal outcome as an absorbing state. These papers do consider aspects of learning. The former paper uses learning as a selection device to rule out the sunspot equilibrium. The latter paper concedes the instability of the sunspot equilibrium under learning, but looks at the impact of fiscal policy on the learning paths to the absorbing state leading the targeted steady state.[2]

We instead consider situations in which, due to some dramatic adverse shock to expectations, the economy is in a region in which expectations are in, or with unchanged policy will enter, the deflation trap region in which adaptive learning reinforces pessimistic expectations. We consider how to avoid this trap, characterized by low output and persistent deflation, by focusing on traditional fiscal policies involving government spending on goods and services, i.e. policies of fiscal stimulus or austerity. We also retain the usual monetary policy of the Taylor interest rate rule that assures the local stability of the economy at the targeted inflation rate.[3]

We consider two types of fiscal policies. The first type is an announced increase or cut in government spending on goods that is tuned to the current macroeconomic situation. Following Evans, Honkapohja, and Mitra (2009) agents are assumed to build into their decision-making the announced path of government spending. We show that a properly tuned fiscal stimulus is effective. With the stimulus policy in place, the economy can escape from the liquidity trap. Surprisingly, in some conditions suitably designed fiscal austerity can also move the economy out of a liquidity trap. The second type of successful fiscal policy is instead rule-based, so that an appropriate increase in government spending is triggered if actual inflation or inflation expectations go below a pre-specified lower threshold for the rate of inflation.

Our analysis relies on the assumption that private agents form their ex-

[2] Christiano and Eichenbaum (2012) and Mertens and Ravn (2014) both use short-horizon learning based on Euler equations, along lines introduced in Evans, Guse, and Honkapohja (2008). In the current paper we use infinite-horizon learning.

[3] We do not consider other more complex monetary policies that have been used in the literature on current crisis to address the problems in the functioning of financial markets.

pectations using adaptive learning.[4,5] In other words, in making forecasts agents act like econometricians who have a forecasting model that in any period is estimated using existing data, and updated as new data becomes available. The state of the economy in any time period is viewed as a temporary equilibrium for given expectations while the learning process is a sequence of temporary equilibria that can converge to rational expectations equilibrium. We explore policies designed to avoid and escape the ZLB in New Keynesian (NK) models with agents who form expectations using adaptive learning rules. We focus on NK models because, from the policy viewpoint the problem with deflation has been associated with declining output, high unemployment and/or stagnation.[6] For some policies the announcements of a sequence of policy moves are a key part of policy. Using the techniques in Evans, Honkapohja, and Mitra (2009) these announcements are assumed to be credible and are thus incorporated into agents' forecasting.

Analytically, the multiple equilibria problem means that, in addition to the targeted steady state at the (gross) inflation rate $\pi = \pi^* \geq 1$, there is a low-inflation unintended steady state. If the (gross) interest rate is at the lower bound $R = 1$, then by the Fisher equation $R = \pi/\beta$ there is a second steady state $\pi_L = \beta < 1$, where β is the discount factor. It turns out that under learning dynamics a persistent deflation trap with $\pi_t \leq \pi_L$ is possible when policy is described by the usual Taylor rule and constant fiscal policy.

The intuition for the deflation trap under learning is that if expected inflation and expected output are below the values corresponding to the low steady state at $\pi_L = \beta$, then aggregate demand will be low because the expected deflation implies high ex-ante real interest rates values. The high real interest rates, especially if combined with low expected output, lead to low actual levels of aggregate output and to actual inflation below expected inflation. Under adaptive learning expectations are revised further

[4]For discussion and analytical results concerning adaptive learning in a wide range of macroeconomic models, see for example Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009). For empirical work on learning, see Milani (2007), Milani (2011), Eusepi and Preston (2011), Slobodyan and Wouters (2012), and as an overview Section 3 of Evans and Honkapohja (2013).

[5]Recently, there has been increasing interest in relaxing the rational expectations hypothesis in the context of macroeconomic policy analysis, see e.g. Taylor and Williams (2010) and Woodford (2013)

[6]Consequences of the interest rate zero lower bound and the liquidity trap under adaptive learning have earlier been studied in Evans and Honkapohja (2005), Evans, Guse, and Honkapohja (2008) and Evans and Honkapohja (2010).

downward, pushing the economy deeper into the deflation trap.

The lack of rational expectations can give scope for wealth effects, like the traditional Pigou effect, as a stabilizing mechanism. Can wealth effects ensure an eventual return to the steady state? The answer depends on specific aspects of the private agents' expectations. Evans and Honkapohja (2010) find deflation traps when agents forecast over the infinite future and perceive that the transversality condition (TVC) is always met along these disequilibrium paths. Such consumers are called "Ricardian," in that they do not perceive bonds and money as net wealth. (In Evans, Honkapohja, and Mitra (2012) it was shown that when expectations are not fully rational, Ricardian equivalence may or may not hold, depending in particular on the assumptions concerning the influence of government financial variables on expectations.) What about the direct wealth effects of real money and bonds when households are non-Ricardian? Would such wealth effects be effective in avoiding deflation traps if households do not have Ricardian consumption functions? We investigate this issue and find that wealth effects can eventually return the economy to the π^* steady state, but that these mechanisms can be slow, and fail in some cases.[7]

Our main focus is on fiscal policies. As indicated above, we first consider policies that implement a temporary fiscal stimulus in the form of government spending, or its converse, a policy of temporary fiscal austerity, under the assumption that future taxes adjust to keep the government solvent in the long-run. We show that a fiscal stimulus can be effective, i.e. deliver convergence of the economy to the targeted steady state, if its magnitude is sufficient and its duration is sufficiently short. Interestingly, a policy of fiscal austerity, i.e. a temporary cut in government spending can also be effective. This requires, however, the fiscal austerity period to be sufficiently long, and the degree of initial pessimism in expectations to be relatively mild.

One disadvantage of fiscal stimulus and fiscal austerity policies is that both their magnitude and duration have to be tailored to the initial pessimistic expectations, so they require swift and precise discretionary action. A second more automatic fiscal policy, a fiscal stimulus "switching rule," can also ensure a return to the intended steady state π^*. This policy eliminates the unintended steady state and guarantees that the economy does not get

[7] Another mechanism that can prevent a deflationary spiral is a lower bound $\underline{\pi}$ on inflation due to asymmetric costs of price adjustment. However, it still can lead to falling output, to stagnation, or to a very slow return to the π^* steady state. See Appendix 2.

stuck in a regime of deflation and stagnation. A significant advantage of this rule is that it is triggered automatically and does not require discretionary fiscal fine tuning.

2 The Model

We start with the same basic economic framework as in Evans, Guse, and Honkapohja (2008). There is a continuum of household-firms, which produce a differentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses both monetary and fiscal policy and can issue public debt as described below.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s}\left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1\right) \tag{1}$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,s} + \frac{P_{t,s}}{P_t}y_{t,s}, \tag{2}$$

where $c_{t,s}$ is the Dixit-Stiglitz consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t, $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,s}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2}\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982).[8] The household decision problem is also subject to the usual "no Ponzi game" condition.

[8] We use the Rotemberg formulation in preference to the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system. See Ascari and Rossi (2012) for a comparison of Rotemberg and Calvo models when there is trend inflation.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha},$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve given by

$$P_{t,s} = \left(\frac{y_{t,s}}{Y_t}\right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. Y_t is aggregate output, which is exogenous to the firm.

The government's flow budget constraint is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected.[9] We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1}, \tag{5}$$

where we will usually assume that $\beta^{-1} - 1 < \kappa < 1$. This restriction on κ means that fiscal policy is "passive" in the terminology of Leeper (1991) and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

Initially we assume that g_t is constant and given by

$$g_t = \bar{g}. \tag{6}$$

From market clearing we have

$$c_t + g_t = y_t. \tag{7}$$

Monetary policy is assumed to follow a global interest rate rule

$$R_t - 1 = f\left(\pi_{t+1}^e, y_{t+1}^e\right). \tag{8}$$

[9] Some of the literature cited above allows for labor income taxes. This could be introduced in our set-up but would complicate our model.

The function $f(\pi, y)$ is taken to be positive and non-decreasing in each argument. The rule (8) is a nonlinear forward-looking Taylor rule, where the nominal rate is set by the central bank as a function of expected inflation and expected output.[10] We assume the existence of π^*, R^* and y^* such that $R^* = \beta^{-1}\pi^*$ and $f(\pi^*, y^*) = R^* - 1$. Here π^* can be viewed as the inflation target of the Central Bank, and y^* is the natural rate of output, i.e. the level of output compatible with steady state inflation π^*. We assume that $\pi^* \geq 1$. In the numerical analysis we will use the functional form

$$f(\pi, y) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)}\left(\frac{y}{y^*}\right)^{\phi_y}, \tag{9}$$

which implies the existence of a steady state at (π^*, y^*). Using $R^* = \pi^*\beta^{-1}$, we obtain $f_{\pi^*}(\pi^*, y^*) = AR^*/\pi^* = A\beta^{-1}$. We assume that $A > 1$. Equations (6), (5) and (8) constitute "normal policy".

