

Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem

Second Edition

Corrections

1. Page 2, line 8: S. C. Lee and the author

2. Page 9, Eq. (1.1.53):

$$|\hat{f}_j(\xi) - \hat{f}_k(\xi)|^2 \leq \frac{1}{2}\epsilon(1+r^2)^{-|t|} \int_{|\xi| \leq r} d\xi$$

3. Page 16: The proof of Lemma 1.2.1 (b) yields the uniform estimate

4. Page 16, the second line of Eq. (1.2.37):

$$\leq \tilde{C}_k(1+|\xi-\zeta|)^{|d|-k}(1+|\zeta|)^{|d|-k}$$

5. Page 17, Eq. (1.2.45):

$$r \sim \Sigma_\alpha d_\xi^\alpha p D_x^\alpha \phi / \alpha! \sim p$$

6. Page 17, last line, on page 18, third line, and on page 59, last line, ∂ has the same meaning as d .

7. Page 19, the first line of Eq. (1.2.57) should end $\phi(y)/\alpha! \sim \bar{q}_1(y, \zeta)\phi(y)$

8. Page 23, fifth line of 1.2.6: is a pseudo-differential operator for ...

9. Page 25, below Eq. (1.2.92): Since $p_j(x, \xi) - \phi(t_j \xi)p_j(x, \xi) \in C_c^\infty$

10. Page 29, Lemma 1.3.3: ... Let $U_i \subset U$.

11. Page 30, line 4: We make a varying linear change of fiber coordinates to define

12. Page 30, Eq. (1.3.23) should begin with $\tilde{P}(\tilde{f})(\tilde{x}) =$

13. Page 31, in the second Definition, the following sentence should be deleted: "We assume that any two points of M belong to at least one of the U_i ." (This assumption would be possible, but is not needed, and it is not compatible with the next assumption.)

14. Page 31, in Eq. (1.3.28): $\phi h_*(p) \in S^d(hO)$

15. Page 32, in the fourth line of the Definition: so $pq - I \in S^{-1}(M)$.

16. Page 33, Eq. (1.3.34):

$$\|f\|_{u,s} = \Sigma_i \|h_{i*}(\phi_i f)\|_s.$$

17. Page 33, the second line of Eq. (1.3.38) should end:

$$\leq C \Sigma_i \|h_{i*}(\phi_i f)\|_s = C \|f\|_{u,s}.$$

18. Page 34, Eq. (1.3.44)

$$\|g_{ij*}(\phi_i P \phi_j f)\|_s \leq C \|g_{ij*}(\phi_j f)\|_{s+d} \leq C \|f\|_{s+d}.$$

19. Page 34, in Eq. (1.3.47), the first constant C can be dropped.

20. Page 34: **Proof:** Let ...

21. Page 35, the second line of Eq. (1.3.51):

$$Q^r: \sim \Sigma_k (I - Q_1 P)^k Q_1.$$

22. Page 39, the line below Eq. (1.4.14): Since C is compact, there exists a convergent subsequence.

23. Page 40, in Eq. (1.4.19) the inverse images should be dropped because they contain the kernels. Equality should be replaced by isomorphism accordingly:

$$\begin{aligned} \mathcal{K}(ST) &\simeq \mathcal{K}(T) \oplus \{\mathfrak{R}(T) \cap \mathcal{K}(S)\} \\ \mathcal{K}(T^*S^*) &\simeq \mathcal{K}(S^*) \oplus \{\mathfrak{R}(S^*) \cap \mathcal{K}(T^*)\} \end{aligned}$$

$$= \mathcal{K}(S^*) \oplus \{\mathcal{K}(S)^\perp \cap \mathfrak{R}(T)^\perp\}.$$