2.1 Optimal decisions for private sector

In Appendix 1 we derive the following optimality conditions for the consumer-producer:

$$\begin{aligned} 0 &= -h_{t,s}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,s} - 1)\pi_{t,s}\frac{1}{h_{t,s}} \\ &\quad +\alpha\left(1 - \frac{1}{\nu}\right)Y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}}c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}}E_{t,s}(\pi_{t+1,s} - 1)\pi_{t+1,s}. \end{aligned} \tag{10}$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s}\left(\pi_{t+1}^{-1}c_{t+1,s}^{-\sigma_1}\right) \tag{11}$$

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_{t,s}\pi_{t+1}^{\sigma_2 - 1}}\right)^{-1/\sigma_2}, \tag{12}$$

where $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$.

For convenience we make the assumptions $\sigma_1 = \sigma_2 = 1$, i.e. utility of consumption and of money is logarithmic. It is also assumed that agents have point expectations, so that their decisions depend only on the mean of their subjective forecasts.

[10] The main results below would also hold in the case of a contemporaneous-data Taylor rule, which is used in Evans, Guse, and Honkapohja (2008).

We now proceed to rewrite the decision rules for c_t and π_t so that they depend on forecasts of key variables over the infinite horizon. The infinite-horizon (IH) learning approach in New Keynesian models was first emphasized by Preston (2005) and Preston (2006), and was used in Evans and Honkapohja (2010) to study the properties of a liquidity trap.[11]

2.2 The infinite-horizon Phillips curve

Defining

$$Q_{t,s} = (\pi_{t,s} - 1)\pi_{t,s}, \tag{13}$$

the price-setting Euler equation (10) becomes

$$\begin{aligned} 0 &= -h_{t,s}^{\varepsilon} + \frac{\alpha\gamma}{\nu} Q_{t,s} \frac{1}{h_{t,s}} \\ &\quad + \alpha \left(1 - \frac{1}{\nu}\right) Y_t^{1/\nu} \frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}} c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu} \frac{1}{h_{t,s}} E_{t,s} Q_{t+1,s}. \end{aligned}$$

Using the production function $y_{t,s} = h_{t,s}^{\alpha}$ we get

$$Q_{t,s} = \frac{\nu}{\alpha\gamma} y_{t,s}^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} Y_t^{1/\nu} y_{t,s}^{(\nu-1)/\nu} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s},$$

and using the demand curve $y_{t,s}/Y_t = (P_{t,s}/P_t)^{-\nu}$ gives

$$Q_{t,s} = \frac{\nu}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} Y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}.$$

It is shown in Appendix 1 that the necessary TVC for optimal price setting implies the condition

$$\lim_{t\to\infty} \beta^t E_{t,s} Q_{t,s} = 0. \tag{14}$$

Defining

$$x_{t,s} \equiv \frac{\nu}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} Y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1},$$

[11] In the literature on learning and bounded rationality it is often assumed that agents have a short (one-period) decision horizon. Then Euler equations provide directly the relevant decision rules. Evans, Guse, and Honkapohja (2008) applies the Euler-equation approach to the analysis of liquidity traps.

iterating the Euler equation yields

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s} \tag{15}$$

by using the limit condition $\beta^j E_{t,s} x_{t+j,s} \to 0$ as $j \to \infty$. This last condition is implied by (14). We remark that the variable $x_{t+j,s}$ is a mixture of aggregate variables and the agent's own future decisions.

At this point there are alternative ways to proceed. One approach emphasized by Eusepi and Preston (2010) is to assume that agents choose $Q_{t,s}$ as part of the optimal plan given expectations about the future values of variables that are exogenous to them.[12] We take a different approach motivated by the agents' knowledge of observed empirical relationships that hold in temporary equilibrium. It is assumed that at time t agents forecast aggregate inflation π_{t+j} and aggregate output Y_{t+j} using an adaptive learning rule that is discussed below. In addition, we make some further adaptive learning assumptions that involve their own future decisions and expected future aggregate variables. In particular, agents are assumed to have learned from experience that, in temporary equilibrium, it is always the case that $P_{t,s}/P_t = 1$ and also $c_{t,s} = Y_t - g_t$ in per capita terms. These two relationships necessarily hold in temporary equilibrium because agents have been assumed to be identical (though agents themselves do not need to know this). Therefore, we assume that agents impose these relationships in their forecasts in (15), i.e. they set $(P_{t+j,s}/P_{t+j})^e = 1$ and $c^e_{t+j,s} = Y^e_{t,t+j} - g^e_{t,t+j}$ for $j \geq 1$. In the case of no policy change the latter assumption becomes $c^e_{t+j,s} = Y^e_{t+j} - \bar{g}$.

Recalling the assumptions of point expectations and log utility of consumption gives

$$\begin{aligned} Q_{t,s} &= x_{t,s} + \sum_{j=1}^{\infty} \beta^j x^e_{t+j,s}, \text{ where} \\ x^e_{t+j,s} &= \frac{\nu}{\alpha\gamma} \left(Y^e_{t+j}\right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} Y^e_{t+j} (Y^e_{t+j} - \bar{g})^{-1} \end{aligned}$$

Finally, assuming homogeneous expectations and imposing symmetry, i.e. all agents are in identical situations so that $y_{t,s} = y_t = Y_t$, $c_{t,s} = c_t$ and in

[12] To implement this approach they linearize the model around the intended steady state.

addition $P_{t-1,s} = P_{t-1}$ for all s (note that $P_{t,s}/P_t = (\pi_{t,s}/\pi_t)(P_{t-1,s}/P_{t-1})$ so that $Q_{t,s} = Q_t$, and $P_{t,s} = P_t$), we obtain

$$\begin{aligned} Q_t &= \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (y_t - \bar{g})^{-1} + \\ & \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j \left(y_{t+j}^e\right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left(\frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}}\right), \end{aligned} \tag{16}$$

which defines the temporary equilibrium value for Q_t. In order to have a monotonic relationship between Q_t and π_t, the appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. We will treat (16), together with (13), as the temporary equilibrium equations that determine π_t, given expectations $\{y_{t+j}^e\}_{j=1}^{\infty}$. Later, we will consider a case where g_t varies over time and then $y_{t+j}^e - \bar{g}$ becomes $nety_{t+j}^e = (y_{t+j} - g_{t+j})^e$ in equation (16).

2.3 The consumption function

To derive the consumption function using the IH-learning approach, the first step is to use the flow budget constraint and the NPG (no Ponzi game) to obtain an intertemporal budget constraint. First, we define the asset wealth

$$a_t = b_t + m_t$$

as the sum of holdings of real bonds and real money balances and write the flow budget constraint as

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1}) m_{t-1}, \tag{17}$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t) y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (17) forward and imposing

$$\lim_{j \to \infty} (D_{t,t+j}^e)^{-1} a_{t+j} = 0, \tag{18}$$

where

$$D_{t,t+j}^e = \prod_{i=1}^{j} r_{t+i}^e,$$

with $r^e_{t+j} = R_{t+j-1}/\pi^e_{t+j}$, we obtain the life-time budget constraint of the household

$$
\begin{aligned}
0 &= r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Phi^e_{t+j} && (19) \\
0 &= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\phi^e_{t+j} - c^e_{t+j}), && (20)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi^e_{t+j} &= y^e_{t+j} - \Upsilon^e_{t+j} - c^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}, \text{ or} && (21) \\
\phi^e_{t+j} &= \Phi^e_{t+j} + c^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}.
\end{aligned}
$$

Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

From the consumer's perspective equation (18) is related to the transversality condition requiring the discounted value of assets a_t to go to 0 as $t \to \infty$. Some earlier papers (see Chapter 2.4 of Woodford (2003) and Benhabib, Schmitt-Grohe, and Uribe (2002)) explore commitments to combinations of fiscal and monetary policies that rule out paths that satisfy Euler equations but that do not converge to the targeted steady state as possible equilibria because they violate the consumers' transversality conditions. Our approach is different. We use fiscal policies involving government spending on goods and services to directly affect aggregate expenditures. In our adaptive learning context these policies generate inflation and interest rate trajectories that exclude paths where inflation falls below a specified threshold, and in particular they can avoid deflationary paths.

Returning to optimization of consumption we have

$$
c_t^{-1} = \beta r^e_{t+1} (c^e_{t+1})^{-1}, \text{ where } r^e_{t+1} = R_t/\pi^e_{t+1}, \text{ and} \tag{22}
$$

under the assumption of a representative agents and identical expectations. The consumption Euler equation (22) implies the relations

$$
c^e_{t+j} = c_t \beta^j D^e_{t,t+j}, \tag{23}
$$

and we obtain

$$
c_t(1-\beta)^{-1} = r_t a_{t-1} + y_t - \Upsilon_t + {\pi_t}^{-1}(1 - R_{t-1}) m_{t-1} + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \phi^e_{t+j}. \tag{24}
$$

As $\phi^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}$, the final term in (24) is

$$\sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}) + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}$$

and using (12) with the representative agent assumption we have

$$\begin{aligned} & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1} \\ = & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(-\chi\beta R^e_{t+j-1}c^e_{t+j-1}) = -\frac{\chi\beta}{1-\beta}c_t \end{aligned}$$

and so

$$c_t\frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}).$$

Finally, we invoke the flow budget identity $b_t + m_t + \Upsilon_t - g_t = m_{t-1}\pi_t^{-1} + r_t b_{t-1}$, see (4), and obtain the consumption function

$$c_t\left[\frac{1+\chi\beta}{1-\beta} - \chi\beta\frac{R_t}{R_t - 1}\right] = b_t + y_t - g_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(z^e_{t+j}), \tag{25}$$

where $z^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j}$.

The derivation of the consumption function (25) has assumed households that do not act in a Ricardian way, i.e. they do not impose the intertemporal budget constraint (IBC) of the government. We next turn to the case of Ricardian consumers.

2.4 The Case of Ricardian Consumers

For Ricardian consumers we modify the consumption function as in Evans and Honkapohja (2010).[13] From (4) one has

$$\begin{aligned} b_t + m_t + \Upsilon_t &= g_t + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= g_t - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

[13] Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

By forward substitution, and assuming

$$\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0, \tag{26}$$

we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j}. \tag{27}$$

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, we assume that agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \Delta_{t+j}^e, \text{ where} \\ \Delta_{t+j}^e &= g_{t+j}^e - \Upsilon_{t+j}^e - m_{t+j}^e + m_{t+j-1}^e (\pi_{t+j}^e)^{-1} \text{ for } j = 1, 2, 3, \ldots. \end{aligned}$$

A Ricardian consumer assumes that (26) holds. His flow budget constraint (17) can be written as:

$$\begin{aligned} b_t &= r_t b_{t-1} + \psi_t, \text{ where} \\ \psi_t &= y_t - \Upsilon_t - m_t - c_t + \pi_t^{-1} m_{t-1} \end{aligned}$$

The relevant transversality condition is now (26). Iterating forward and using (23) together with (26) yields the consumption function

$$c_t = (1-\beta)\left(y_t - g_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (nety_{t+j}^e) \right), \tag{28}$$

where $nety_{t+j} = y_{t+j} - g_{t+j}$. For more details see Evans and Honkapohja (2010).

3 Temporary Equilibrium and Learning

3.1 Equilibrium Conditions

We now come to the formulation of learning (see footnote 3 for general references on adaptive learning). In general, in adaptive learning it is assumed that each agent has a model for perceived dynamics of state variables, also called the perceived law of motion (PLM), to make his forecasts of relevant variables. In each period the PLM parameters have been estimated using available data and the estimated model is used to compute forecasts. The PLM parameters are then re-estimated when new data becomes available in subsequent periods. A common formulation is to postulate that the PLM is a linear regression model where endogenous variables depend on intercepts, observed exogenous variables and possibly lags of endogenous variables. The estimation would then be based on least squares or related methods. The regression formulation cannot be applied here because there would asymptotic perfect multicollinearity in the current non-stochastic setting.[14] We therefore assume that agents form expectations using so-called steady state learning, which is formulated as follows.

Steady-state learning with point expectations is formalized as

$$s_{t+j}^e = s_t^e \text{ for all } j \geq 1, \text{ and } s_t^e = s_{t-1}^e + \omega_t(s_{t-1} - s_{t-1}^e) \tag{29}$$

for $s = y, z, nety, \pi$. Here ω_t is called the "gain sequence," and measures the extent of adjustment of estimates to the most recent forecast error. In stochastic systems one often sets $\omega_t = t^{-1}$ and this "decreasing gain" learning corresponds to least-squares updating. Also widely used is the case $\omega_t = \omega$, for $0 < \omega \leq 1$, called "constant gain" learning. In this case it is usually assumed that ω is small. Stability of the steady states is examined below using the simple learning rules just described.

The temporary equilibrium equations with steady state learning are as follows. In presenting them we must distinguish between the cases of Ricardian and non-Ricardian consumers.

1. The aggregate demand relation. In the non-Ricardian case

$$\begin{aligned} y_t &= g_t + \left[\frac{1+\chi\beta}{1-\beta} - \chi\beta\frac{1+f(\pi_t^e, y_t^e)}{f(\pi_t^e, y_t^e)}\right]^{-1}\left[b_t + y_t - g_t + \sum_{j=1}^{\infty}(D_{t,t+j}^e)^{-1} z_t^e\right] \\ &= g_t + \left[\frac{1+\chi\beta}{1-\beta} - \chi\beta\frac{1+f(\pi_t^e, y_t^e)}{f(\pi_t^e, y_t^e)}\right]^{-1}\left[b_t + y_t - g_t + \frac{\pi_t^e}{1+f(\pi_t^e, y_t^e) - \pi_t^e} z_t^e\right] \\ &\equiv g_t + C(\pi_t^e, y_t^e, z_t^e, b_t, y_t), \end{aligned} \tag{30}$$

[14] See Evans and Honkapohja (1998) or Section 7.2 of Evans and Honkapohja (2001) for discussions of learning in deterministic and stochastic models.

where we have assumed that agents know the interest rate rule.

For the case of Ricardian consumers this equation is replaced by $y_t = g_t + c_t$, where c_t is given by (28). This leads to equation (40), given in the next section.

2. The nonlinear Phillips curve

$$\begin{aligned} \pi_t &= Q^{-1}[\tilde{K}(y_t, y^e_{t+1}, y^e_{t+2}...)] \\ &\equiv Q^{-1}[K(y_t, y^e_t)] \\ &\equiv G_2(y_t, y^e_t), \end{aligned} \tag{31}$$

where

$$Q(\pi_t) \equiv (\pi_t - 1)\,\pi_t \tag{32}$$

$$\begin{aligned} K(y_t, y^e_t) \equiv\ & \frac{\nu}{\gamma}\left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1-\nu^{-1}\right)\frac{y_t}{(y_t - g_t)}\right) \\ & + \frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y^e_t)^{(1+\varepsilon)/\alpha} - \left(1-\nu^{-1}\right)\frac{y^e_t}{nety^e_t}\right)\right), \end{aligned} \tag{33}$$

and where until Section 4 we assume that $nety^e_t = y^e_t - \bar{g}$.

3. Bond dynamics

$$b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}. \tag{34}$$

4. Money demand

$$m_t = \chi\beta \frac{R_t}{R_t - 1} c_t. \tag{35}$$

5. Interest rate rule

$$R_t = 1 + f\left(\pi^e_t, y^e_t\right).$$

The state variables are b_{t-1}, m_{t-1}, and R_{t-1}. The system in general has four expectational variables: output y^e_t, inflation π^e_t, income net of taxes z^e_t and net output $nety^e_t$. In cases where government spending is constant we have $nety_t^e = y_t^e - \bar{g}$, so that it is not necessary to introduce expectations of net output separately. The evolution of expectations is given by

$$\begin{aligned} y_t^e &= y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e) && (36) \\ \pi_t^e &= \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e) && (37) \\ z_t^e &= z_{t-1}^e + \omega(z_{t-1} - z_{t-1}^e) && (38) \\ nety_t^e &= nety_{t-1}^e + \omega(nety_{t-1} - nety_{t-1}^e), && (39) \end{aligned}$$

where equation (38) is used only in cases when the households are Non-Ricardian.

3.2 Dynamics under standard policies

3.2.1 The case with Ricardian consumers

We now consider the case where government spending is constant $g_t = \bar{g}$. In this case we can assume that $nety_{t+j}^e = z_{t+j}^e = y_{t+j}^e - \bar{g}$. For simplicity, in this section we drop the dependence of the interest rate rule on expected output so that $\phi_y = 0$ and $R_t = 1+ f(\pi_t^e)$. Using this and the steady-state learning assumption in (28), the market-clearing equation $y_t = g_t + c_t$ gives the aggregate output equation

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g}) \left(\frac{\pi_t^e}{1 + f(\pi_t^e) - \pi_t^e} \right) && (40) \\ &\equiv G_1(y_t^e, \pi_t^e). \end{aligned}$$

The temporary equilibrium is now given by the Phillips curve (31), the output equation with Ricardian consumption function (40) and the independent equation for the evolution of debt and money. Note that the Ricardian system just depends on expectations of output and inflation, so that the paths of inflation and output do not depend on the evolution of bonds and real balances. The (small gain) dynamics can therefore be described by the E-stability differential equation using a two-dimensional phase diagram. (See e.g. Evans and Honkapohja (2001) for a discussion of E-stability.)

The E-stability differential equations are given by

$$\begin{aligned} \frac{dy^e}{d\tau} &= G_1(y^e, \pi^e) - y^e && (41) \\ \frac{d\pi^e}{d\tau} &= G_2(y^e, \pi^e) - \pi^e, \end{aligned}$$

where using (31) we define $G_2(y^e, \pi^e) = G_2(G_1(y^e, \pi^e), y^e)$. The steady state equations for h, c and π are

$$c = h^{\alpha} - \bar{g},$$
$$-h^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi-1)\pi + \alpha\left(1-\frac{1}{\nu}\right)h^{\alpha}c^{-1} = 0$$
$$1 + f(\pi) = \beta^{-1}\pi.$$

Steady states are defined by $R = 1 + f(\pi)$ together with the the Fisher relationship $R = \pi\beta^{-1}$. For $A > 1$ there are two steady states, (y^*, π^*) and (y_L, π_L) with $\pi_L < \pi^*$. Local E-stability results for the Ricardian case are given by Proposition 2 of Evans and Honkapohja (2010): the π^* steady state is locally stable under learning, while for small γ, the π_L steady state is locally unstable under learning, with the local learning dynamics taking the form of a saddle.[15]

One can also look at the global learning dynamics using a phase diagram of system (41). The dynamics in the phase diagram approximate the discrete real-time paths of steady state learning when the gain ω is small. For typical parameter values the learning dynamics are shown below in Figure 1. The figure is constructed with the following parameter values $A = 2.5$, $\pi^* = 1.02$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 350$, $v = 21$, $\varepsilon = 1$, and $g = 0.2$. While $A = 1.5$ is the usual value for the interest rate rule in the literature, we choose $A = 2.5$ to clearly separate the intended and unintended steady states in the numerical analysis (our results are robust to using $A = 1.5$).