24. Page 41: Since \tilde{S} is invertible, $\text{Index}(\tilde{S}) = 0$.
25. Page 43, in the third Definition: We say $(\mathcal{P}, \mathcal{V})$ is elliptic if ...
26. Page 44, Eq. (1.5.8) should end $\bigoplus_k \delta_{2k+1} = q_{ev} \cdot q_{od}$.
27. Page 46, in Eq. (1.5.24) a factor f_l is missing in the middle term.
28. Page 49, Eq. (1.6.5): $\text{Spec}(T)$...
29. Page 50, in Eq. (1.6.9): $=: x$
30. Page 51, in Lemma 1.6.3 (b): $\|\phi_n\|_{\infty, k}$
31. Page 51, Eq. (1.6.18)
- $$\lim_{n \rightarrow \infty} n|\lambda|^{-m/d} = C(P).$$
32. Page 53, below Eq. (1.6.27): We integrate (1.6.27) over M to prove (c) in this special case:
33. Page 53, in Eq. (1.6.29) and likewise in the first line of Eq. (1.6.30):
- $$\begin{aligned} \|\phi(x)\|^2 &= \dots \\ &\leq C^2(1 + |\lambda_n|)^2 \dots \end{aligned}$$
34. Page 53, in Eq. (1.6.35) the factor C should be dropped in the first line.
35. Page 54, first line: for $k \geq d\ell$ and ...
36. Page 56, above Eq. (1.6.51): We complete the proof by computing that
37. Page 57, in Eq. (1.6.56) Δ_k should be replaced by Δ_i at two places. The last line should be: $= \text{Index}(\mathcal{P}, \mathcal{V})$.
38. Page 57: **Remark:** There is an equivariant version we will use ...
39. Page 61, in the second line of Lemma 1.7.2: *which is polyhomogeneous* ...
40. Page 63, Lemma 1.7.4: Let k be given.
41. Page 64, Lemma 1.7.5 (a): $h(x, t)$ is C^∞ in (x, t) .
42. Page 65, in Eqs. (1.7.44), (1.7.45) and (1.7.46) e^{-t} should be replaced by 1. On the left-hand side of the inequality below Eq. (1.7.45) the exponent -1 from Eq. (1.7.45) is missing. In the middle term of Eq. (1.7.46) a factor C_s should be inserted. The sentence “Consequently ...” is now redundant.
43. Page 65, in the last line, on page 74 above Eq. (1.9.1), on page 76 in Lemma 1.9.2 (b), and on page 80 in Theorem 1.9.4: $P \in \text{Ell}^d(\mathcal{R}, M, V)$
44. Page 67, in the second line of Eq. (1.8.8) the constant C_k should be C .
45. Page 69, in the third line of Eq. (1.8.15) a factor $(-1)^n$ should be provided and the factor t in the exponent be dropped.
46. Page 69, in the fifth line of the Remark: $P = ad\delta + b\delta d - \phi$
47. Page 70, Lemma 1.8.3 (d): $\text{Tr}_{L^2}(e^{-tP}) \sim \sum_n a_n(P) t^{(n-m)/d}$. **Remark:** This
48. Page 71, below Eq. (1.8.23): the total symbol of P .
49. Page 71, in Eq. (1.8.26), and on page 75 in Eq. (1.9.6) factors $v(x)^{-1}$ and $v(y)^{-1}$ respectively should be provided on the right-hand sides.
50. Page 72, Eq. (1.8.29):
- $$\det(p_d - \lambda)^{-\nu-2} \tilde{q}$$
51. Page 73, in Eq. (1.8.35) the index n should be replaced by m .
52. Page 75, Eq. (1.9.5):
- $$|L(t, x, y, P, Q) - \sum_{n \leq N} L_n(t, x, y, P, Q)|_{\infty, k} \leq C_N t^k.$$
53. Page 76, in the first line: is admissible as defined in §1.8,
54. Page 79, in Eq. (1.9.28) the exponent should be $k - |\beta| - d|\gamma|$.
55. Page 79, in Eq. (1.9.32) the norm should be $\|\cdot\|_{\infty, k}$.

56. Page 80: **Remark:** We
57. Page 81: **Example 1.9.1:** Let $\Delta_0 = \delta_0 d_0$ be the scalar Laplacian ... Let $f \in C^\infty(M)$ and let $g(a) = e^{2af} g_0$ be a smooth 1-parameter family of
58. Page 83, the second **Definition:** Let
59. Page 84: **Lemma 1.10.1:** $\mathcal{L}(\mathcal{T}, \mathcal{P}, \mathcal{V}) = \dots$
60. Page 84, the second line of Eq. (1.10.21):