The calibrations of the target inflation rate π^*, the discount factor β, the labor share α, and the approximate GDP share of government spending, g are standard. We set the labor supply elasticity $\varepsilon = 1$. To calibrate γ, we exploit the relation of the Rotemberg and the Calvo models of costly and sticky price adjustments via their reduced form implications for the linearized Phillips curve. As shown by Keen and Wang (2007), using our notation we can express $\gamma = \frac{(\nu-1)\phi}{(1-\phi)(1-\beta\phi)}$ where $1-\phi$ is the fraction of firms changing their price during the quarter. Following Basu and Fernald (1997) we calibrate $\nu = 21$, implying a conservatively estimated 5% markup. To calibrate ϕ we use the estimate of Kehoe and Midrigan (2010), p. 8 for the frequency of

[15] Instability of the low inflation steady state under learning and the divergent paths were earlier described in McCallum (2002), Eusepi (2007), and Evans, Guse, and Honkapohja (2008). Bullard and Cho (2005) show the possibility of "escape paths" toward the low-inflation outcome.

regular price changes from BLS data, excluding temporary changes in price that quickly revert to their older trend level. Kehoe and Midrigan find this frequency to be 14.5 months or 4.8 quarters, implying that the percentage of firms not changing prices is during a quarter is $\phi = 0.7916$. Using the formula above we obtain $\gamma \approx 350$ for our calibration. The literature contains a range of estimates for the value of ν and ϕ and thus one could have alternative calibrations to our model. Our qualitative results are robust to different calibrations.

We also assume that interest rate expectations $r^e_{t+j} = R_{t+j-1}/\pi^e_{t+j}$ revert to the steady state value β^{-1} for $j \geq T$. This truncation is needed for technical reasons to prevent agents forecasting negative real interest rates indefinitely, which would imply unbounded consumption. For the long run, it is also plausible that consumers would make this assumption. In Figure 1 we use $T = 28$, which under a quarterly calibration corresponds to 7 years.[16]

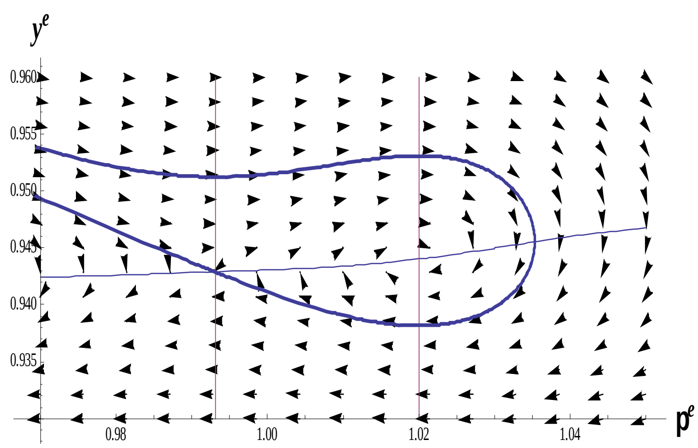

Figure 1: Global learning dynamics – the Ricardian case.

The main features that stand out are, first, the local stability of the targeted steady state at $(\pi^*, y^*) \approx (1.02, 0.9440)$. There is in fact a "corridor of stability" defined by a set of initial expectations that converge to the π^* steady state. (The term "corridor" is due to Leijonhufvud (1973).) This corridor is defined by the region enclosed within the stable manifold of the unintended steady state $(y_L, \pi_L) \approx (0.9931, 0.9429)$. The intuition for the local stability of the targeted steady state under learning is that if, say, (y^e, π^e)

[16] This choice is roughly in line with data on the aftermath of financial crises. See Reinhart and Rogoff (2009).

is somewhat below steady-state values then the locally active Taylor rule reduces interest rates enough to reduce ex-ante real interest rates, stimulating output, which increases inflation. Under adaptive learning expectations will then be revised upward.

Second, we see that convergence to π^* is locally cyclical: when expectations differ from the intended steady state, the adjustment under normal policy, with adaptive learning, has gradually convergent cyclical dynamics. We will see that this phenomenon cannot eliminated by the fiscal policies that we consider. The extent of cycling does vary with alternative policies for avoiding deflation traps. However, the design of policies to minimize this cyclical dynamics is not our objective in the current paper. Third, it can be seen that there is a heteroclinic orbit connecting the π_L steady state with the π^* steady state.

Fourth, and most strikingly, we observe that for initial points outside the corridor of stability the trajectory of expectations is (at least eventually) led into a deflation trap in which (y^e, π^e) fall steadily over time. Along these paths we have falling actual output and inflation, intensifying as deflation sets in. The intuition for these paths is that if, say, (y^e, π^e) are somewhat below the low steady-state values (y_L, π_L), then we are in the liquidity trap region near the ZLB in which there is negligible room to reduce nominal interest rates. However, the real rate is positive and indeed above β^{-1} due to the expected deflation. These high real interest rates, combined with low y^e, lead to low levels of aggregate demand and low output, and through the Phillips curve to actual inflation below expected inflation. Under adaptive learning expectations of inflation and output are revised further downward, preventing escape from the deflation trap.

Finally, note that even though the financial wealth of agents is getting very large over time along such a deflationary path, Ricardian agents do not respond by sufficiently increasing consumption, as they expect that this increase in wealth will be offset by future growth in taxes. Thus in the Ricardian case, wealth effects do not lead to an escape from the deflation trap.

3.2.2 Wealth Effects and Non-Ricardian Consumers

We next consider Non-Ricardian consumers. A traditional argument against the liquidity trap dates back to Pigou (1943) and Patinkin (1965). In principle, wealth effects could prevent a deflation trap: if declining prices lead to higher perceived wealth, agents will increase their spending. This can be investigated numerically. Our simulations indicate that wealth effects can indeed stabilize the economy at π^*.

The dynamics under learning when consumers are not Ricardian are given in Section 3.1. These describe the temporary equilibrium, and the adjustment of expectations. Taken together they constitute the dynamic system that determines the real-time evolution of the economy. Because government bonds and real balances are state variables that affect consumption and output, expectations y^e, π^e are no longer sufficient statistics for the economy and it is now not possible to characterize the dynamics of the system using a phase diagram as in (41) and Figure 1. We therefore directly simulate the real-time dynamics of the system under learning.

To illustrate the possibility of wealth effects successfully leading the economy back to the targeted steady state we provide a numerical simulation. Assume that initial expectations are pessimistic, with $\pi^e(0) = 0.9925$ and $y^e(0) = 0.9425$. These expectations are below the low inflation steady state values and therefore in the deflation trap region when households are Ricardian. In the case of non-Ricardian households the evolution of output and inflation also depends on wealth dynamics. We are interested in whether these wealth dynamics can lead the economy to the targeted steady state. We find that this indeed is possible, but that there is sensitivity to the tax policy parameters and to the initial wealth of the households.

In the non-Ricardian case we slightly change the interest-rate rule (9) to

$$f(\pi, y) = \eta + (R^* - 1 - \eta)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)}\left(\frac{y}{y^*}\right)^{\phi_y},$$

for small $\eta > 0$ so that R_t is bounded above $1 + \eta$. This prevents money demand from becoming unbounded for large deflation rates and low levels of output. This issue is irrelevant in the Ricardian case but is important in the non-Ricardian case because of perceived wealth effects. In the numerical examples we set $\eta = 0.001$, which corresponds to a floor on net interest rates of one-tenth of one percent.

As an illustration consider the tax function (5) with $\kappa_0 = 0.05$ and $\kappa = \beta^{-1} - 1 + 0.001$, so that fiscal policy is passive in the sense of Leeper (1991).[17]

[17] The other parameters are set at their previous values. We also set $\pi(0) = \pi^e(0)$ and $y(0) = y^e(0)$. The value of $\phi_y = 50$ corresponds to the output coefficient of linearized Taylor rule of 1.5 at the intended steady state.

We set $\chi = 0.03$ to match the fraction of real balances to consumption in the targeted steady state (see (35)), and we set the gain parameter $\omega = 0.01$. The initial values of real balances and real bonds are $m(0) = 0.75$ and $b(0) = 0.77$, which are close to the values of m and b at the targeted steady state for this tax function. Figure 2 illustrates the dynamics of inflation and output from this starting point.

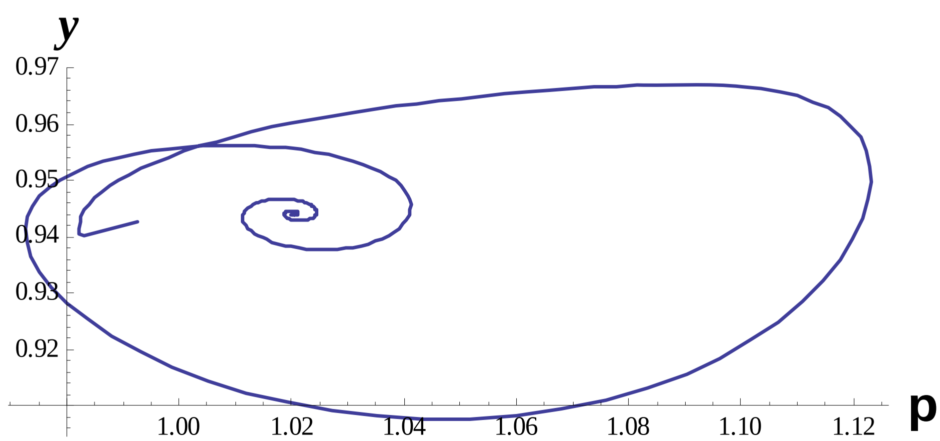

Figure 2: Inflation, output dynamics with non-Ricardian consumers

Figure 2 shows actual inflation and output on horizontal and vertical axes, respectively. There is a wide clockwise cycle where inflation and output at first overshoot (π^*, y^*), then spiral below (π_L, y_L) and finally follow a cyclical convergent path to (π^*, y^*).[18] In this example wealth effects do lead to eventual convergence to the targeted steady state, in contrast to the divergent deflationary path that would arise with Ricardian consumers. However, the path in Figure 2 is highly cyclical, and has extended periods of low output and substantial deflation with big swings in inflation and output.

Convergence from pessimistic initial expectations to the targeted steady state appears to be generally robust to starting points for expectations and initial real bonds and real balances.[19] This finding is, however, sensitive to the value of κ, in that if κ is decreased, for example to $\kappa = \beta^{-1} - 1 - 0.001 \approx$

[18] Time paths of m_t and b_t also asymptotically converge to their steady state values.

[19] For brevity, we omit the details. For initial $m(0)$ and $b(0)$ at levels that are very high, for example 15 times GDP or higher, we see an extended period of cycling aroung the low steady state before eventual convergence to the targeted steady state.

0.0091, then the level of bonds eventually explodes. The reason is that now fiscal policy is active in the sense of Leeper (1991). At the unintended steady state monetary policy is passive and learning dynamics lead the economy towards the intended steady state where, however, both fiscal and monetary policies are active and financial wealth levels will diverge. This leads to instability under learning: the economy appears to move around the targeted steady state for a period but eventually bonds follow an explosive path and the economy diverges.[20]

From a policy perspective, we see that it is indeed possible for wealth effects to provide a mechanism for the economy to escape from a deflationary situation and to return eventually to the targeted steady state. However, this mechanism relies on consumers being non-Ricardian and on appropriate tax policy. Furthermore, the path back to the targeted steady state is cyclical with wide swings in inflation and output.

4 Fiscal Policies

We now examine the role of fiscal policy when large adverse expectation shocks make deflation traps and stagnation a serious risk.[21] We focus on changes in government purchases of goods and services, rather than tax changes with unchanged government spending, because in our set-up tax changes by themselves are neutral if households are Ricardian. In practice, tax changes financed by changes in government debt can have macroeconomic effects, e.g. if some households are liquidity constrained or are non-Ricardian.[22] However, our objective is to demonstrate that suitable fiscal rules, based on temporary increases in government spending, can prevent the economy from falling into or becoming stuck in the deflation trap and can return the economy to the targeted steady state even if tax changes

[20] These results are not surprising in view of the (flexible-price, short decision-horizon) results in Evans and Honkapohja (2005). In that paper under steady state learning there is convergence to π^* but with debt exploding under active fiscal policy. In the current paper with non-Ricardian households the explosive debt path eventually destabilizes inflation and output as well.

[21] Evans and Honkapohja (2010) show that for some points within the deflation trap region, even committing to zero net nominal interest rates forever may be insufficient for escaping the deflation trap.

[22] There is empirical evidence of positive impacts of tax reductions on aggregate output, see Romer and Romer (2010).

by themselves are neutral. Therefore in this section we focus on Ricardian households. We will return to the case of non-Ricardian consumers in the next section.

4.1 Temporary Fiscal Stimulus

A traditional countercyclical policy for an economy facing deflation with declining or stagnant output is a fiscal stimulus taking the form of increased government expenditures above their normal levels for a finite time horizon, after which they revert back to lower levels. Analysis of this type of policy has not been done before when the economy is in a liquidity trap and the dynamics are assumed to evolve in accordance with adaptive learning.[23] We want to study the effectiveness of such a policy under the Ricardian assumption that the government remains solvent in the long run, and that consumers know and expect this. In this IH learning framework agents know the trajectory of government expenditures, including the date at which the expenditures will return to lower levels, and they incorporate this knowledge into their optimal consumption and pricing decisions. The consumption function, aggregate demand and the Phillips curve reflect these forward-looking expectations of the agents.

More explicitly, consider a simple case of anticipated changes in government policy. Suppose that there is an initial pessimistic expectations shock that has lowered $\pi^e(0)$ and $y^e(0)$ sufficiently so that the economy is in the deflation trap region. Under normal policy the economy will fail to return to the targeted steady state. We therefore consider fiscal policies in which there is a temporary increase in $\bar{g}$ (from its initial steady state level $\bar{g} = \bar{g}_1$), taking the form

$$g_t = \begin{cases} \bar{g}_0 \text{ for } t = 0, ..., T_0 \\ \bar{g}_1 \text{ for } t = T_0 + 1, ... \end{cases},$$

where $\bar{g}_0 > \bar{g}_1$. Here we assume that the policy is announced and started at $t = 0$ and it is credible. Agents understand that government spending will be continued at the higher level $\bar{g}_0$ through period T_0 and that it will be reduced to its previous level beginning at $T_0 + 1$.

For gross output agents are assumed to have expectations given by the simple adaptive rules described in Section 3. For net output, however, ex-

[23] Anticipated future policy changes are discussed in Evans, Honkapohja, and Mitra (2009) in the context of a Ramsey model with flexible prices and without money.

pectations are given by

$$nety_j^e = \begin{cases} y_j^e - \bar{g}_0 \text{ for } j = t, ..., T_0 \\ y_j^e - \bar{g}_1 \text{ for } j = T_0 + 1, ... \end{cases}, \tag{42}$$

so that agents incorporate the known future path of government spending into their forecasts.

The variables $nety_j^e$ that appear in the Phillips curve (16), and in the consumption function (28) are now defined according to (42). This requires evaluating the weighted sums of $nety_j^e$ using the appropriate value of government expenditures for each j. The computations are straightforward, and the consumption function is now given by:

$$c_t = (1-\beta)\left(y_t - \bar{g}_0 + (y_t^e - \bar{g}_0)\frac{1-(r_t^e)^{t-T_0}}{(r_t^e)-1} + (y_t^e - \bar{g}_1)\frac{(r_t^e)^{t-T_0}}{(r_t^e)-1}\right),$$

where r_t^e and y_t^e are the time t forecasted (constant) value of future real interest rates and output.

For the interest rate rule (9) we set $A = 2.5$ and $\phi_y = 50$, a calibration broadly consistent with the standard Taylor-rule parameters.

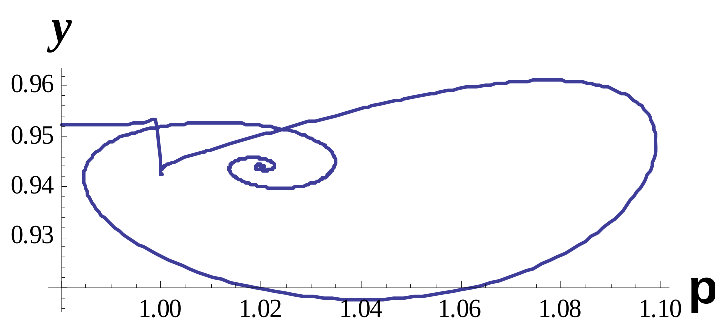

Figure 3: y and π under a fiscal stimulus.

Given a specific fiscal stimulus, we can proceed as in Section 2.4, except that we now report real-time dynamics based on the adaptive learning rules of Section 3. Figure 3 illustrates one example of the dynamics of output and inflation for $T_0 = 6$, and with $\bar{g}_0 = 0.21$, $\bar{g}_1 = 0.2$. Thus there is a fiscal stimulus, taking the form of a 5% increase in government spending for six periods. We set initial expectations at $y^e[0] = 0.9425$ and $\pi^e[0] = 0.993$.

These are in the deflation trap region, and without the fiscal stimulus there would be falling inflation and output (compare to the steady state values at the end of Section 3.2.1). Under the fiscal stimulus the economy instead converges to the intended steady state, though after a wide swing that takes inflation well above the intended steady state. As noted in connection with Figure 1, under normal policy the convergence dynamics inside the corridor of stability are inherently cyclical. This feature also appears after the end of the temporary stimulus, when expectations overshoot the values of the targeted steady state.

An important feature of the policy is that the length of the temporary fiscal stimulus is crucial for its efficacy. For example, if, holding $\bar{g}_0 = 0.21$, $\bar{g}_1 = 0.2$, we set $T_0 = 1, 2$ or $T_0 \geq 37$ then the fiscal stimulus does not enable the economy to return to the targeted steady state. In fact, the size of the stimulus and the degree of pessimism of expectations also matter for the efficacy of fiscal stimulus. We now examine this more systematically.[24]

We consider four different degrees of pessimism of expectations as follows:

Mild: $\pi^e = 0.993$ and $y^e = 0.9425$.

Large: $\pi^e = 0.991$ and $y^e = 0.9425$

Severe: $\pi^e = 0.985$ and $y^e = 0.9425$

Extreme: $\pi^e = 0.985$ and $y^e = 0.9$.

We find that a temporary fiscal stimulus always works for a range of government spending $\bar{g}_0$ and length of stimulus T_0. For $T_0 = 1$, a temporary fiscal stimulus works for sufficiently large $\bar{g}_0$. Often, increasing length of stimulus T_0 somewhat allows the use of a smaller value of $\bar{g}_0$ to achieve convergence to the intended steady state.

Some specific results are as follows:

Mild pessimism: $\bar{g}_0 = 0.205$ yields desired convergence for stimulus of length $T_0 = 11, \ldots, 22$, while with this g_0, the policy fails if T_0 is outside this range. A smaller value of $\bar{g}_0 = 0.204$ is never effective while $\bar{g}_0 = .25$ makes the T_0 range larger.