$$= \sum_k (-1)^k \text{Tr}(\mathcal{T}_k(0)) = \mathcal{L}(\mathcal{T}, \mathcal{P}, \mathcal{V}).$$
61. Page 89, in the first line of Eq. (1.10.49), ω should be replaced by w .
62. Page 89, in Lemma 1.10.3, $\text{Tr}(\mathcal{T}e^{tP}) \sim \dots$
63. Page 89, in Eq. (1.10.52), $\text{Tr}(\mathcal{T}(y))$ is actually $\text{Tr}(\mathcal{T}(x(y)))$ but this likewise reduces to $\text{Tr}(\mathcal{T}(0))$ in Eq. (1.10.55).
64. Page 90, in the last line of Theorem 1.10.4: sign det
65. Page 93, in Eqs. (1.11.13) and (1.11.16), W should be replaced by \mathcal{W} .
66. Page 93, below Eq. (1.11.18): The associated boundary operator ...
67. Page 94, the last line of the Remark: $\mathcal{K} = \mathbb{C} - \mathbb{R}^+$ or $\mathcal{K} = \mathbb{C} - \mathbb{R}^- - \mathbb{R}^+$.
68. Page 94, Eq. (1.11.25): $B\bar{\gamma}f =$
69. Page 94, Eq. (1.11.26): $p_d(y, 0, \zeta, D_r)f(r) = \dots$
70. Page 96, Eq. (1.11.46): $\sigma_g(B_P)(y, \zeta)\bar{\gamma}(f) = \dots$
71. Page 99, the Remark should end: the eigenvalues so that $0 \leq |\lambda_1| \leq |\lambda_2| \leq \dots$
72. Page 106, in the Definition: (b) There exists $\delta > 0$ so that $|h(t, \epsilon)| \leq C e^{-t\delta}$ for $t \geq 1$. Otherwise in the Remark not every trace would be admissible. Accordingly the constant C should be inserted in the line below Eq. (1.12.6).
73. Page 106, in Eq. (1.12.8) the exponent $-k$ should be k .
74. Page 106, in the last line: This gives a meromorphic extension to the halfplane
75. Page 109, at the end: we use analytic continuation to see that (1.12.27) holds for all ϵ .
76. Page 110, above Eq. (1.12.31): to suppose $A(r) = r^a$.
77. Page 111, in the exponent on the left-hand side of the first line of Eq. (1.12.43): λ^b
78. Page 113, below Eq. (1.12.51): ... ζ has a meromorphic extension to \mathbb{C} with isolated poles. The poles need not be simple; double poles can occur.
79. Page 113, the Eq. in Lemma 1.12.6:

$$\lim_{n \rightarrow \infty} n |\lambda_n|^{-m/d} = \frac{d}{m} \Gamma\left(\frac{m}{d}\right)^{-1} a_0(P).$$
80. Page 115, above Eq. (1.13.7): $-a \notin \text{Spec}(P(0))$
81. Page 116, below Eq. (1.13.11): We use Lemma 1.9.1
82. Page 117, first line: If $-u \notin \text{Spec}(P(\epsilon))$
83. Page 118, in the third line of Eq. (1.13.21) the curly bracket is closed by a round bracket:

$$\{ \dots (\dots) \} dv$$
84. Page 118, in Eq. (1.13.23), Σ_n and s should be interchanged, such that the following statement can be rectified: We use Lemma 1.12.1 to see the sum has a simple pole at $s = 0$...
85. Page 126, the first line of Eq. (2.1.20) should end $-\omega_j^i \wedge \omega_k^j \otimes s^k$,
86. Page 127, in the third Definition: let $\nabla \in \text{Con}^G(V)$

87. Page 128, below Eq. (2.1.36): so $\theta \in C^\infty(T^*M \otimes \mathfrak{g})$ is invariantly defined
88. Page 129, the last but one line: it follows that the restriction of $\tilde{\Omega}$ to
89. Page 132, at the end: such bundles played an important role in Lemma 1.11.2
90. Page 133, below Eq. (2.1.61): Expand $z = z_i s^i \dots$ (as this complex variable is different from the real x above and below)
91. Page 135, in Eq. (2.1.82), V_+ should be replaced by ∇_+ .
92. Page 135, at the end of Example 2.1.2: We use the criteria of Lemma 1.11.2 to see
93. Page 136: **Definition:** Let $k = 2j$ or $k = 2j + 1$. We define the total Pontrjagin polynomial $P(A)$ for $A \in \mathfrak{o}(k, \mathbb{R})$ by:
94. Page 137, in the Remark: for example
95. Page 139, below (2.2.14): Since $\mathcal{P}(1) = 1$,
96. Page 139, in Lemma 2.2.2 (b): *If $k = 2j$, then $e_k^2 = p_j, \dots$*
97. Page 140, below Eq. (2.2.21): Since $e_{2j}(A)$ is continuous,
98. Page 142, in the second line of section 2.2.3: and let $\nabla \in \text{Con}^G(V)$.
99. Page 152, in the third line: for any point of the sphere.
100. Page 157, Eq. (2.3.87) and below:

$$\text{ch}_\tau[\mathbb{C}P_C(\rho)] = 0$$

if $\ell(\tau) > \ell(\rho)$

101. Page 157, the last two lines of Eq. (2.3.94):
- $$\begin{aligned} &= (2n + 1)(\text{ch}_{2n}(L) + \text{ch}_{2n}(L^*)) \\ &= (2n + 1)(x^{2n} + (-x)^{2n})/(2n)!. \end{aligned}$$
102. Page 159, in the two lines below Eq. (2.3.102), k should be replaced by n .
103. Page 159, the last line of Eq. (2.3.105) should read
- $$= \text{Res}_{x=0}(1 - e^{-x})^{-1} = 1.$$
104. Page 162, in Eq. (2.4.11), the factor 2 should be dropped.
105. Page 164, the line in the middle: the previous notion of order and the Lemma follows from Theorem 1.8.4.
106. Page 169, Eq. (2.4.59) should read

$$\deg_i(g_{i_k j_k}) = \deg_i(g_{p_k q_k}) \text{ and } \alpha_k(i) = \beta_k(i).$$

The second **Example:** We

107. Page 170, **Remark:** The
108. Page 170, Eq. (2.4.63) should end with $P^{(v)}$. In Eq. (2.4.64), n should be replaced by j .
109. Page 171, the first line of Eq. (2.4.65)

$$P^{(1)} = \sum_{A_1} c(A_1, P) A_1^{(1)}$$

110. Page 171, Eq. (2.4.66)

$$\beta(k) = \begin{cases} \alpha(1) + 1 & \text{if } k = 1, \\ \alpha(2) - 1 & \text{if } k = 2, \\ \alpha(k) & \text{if } k \geq 3. \end{cases}$$

111. Page 173, above Eq. (2.4.72): monomial A_1 of P with

112. Page 173, above Eq. (2.4.73): where $\beta_1(i) = 0$ for $2 < i \leq k$
 In Eq. (2.4.73) \deg_k should be \deg_i .
 In Eq. (2.4.74), the variables u and v should get an index 1, $P_{1,l}$ should be P_l .
113. Page 179, above Eq. (2.5.35): there exists a universal constant $\tilde{c}(m)$
114. Page 181, at the top (a) and (b) should both end $\sum_k \deg_k A = 2m + 2$. (c) should end $\sum_k \deg_k A = 2m + 4$.
115. Page 181, in Eq. (2.5.46), I and J should be lower indices of ϵ as in Eq. (2.5.45).
 Theorem 2.5.5: (b)
116. Page 182, at the end of the second line: If $k = 0$, $c(m, k, p) = (-1)^p$
117. Page 182, above Eq. (2.5.55): Then (2.5.52) follows by noting:
118. Page 182, the third line of Eq. (2.5.59)
- $$= (4\pi)^{-1/2} \sum_p c(m-1, k-1, p) a_n(\cdot, \Delta_p^{m-1})$$
119. Page 184, the seventh line: local orthonormal frame $\vec{s} = (s^1, \dots)$
120. Page 186, above Eq. (2.6.22): The Bianchi identities show:
121. Page 188, Eq. (2.6.37): $Q_p(V) = \Lambda^{m-4|\rho|}(N)$
122. Page 189, in Eq. (2.6.38), in the first two lines $Q_\tau(V)$ should be $Q_\rho(V)$ which only in the third line turns out to be $Q_\tau(V)$.
123. Page 189, below Eq. (2.6.40): Let L_j be the line bundles
124. Page 191, above Eq. (2.7.7): the inward unit normal.
125. Page 192, below Eq. (2.7.12): Conversely, suppose that for some ϕ (2.7.12) vanishes ...
126. Page 192, in Eq. (2.7.14) the factor $\sqrt{-1}$ should be deleted.
127. Page 193, above Eq. (2.7.17): Since $d + \delta$ is self-adjoint,
128. Page 197, above Eq. (2.7.39): Thus by Theorem 1.11.4,
129. Page 198, in the second line of Eq. (2.7.45), the range of the integral should be ∂M as in the first line.
130. Page 199, the third line: only if every index $1 \leq i \leq m-1$... In the last line a comma appears twice.
131. Page 206, in the first line: Thus $p \leq 2\nu + \mu$. In the third line, ω_a should be w_a . In the ninth line: $\mathfrak{F}_{m,n,n}$
132. Page 206, below (2.8.19), in Eq. (2.8.23) and Eq. (2.8.24), the argument $[M]$ in fact means $[\partial M]$ as will be defined in Theorem 2.8.2 (c).
133. Page 206, Eq. (2.8.21) should be
- $$G(\varepsilon, h) = G_0 + e^{\varepsilon h} \tilde{G}.$$
134. Page 207, in Eq. (2.8.25) the factor ν^{-1} from the middle should also appear on the right-hand side: $d_\tau(\nu^{-1} \text{int}$
135. Page 210, at the end of the first line of the proof: for $P_i \in \mathfrak{F}_{m,i}^g$.
136. Page 210, the line above Eq. (2.9.7) should read: Since $P[M]$ is independent of the length of the circle,
137. Page 211, at the end of the second line of Eq. (2.9.11), e_{i_j} and the bracket $\}$ should be interchanged: $e_{i_j} \}$
138. Page 211, Eq. (2.9.12) should begin $0 = \frac{d}{d\varepsilon} \Big|_{\varepsilon=0} P[M, e^{2\varepsilon f} G] =$
139. Page 212, Eq. (2.9.19) and below:

$$Q_\mu := Q_{i_1 \dots i_\mu} e_{i_1} \circ \dots \circ e_{i_\mu} \in S_\mu M \otimes \Lambda^p M$$

and let $\nabla^\mu f \in \otimes^\mu TM$.

140. Page 213, the second line of Eq. (2.9.24) should end $e_{i_\sigma}^{-1} e_{j_\mu} \text{int}^l(e_{i_\sigma}) \text{ext}^l(e_{j_\mu}) \theta$.
141. Page 213, the first line of Eq. (2.9.25) should end $\text{int}^l(e_{i_\sigma}) \omega_I$,
142. Page 213, in the middle of the page, case (3) should end $\Sigma_\sigma a_\sigma \theta = \nu \theta$.
143. Page 213, above Eq. (2.9.28): We use (2.9.20) to see
144. Page 214, Theorem 2.9.5 (b): *a polynomial*
In the first line of the proof: for $P_i \in \mathfrak{P}_{m,i,p}^g$.
145. Page 217, Eq. (3.1.9): $\gamma(v)^2 = -|v|^2 I_W$.
146. Page 218, the last line of Eq. (3.1.16): $= \text{Pin}(V) \cap \text{Clif}(V)^e$.
147. Page 221, below Eq. (3.1.35): We complete the proof by checking ...
148. Page 222, Eq. (3.1.43):

$$\alpha_j := \sqrt{-1} e_{2j-1} * e_{2j} \text{ for } 1 \leq j \leq n.$$

149. Page 222, Eq. (3.1.44):

$$W(\epsilon) = \{w \in W : \gamma(\alpha_j)w = \epsilon_j w \text{ for } 1 \leq j \leq n\}$$

150. Page 223, the second line of Eq. (3.1.50):

$$= \frac{1}{4} \Gamma_{ikl} (-2\gamma_k \delta_{jl} + 2\gamma_l \delta_{kj})$$

Lemma 3.1.4: There exists a unique connection ...