Large pessimism: A large value of spending $\bar{g}_0 = 0.25$ delivers desired convergence for $T_0 = 1, \ldots, 37$. A smaller value $\bar{g}_0 = 0.21$ fails.

Severe pessimism: With $T_0 = 1$, $\bar{g}_0 = 0.34$ is effective.

[24] Also the parameter T describing statistical forecasting horizon affects the quantitative results. Through period $t + T$ agents use their forecasts $\pi^e(t)$, whereas after $t + T$, they assume that the real interest rate has reverted to normal and set $r^e_{t+j}(t) = \beta^{-1}$ for $j > T$. As indicated earlier, we set $T = 28$, i.e. agents think it will take 7 years for real interest rates to return to normal steady state.

Extreme pessimism: With $T_0 = 5$, $\bar{g}_0 = 0.8$ is effective.

Thus, the fiscal stimulus must be adequate in size and length to push the economy out of the deflation trap region. The intuition for these results is that the demand stimulus from a temporary increase in g outweighs the partially offsetting reduced consumption from the higher present value of taxes, which for Ricardian households equals the present value of government spending. A *permanent* increase in $\bar{g}$ in this set-up does not lift the economy out of the deflation trap, because the permanently higher taxes exactly offset the increase in government spending. In contrast, a large enough increase in government spending for a limited period will add enough stimulus to lead the economy back to the targeted steady state. We note that the tax rule (5) implies that the long-run debt to GDP ratio is unaffected by the temporary stimulus.

4.2 Fiscal Austerity

Perhaps surprisingly, it turns out that a carefully designed restrictive fiscal policy can in certain cases lift the economy out of the liquidity trap, provided it is applied for a sufficient long period of time. We now examine this possibility for the different degrees of pessimism of expectations.[25] The results for the different degrees of pessimism are as follows:

Mild pessimism: cutting government spending to $\bar{g}_0 = 0.19$ is effective in moving the economy out of the deflation trap when the length of the policy is in the range $T_0 \geq 33$ but this policy fails for smaller values of T_0.[26] A more severe policy $\bar{g}_0 = 0.15$ is effective also for $T_0 \geq 28$.

Large pessimism: $\bar{g}_0 = 0.19$ is effective for length $T_0 \geq 67$.

Severe pessimism: $\bar{g}_0 = 0.15$ is effective for length $T_0 \geq 100$.

Extreme pessimism: Fiscal austerity is never effective.

In terms of the length of policy T_0, stimulus and austerity policies have an interesting contrast. The efficacy of the former requires a limited duration whereas a very long period of the latter is necessary. In all our examples the efficacy of stimulus policies imply that the austerity policies of same absolute

[25] In this section the forecasting horizon is set at $T = 60$. For shorter horizons, for example for $T = 28$, fiscal austerity seems to be ineffective. On the other hand, temporary fiscal stimuli continue to be effective for large values of T.

[26] We note that if $\bar{g}_0 = 0.19$ and $T_0 = 50$ the policy induces fairly large fluctuations in output and inflation compared to a corresponding case of stimulus shown in Figure 3. The ranges of fluctautions are $y \in (0.886, 0.990)$ and $\pi \in (0.986, 1.142)$.

magnitude and duration are not effective and *vice versa*. However, there are also cases for which neither policy is effective for certain intermediate durations. As an example consider the stimulus policy $\bar{g}_0 = 0.25$ under mild pessimism for a forecasting horizon $T = 60$. A stimulus policy with $T_0 \geq 25$ is ineffective in lifting the economy out of the deflation trap as is an austerity policy of $\bar{g}_0 = 0.15$ for $T_0 < 28$.

In general, efficacy of austerity policies is more sensitive to the degree of pessimism of expectations as suggested by the following subtle intuition. If the economy is in a region in which the ex-ante real interest rate factor is less than β^{-1} then the consumption function dictates an increase in consumption flow, stemming from a fixed permanent decrease in taxes, that is larger than the decrease in g. The present value is the same when measured by r^e, but because $r^e < \beta^{-1}$, households will substitute toward current consumption. Formally consider a permanent change in government spending to $\bar{g}_0 < \bar{g}_1$. Then actual output, for given expectations, is given by

$$y_t = \bar{g}_0 + (\beta^{-1} - 1)(y_t^e - \bar{g}_0)/(r_t^e - 1) > \bar{g}_1 + (\beta^{-1} - 1)(y_t^e - \bar{g}_1)/(r_t^e - 1),$$

provided $\beta^{-1} > r_t^e$. This effect only holds for a range of π^e in which monetary policy delivers a low r^e. For larger deflation rates, however, i.e. $\pi^e < 0.985$, this policy cannot work for initial expectations in which $\pi^e(t)$ falls over time under normal policy. Thus for sufficiently pessimistic initial expectations we would expect permanent or very long cuts in government spending to fail as a policy that takes the economy to a steady state.

The above analysis also implies that under adaptive learning, whether households are Ricardian or not, a fiscal stimulus can give rise to a “fiscal multiplier” that is quite different from the multiplier under a policy of fiscal austerity, depending on the magnitude and duration of the policy and on the initial expectations. This suggests that in an adaptive learning context, results of empirical studies of the fiscal multiplier will be sensitive to initial expectations and to the duration and magnitude of policies.

In this section we have seen that the success of the temporary fiscal policy in general depends on fine tuning the magnitude, direction and duration of the policy. We next look at an endogenous switching rule for government spending that eliminates deflation and stagnation and that also appears to have reasonable performance overall.

5 A Fiscal Switching Rule

In Section 4.1 we found that a suitably designed temporary stimulus is effective in getting the economy out of the deflation trap. This is in line with testimony by Lawrence Summers to the Joint Economic Committee hearing on January 16, 2008, that fiscal "stimulus program should be timely, targeted and temporary." Section 4.1 showed that for a successful policy it is essential to get the parameters in the right range. This leads us to a discussion of whether a more "automatic" policy rule can be designed for this purpose.

To prevent deflationary spirals or deflation with declining or stagnant output, we now explore a temporary fiscal stimulus policy designed to ensure that expected inflation eventually exceeds some threshold $\tilde{\pi} > \pi_L$. Here the length of the stimulus is dictated by the state of the economy. Specifically, if $\pi_t^e < \tilde{\pi}$ the government sets $g_t \geq \bar{g}$ as needed to achieve an output level y_t such that realized inflation π_t exceeds expected inflation π_t^e. In addition, if $\pi_t^e \geq \tilde{\pi}$, the government sets $g_t \geq \bar{g}$ as needed to ensure that π_t exceeds the threshold $\tilde{\pi}$.[27]

We remark that the idea of a lower threshold for inflation and increased government spending to ensure that actual inflation stays above the threshold was suggested in Evans, Guse, and Honkapohja (2008) and Evans and Honkapohja (2010). The rule proposed here improves upon the earlier ideas in that it focuses squarely on inflation expectations and the new rule leads to less extreme fluctuations in g_t than rules used in the cited earlier papers.[28]

To implement this fiscal switching policy, we assume that the government monitors expectations. Given expectations, it can set g to achieve a level of y using equation (30). In effect the government observes inflation monthly, and would be able to adjust spending in order to maintain $\tilde{\pi} < \pi$ on a quarterly basis. Automatic stabilizers, like unemployment benefits and other income subsidy programs triggered by output thresholds may be useful, but may also

[27] If $\pi_t^e < \tilde{\pi}$ we set g_t to ensure that $\pi_t \geq \pi_t^e + \varepsilon_\pi$ and if $\pi_t^e \geq \tilde{\pi}$ we set g_t to ensure that $\pi_t \geq \tilde{\pi} + \varepsilon_\pi$, for some small $\varepsilon_\pi > 0$. In numerical illustrations we set $\varepsilon_\pi = 0.005$. Note that if $\pi_t^e \geq \tilde{\pi}$ and $\pi_t \geq \tilde{\pi}$ for $g = \bar{g}$ then the rule sets $g_t = \bar{g}$.

[28] We require a much smaller increase in g when the trigger is activated than Evans, Guse, and Honkapohja (2008) and Evans and Honkapohja (2010). In the latter the trigger was simply a lower bound on actual inflation as opposed to expected inflation, and g_t was raised to achieve this bound. In our case with Ricardian consumers the g_t/y_t ratio goes up from 0.2 to 0.26 while in Evans and Honkapohja (2010) the ratio is much higher, going from 0.2 to 0.34 if identical calibrations are used.

be insufficient. It should[29] be emphasized that if expectations turn substantially pessimistic, government expenditures triggered by an output threshold may not be able to prevent deflation traps (see Evans, Guse, and Honkapohja (2008)). This could happen even at zero nominal rates if the ex-ante real interest rate rises and depresses private consumption as a result of substitution between private consumption and government spending with gross output remaining at the threshold level. Therefore we focus here on fiscal switching rules based on thresholds for inflationary expectations. Triggered government expenditures could involve for example infrastructure or research projects activated at times of deflationary expectations and designed to avoid fiscal lags.

From equations (31), (32) and (33) it is apparent that y can be chosen to attain the required level of inflation. This procedure ensures that eventually $\pi^e \geq \tilde{\pi}$. We simulate this economy using the same parameters used in Figure 3 above for Ricardian consumers, except that we now use the fiscal switching rule.[30] For the numerical results in this section we set $\tilde{\pi} = \pi_L + 0.005 = 0.9981$.