151. Page 226, the last line of Eq. (3.2.5):

$$= \Gamma_{ii_1 k} e_k * e_j + e_{i_1} \omega_i(e_j)$$

152. Page 228, Eq. (3.2.19) should end: $:= \alpha_n^s(x, G, \nabla^1)$.
153. Page 230, below Eq. (3.2.28): be the generator discussed in Lemma 2.3.1.
154. Page 231, Lemma 3.2.7 (a): *There exists a universal constant C_2 so if $m = 2$ and if V belongs to*
155. Page 232, **Remark:** The
156. Page 234, above section 3.3.1: not every orientable manifold admits a spin structure.
In the middle line of Eq. (3.3.5) the = sign is missing.
157. Page 235, in Eq. (3.3.14), the indices of V_3 and V_4 should be interchanged.
158. Page 236, below the second Definition: The intersection of geodesically convex sets is again geodesically convex and every geodesically convex set is contractible.
159. Page 237, below Eq. (3.3.22): if $U_{\alpha_0} \cap \dots \cap U_{\alpha_{j+1}} \neq \emptyset$. Eq. (3.3.26) should read

$$(-1)^{\delta f_\phi(\alpha, \beta, \gamma)} = \det(\phi_{\alpha\beta} \phi_{\beta\gamma} \phi_{\gamma\alpha}) = \det(I) = 1.$$

160. Page 238, in the last but two line: If V_c is a complex vector bundle over M , let V_r be the underlying real vector bundle.
161. Page 239, above Eq. (3.3.36): into three sectors.
162. Page 243, in Lemma 3.3.5 (a) and (b) the last - and + signs should be interchanged such that they coincide with Lemma 3.4.4 (a) (order reversed).
Above the last unnumbered Eq.: Since $w \in \text{Clif}_c^e(M)$ if and only if
163. Page 247, in the first line of Lemma 3.4.3: $\tilde{A}_s \in \mathfrak{P}_{2s}(O)$

164. Page 247, at the end of the last line: Then $C_1 = 1$ and $C_2 = 2$.
 165. Page 248, in the second line of Eq. (3.4.10) the last argument Spin should be dropped.

In the Remark: and thereby of Theorem 3.2.8

In Theorem 3.4.5 (b), the argument Spin should be an upper index.

166. Page 249, Eq. (3.4.15) should start with a_m^{Spin}
 167. Page 251, the last line of Eq. (3.4.29) should be

$$-2 \int_M c_2(V).$$

168. Page 254, the two lines of Eq. (3.5.14) should begin with $\text{ext}_c^l(\xi)$ and $\text{int}_c^l(\xi)$.
 169. Page 254, the first line of Eq. (3.5.15) should read

$$\gamma_c^l(\tau)1 = \sqrt{-1}\gamma_c^l(e^1)\gamma_c^l(f^1)1 = \frac{1}{\sqrt{2}}\sqrt{-1}\gamma_c^l(e^1)(f^1 + \sqrt{-1}e^1)$$

170. Page 255, the second line of Eq. (3.5.19) should end
 $\rightarrow C^\infty(\Lambda^{0,e}M \otimes V)$

The adjoint of $A_V^{j,e}$ is $A_V^{j,o}$.

In Eq. (3.5.20), the upper indices j should be J .

171. Page 257, in the second line of Eq. (3.5.31), the minus sign in the argument of the Index should be deleted.

In the second line of Eq. (3.5.37), the index I of f should be deleted.

In Eq. (3.5.38), the = sign behind the colon is missing.

172. Page 258, in the second line of Eq. (3.5.39), the left-hand side $\text{Index}^J(M, V) =$ should be deleted because this equality is only established below.

The second line of Eq. (3.5.40) and the lines below should read

$$= \sqrt{-1}\text{ext}_c^l(\xi)$$

and dually $\sigma_L(\bar{\partial}^*)(\xi) = -\sqrt{-1}\text{int}_c^l(\xi)$. This shows that

$$\sqrt{2}o_L(\bar{\partial}_V + \bar{\partial}_V^*) = \sqrt{-1}\gamma_c^l(\xi) \otimes I_V = \sigma_L(\gamma_c^l \otimes I_V \diamond \nabla).$$

173. Page 259, the first line of Eq. (3.5.44):

$$\partial^{p,q} = \pi^{p+1,q} \circ d: C^\infty \Lambda^{p,q} \rightarrow C^\infty \Lambda^{p+1,q}$$

174. Page 260, below Eq. (3.5.45): i does not lift to Spin $(2n)$.

The right-hand side of Eq. (3.5.52) should be $\det(U)$

Three lines below: (3.5.51) is independent of the particular spectral resolution chosen;

The last line but two: If we replace θ_j by $\theta_j + 2\pi$, both components of Eq. (3.5.50) change sign so f does not depend upon the choice of angles and is invariantly defined.