Two points should be noted about this form of fiscal policy. First, it is not necessary to decide in advance the magnitude and duration of the fiscal stimulus. Second, in contrast to the preceding section we now do not assume that agents know the future path of government spending. Instead agents use adaptive learning to forecast the future values of their net income in addition to forecasts of inflation and output.

[29] small change

[30] In the section we use net rather than gross output in the interest-rate rule, because of the potential large variation in gross output due to government spending. The results presented here are not significantly affected by this issue.

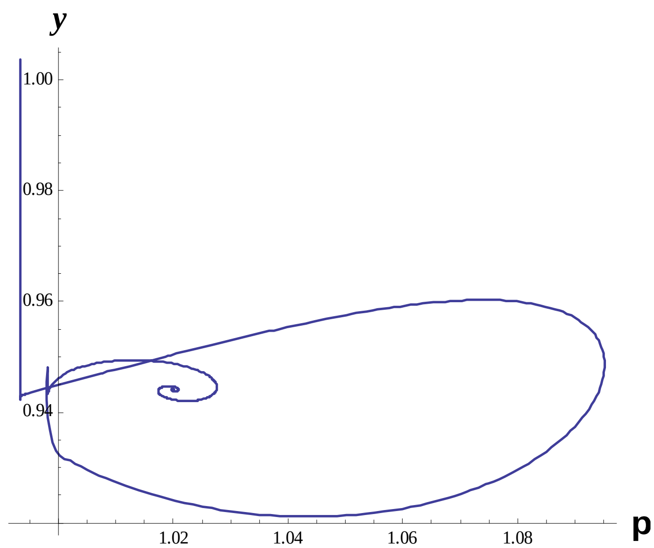

Figure 4: y and π under a fiscal switching rule, Ricardian households

We start with the case in which consumers are Ricardian. In contrast to the economy depicted in Figure 1, the fiscal switching rule eliminates the unintended steady state with the inflation rate π_L: the path starting in the vicinity of π_L converges to the intended steady state. This is illustrated in Figure 4. A strong fiscal stimulus generates a steep rise in output and lifts the economy out of the deflation trap and the economy eventually converges to the intended steady state. For initial expectations in Figure 4, which are the same as in Figure 3, the dynamics would be unstable without the fiscal switching rule. Compared to the policy used in Figure 3, the main difference is that there is a much stronger but shorter fiscal stimulus under the fiscal switching rule. There is a also a brief small fiscal stimulus used at a later date when inflation again is a low values.

In all four cases of pessimism illustrated in Section 4 our switching rule generates paths that converge to the targeted steady state, and the performance of these rules is comparable or somewhat better. The main advantage of the fiscal switching rule is that it provides a robust policy for ensuring that the economy does not get stuck in the deflation trap, and it does so using an “automatic” fiscal policy that does not require tuning to the economic situation.

The results with non-Ricardian consumers are similar: the fiscal switching rule eliminates the unintended steady state π_L and ensures convergence to the targeted steady state. Figure 5, using the same parameters used for the non-Ricardian case of Figure 3, illustrates these results.

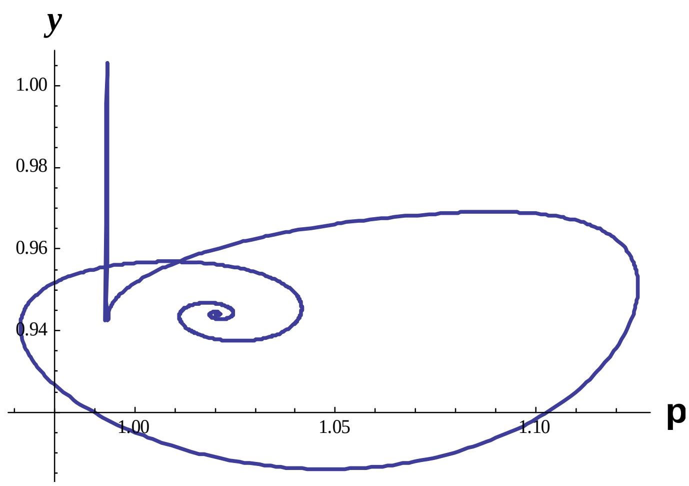

Figure 5: y and π dynamics under fiscal switching rule, non-Ricardian households

Although in the non-Ricardian case, both paths with and without policy show cyclical convergence, the path without policy is more volatile. Table 1 illustrates these results for the case of extreme pessimism, i.e. : $\pi^e = 0.985$ and $y^e = 0.9$.

Table 1: Non-Ricardian Households

	Without policy	With policy
maximum inflation, π	1.371	1.140
minimum output, y	0.805	0.900

Thus, even in the non-Ricardian case in which wealth effects do eventually return the economy to the intended steady states, the fiscal switching policy improves performance.

As illustrated for both the Ricardian and Non-Ricardian cases examined, the fiscal switching rule, together with our interest rate rule, yields convergence to the targeted steady state after an initial overshooting of inflation and output.[31] The overshooting arises from the necessary big initial policy responses that are needed to counteract the initial pessimistic expectations

[31] We also checked that with this combination of rules there is convergence to the targeted steady state from even more pessimistic initial expectations.

which tend to be inertial under adaptive learning. While our focus in this section is fiscal policy, we can explore whether more aggressive monetary policies working in conjunction with fiscal policy can improve stabilization. If we modify the coefficients of the Taylor rule we can dampen the fluctuations of output and inflation seen in Figure 4. This is achieved with significant departures from coefficients typically used in the literature and policy practice, in particular by dramatically increasing the response of the nominal interest rate to the output gap. For example, if we set $\phi_y = 1500$, for the pessimistic initial expectations $\pi^e = 0.993$ and $y^e = 0.9425$ used in Figure 4, we can reduce the overshooting of the inflation rate to about one percentage point above target and eliminate almost entirely the undershooting of output.[32]

In summary, our analysis suggests that one policy which might be used to combat stagnation and deflation, in the face of pessimistic expectations, would consist of a fiscal switching rule combined with a Taylor-type rule for monetary policy. The fiscal switching rule applies when expected or actual inflation falls below a critical value. The rule specifies increased government spending in such a way that expected inflation is ensured to exceed eventually the critical threshold. This part of the policy eliminates the unintended steady state and makes sure that the economy does not get stuck in a regime of deflation and stagnation. Furthermore, unlike the temporary fiscal policies discussed in the previous section, the switching rules do not require fine tuning and are triggered automatically. Remarkably, our simulations indicate that this combination of policies is successful regardless of whether the households are Ricardian or non-Ricardian.

6 Conclusion

We have studied how the an economy can fall into a deflation or low inflation trap with declining or stagnant output, and explored the design of policies to avoid such outcomes. Under the perfect foresight view, simply announcing appropriate money growth and/or fiscal policies can in principle avoid low inflation. The effectiveness of such policies however depends on the assumption of perfect foresight, on policy credibility, and on wealth effects to eliminate all equilibria except the targeted π^* steady state. Furthermore such policies are "too powerful" under perfect foresight: bad outcomes never happen.

[32]In terms of the Taylor rule linearized at the targeted steady state, the coefficient on the output gap corresponding to $\phi_y = 50$ is 1.5, and for $\phi_y = 1500$ it is 45.

If we adopt a more plausible adaptive learning view, outcomes with low inflation and output are still possible. We find that policies of temporary fiscal stimulus, and in some cases fiscal austerity, can eliminate liquidity traps and can lead the economy back to its intended steady state. However, such policies require careful fine tuning of the magnitude, direction and duration of the policy. A "fiscal switching rule" that automatically triggers a stimulus of high government expenditures when inflation or expected inflation falls below a critical threshold is equally effective in stabilizing the economy, but does not require complicated and discretionary fine tuning, and therefore seems preferable.

7 Appendix 1: Private sector optimization

Recall the form of the utility function for household-producer

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,s}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2}\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)^2$$

and the constraints

$$\begin{aligned} c_{t,s} &= -m_{t,s} - b_{t,s} - \Upsilon_{t,s} + m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,s} + \frac{P_{t,s}}{P_t}y_{t,s}, \\ h_{t,s} &= Y_t^{1/\alpha}\left(\frac{P_{t,s}}{P_t}\right)^{-\upsilon/\alpha} \text{ and } y_{t,s} = Y_t\left(\frac{P_{t,s}}{P_t}\right)^{-\upsilon}. \end{aligned}$$

We compute the derivatives with respect to $(t-1)$-dated variables

$$\begin{aligned} \frac{\partial U_{t,s}}{\partial m_{t-1,s}} &= c_{t,s}^{-\sigma_1}\pi_t^{-1} + \chi(m_{t-1,s}\pi_t^{-1})^{-\sigma_2}, \\ \frac{\partial U_{t,s}}{\partial b_{t-1,s}} &= c_{t,s}^{-\sigma_1}R_{t-1}\pi_t^{-1}, \\ \frac{\partial U_{t,s}}{\partial P_{t-1,s}} &= \gamma\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)\left(\frac{P_{t,s}}{P_{t-1,s}}\right)\frac{1}{P_{t-1,s}} \end{aligned}$$

and with respect to t-dated variables

$$\begin{aligned} \frac{\partial U_{t,s}}{\partial m_{t,s}} &= \frac{\partial U_{t,s}}{\partial b_{t,s}} = -c_{t,s}^{-\sigma_1}, \\ \frac{\partial U_{t,s}}{\partial P_{t,s}} &= -c_{t,s}^{-\sigma_1}Y_t(1-\upsilon)\left(\frac{P_{t,s}}{P_t}\right)^{-\upsilon}\frac{1}{P_t} - \frac{\upsilon}{\alpha}h_{t,s}^{1+\varepsilon}\frac{1}{P_{t,s}} - \gamma\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)\frac{1}{P_{t-1,s}}. \end{aligned}$$

The Euler equations are

$$
\begin{aligned}
\frac{\partial U_{t,s}}{\partial m_{t,s}} + \beta E_{t,s} \frac{\partial U_{t+1,s}}{\partial m_{t,s}} &= 0, \\
\frac{\partial U_{t,s}}{\partial b_{t,s}} + \beta E_{t,s} \frac{\partial U_{t+1,s}}{\partial b_{t,s}} &= 0, \\
\frac{\partial U_{t,s}}{\partial P_{t,s}} + \beta E_{t,s} \frac{\partial U_{t+1,s}}{\partial P_{t,s}} &= 0.
\end{aligned}
$$

The second equation is just the consumption Euler equation (11), while combining the first and second equations yields the money demand function (12). The third Euler equation implies one-step nonlinear the Phillips curve (10).

Next, we examine the transversality condition for optimal price setting. Using Kamihigashi (2003), the transversality condition

$$\lim_{T\to\infty} \beta^T E_{T,s}[\Psi_{T,s}(P_{T,s} - \hat{P}_{T,s})] \leq 0,$$

where $\{\hat{P}_{T,s}\}$ denotes the optimal pricing policy and $P_{T,s}$ is a perturbation from the optimum, is a necessary condition for optimality under some regularity conditions (in particular, an interior optimum is required).[33] Here we use the short-hand notation

$$
\begin{aligned}
\Psi_{T,s} &= (1-\upsilon)\,(c_T)^{-\sigma}\, Y_T P_T^{\upsilon-1}\left(\hat{P}_{T,s}\right)^{-\upsilon} \\
&\quad - \left(\frac{\upsilon}{\alpha}\right) Y_T^{(1+\varepsilon)/\alpha} P_T^{(1+\varepsilon)\upsilon/\alpha}\left(\hat{P}_{T,s}\right)^{-(1+\varepsilon)\upsilon/\alpha-1} - \gamma\left(\frac{\hat{P}_{T,s}}{\hat{P}_{T-1,s}} - 1\right)\frac{1}{\hat{P}_{T-1,s}}.
\end{aligned}
$$

Note that $\Psi_{T,s} = \frac{\partial U_{T,s}}{\partial P_{T,s}}$ at the optimum. Using the Euler equation for price setting we have

$$
\begin{aligned}
&\lim_{T\to\infty} \beta^T E_{T,s}[\Psi_{T,s}(P_{T,s} - \hat{P}_{T,s})] \qquad (43) \\
&= \lim_{T\to\infty} \beta^T E_{T,s}\left[-\beta\gamma\left(\frac{\hat{P}_{T+1,s}}{\hat{P}_{T,s}} - 1\right)\frac{\hat{P}_{T+1,s}}{\hat{P}_{T,s}}\right]\left(\frac{P_{T,s}}{\hat{P}_{T,s}} - 1\right) \leq 0
\end{aligned}
$$

for all permitted $P_{T,s}$. Since we are dealing with necessary condition for local max, the perturbations $P_{T,s}$ can be taken to be in a sufficiently small

[33] For brevity, we do not consider these conditions in detail.

neighborhood of $\hat{P}_{T,s}$. We require that $\frac{P_{T,s}}{\hat{P}_{T,s}}$ remains close to 1 but it can be smaller or bigger than 1. This leads to the requirement

$$\begin{aligned} 0 &= \lim_{T\to\infty} \beta^T E_{T,s}\left[\beta\gamma\left(\frac{\hat{P}_{T+1,s}}{\hat{P}_{T,s}}-1\right)\frac{\hat{P}_{T+1,s}}{\hat{P}_{T,s}}\right] \\ &= \beta\gamma \lim_{T\to\infty} \beta^T E_{T,s}\hat{Q}_{T+1,s} = 0, \end{aligned}$$

which gives the TVC (14) in the text.

8 Appendix 2: Asymmetric Price Adjustment

If the costs of price adjustment are asymmetric and are higher for reductions in prices, then this can provide a lower bound on deflation.[34] Consider for example the case where the cost of price adjustment in the utility function takes the form

$$Cost = \begin{cases} \frac{\gamma}{2}\left(\pi_{s,t}-1\right)^2 \text{ for } \pi_{s,t} \geq \underline{\pi} \\ +\infty \text{ for } \pi_{s,t} < \underline{\pi}, \end{cases}$$

where $\pi_{s,t} = P_{s,t}/P_{s,t-1}$. To examine the implications of asymmetric price-adjustment costs, we return to the case of Ricardian consumers discussed in Section 2.4. The temporary equilibrium map for inflation is modified to

$$\pi_t = \begin{cases} G_2(y_t, y_t^e) \text{ for } G_2(y_t, y_t^e) \geq \underline{\pi} \\ \underline{\pi} \text{ for } G_2(y_t, y_t^e) < \underline{\pi}. \end{cases}$$

Because the Ricardian case is a forward-looking two-dimensional system with adaptive learning, one can illustrate the possible results using phase diagrams showing the expectational learning dynamics. There are three cases:

1. $\underline{\pi} > \pi_L$. In this case π^* is globally stable, since $\pi < \pi_L$ is no longer possible.

2. $\underline{\pi} < \pi_L$. The deflation trap continues to exist. If $\pi_L - \underline{\pi}$ is small, however, in the region $\underline{\pi} < \pi^e < \pi_L$ there is gradually falling output.

3. $\underline{\pi} = \pi_L$. The stagnation regime. In this case there can be convergence to any $0 < y < y_L$ with $\pi = \pi_L$.

[34] See Evans (2013) for the stagnation regime.

Figures 6 illustrates the phase diagram for the E-stability differential equations in (π^e, y^e)-space for the case $\underline{\pi} < \pi_L$ in which a deflation trap continues to exist. In this case the targeted steady state π^* is locally stable. However, if output expectations are low, the economy may converge to the trap even if initially inflation expectations are low but above π_L. The main difference from the symmetric price-adjustment cost set-up examined in the paper is that deflation is now bounded from below at rate $\underline{\pi}$. Thus, in this case, persistently low and falling output is compatible with steady deflation at low levels.

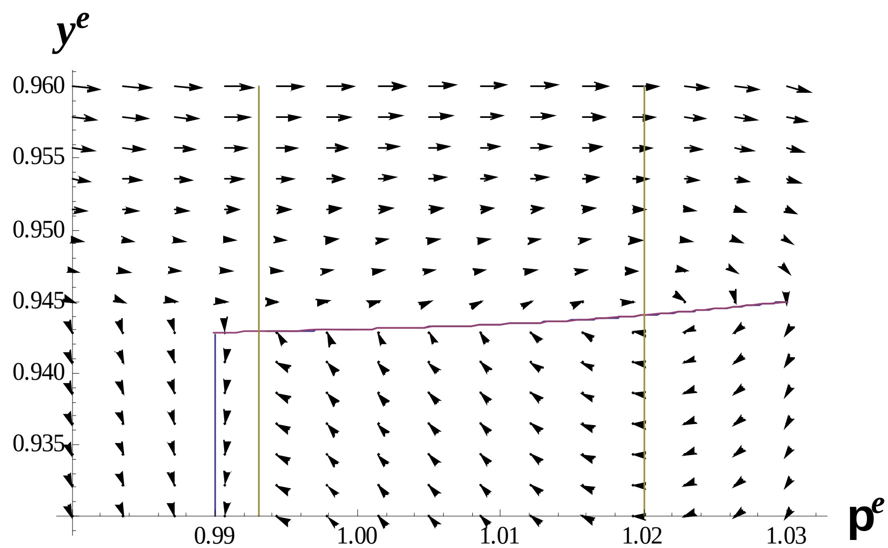

Figure 6: asymmetric cost adjustment with $\underline{\pi} < \pi_L$.

We briefly describe the other two cases of asymmetric adjustment costs. In all cases the targeted steady state is locally stable under learning. If $\underline{\pi} = \pi_L$, there is also a locally stable continuum of steady states at $\pi = \pi_L = \underline{\pi}$ and $y < y_L$, where y_L is the level of output associated with the usual π_L steady state. E-stability dynamics indicate that under learning the economy can converge to any point on the continuum from initial conditions $\pi^e(0) \gtrsim \underline{\pi}$ and $y^e(0)$ sufficiently low. Similar convergence to the continuum can happen for initial $\pi^e(0) \lesssim \underline{\pi}$ and $y^e(0)$ sufficiently low. In the case $\underline{\pi} > \pi_L$ the economy under learning is globally stable at the targeted steady state π^*. However, for $\underline{\pi}$ only slightly above π_L, pessimistic initial expectations

$(\pi^e(0), y^e(0))$ can lead to extended periods of low output and mild deflation before inflation expectations are pulled up towards $\underline{\pi}$ and a recovery begins.

As noted, for example, by Bullard (2010), we do observe economies exhibiting extended periods of very low inflation or mild deflation. The cases $\underline{\pi} = \pi_L$ and $\underline{\pi} < \pi_L$ show that steady mild deflation is consistent with a deflation trap region that leads to persistently falling or persistently low levels of output. The analysis of fiscal policy provided in this paper could easily be extended to the various cases of asymmetric price adjustment.

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