175. Page 261, three lines above Eq. (3.5.53): if and only if it is possible

176. Page 261, below Eq. (3.5.57): the simultaneous -1 eigenspace for right multiplication

177. Page 262, Eq. (3.5.62) should begin $f(U)\bar{\beta}_j =$

178. Page 262, Eq. (3.5.65) and below should read

$$\gamma_c(e_1)\bar{\beta}_j * \gamma = \begin{cases} \bar{\beta}_1 * \bar{\beta}_j * \gamma/2 & \text{if } j_1 > 1, \\ -2\bar{\beta}_K * \gamma & \text{if } j_1 = 1. \end{cases}$$

Let

$$\Psi(\bar{\beta}_j) := 2^{-q/2}\bar{\beta}_j * \gamma.$$

179. Page 264, in Eq. (3.6.2) and above Eq. (3.6.5), $K(X)$ should be replaced by $K(M)$.
180. Page 265, in Eq. (3.6.12), $A \in \text{End}(\mathbb{C}^n)$
181. Page 266, the last line of Eq. (3.6.20) should be $= q^*V(2^{n-1}, 2^n)$.
182. Page 267, in the first line of Eq. (3.6.25) the curly bracket is not closed at the end.
In the second line: $\pm u \geq 0$
183. Page 268, in Eq. (3.6.34) and above, $\pi^-(x, 0, -1)$ is actually not the projection on Π^- as before but the projection on V^- .
184. Page 270, in Eq. (3.6.46), rather End than Hom , in the first line of Lemma 3.6.8:
 θ_i
185. Page 271, the right-hand side of Eq. (3.6.49) should be $\Delta^- := \Delta^{-,+} \oplus \Delta^{+,-}$.
186. Page 271, in the first two lines of Eq. (3.6.52), π^- and π^+ should be interchanged.
In the fourth line \tilde{z}_2 should be z_2 .
187. Page 272, in Eq. (3.6.60): $\Sigma_{|I|=m}$
In the first two lines of Eq. (3.6.63), $c_{I,J}$ should be $\tilde{c}_{I,J}$, in the second and third line, g should be replaced by ψ and in the third line S^x by $\Sigma(x)$.
188. Page 274, Lemma 3.7.2: (a) *There is a linear map*
189. Page 276, above Eq. (3.7.17): independent of the orientation
Eq. (3.7.18) should read
- $$\{\mathcal{T} \wedge \mathcal{Q}\}_{4s} = 2^{(m-4s)/2} L_s$$
190. Page 277, in Eq. (3.7.22): $\text{ch}(W)$
191. Page 278, in Lemma (3.7.6) (c): $\int_{\Sigma M}$
192. Page 281, below Eq. (3.7.48): Let $\nabla \in \text{Con}^U(V)$; we extend ∇ to ΛV
Above Eq. (3.7.56): shows in this more general setting, that the curvatures
193. Page 285, the last sentence of the proof of Lemma 3.8.2: The proof for the general case is essentially the same
In the first line of section 3.8.2: SM plays much the same role
In Eq. (3.8.5) P instead of P_0
194. Page 288, on the right-hand side of Eq. (3.8.24), both V should be V_2 .
195. Page 289, in Eq. (3.8.32), $P_{a,b}$ and $P_{a,b}^*$ should be $P_{3,a}$ and $P_{3,a}^*$.
Lemma 3.8.5: *a self-adjoint*
Above Eq. (3.8.35): Let $\{\theta_j, \mu_j\}$ be a spectral resolution
Above Eq. (3.8.36): Then $\{P_{3,a}\theta_j/\sqrt{\mu_j}, \mu_j\}$ is a spectral resolution
196. Page 290, the proof of Lemma 3.8.6: We generalize the operators of Lemma 3.8.5.
197. Page 291, above Eq. (3.8.50): Let $\{\phi_\nu, \lambda_\nu\}$ be a spectral resolution
Eq. (3.8.50): $E_\nu = \phi_\nu \cdot (L^2(S^1) \oplus L^2(S^1))$.
Below Eq. (3.8.51): for $a = \lambda_\nu$
In Eq. (3.8.52): $\sum_\nu \eta(s, R_\nu)$
Above Eq. (3.8.53): let $\pi: SM \rightarrow PM$ be the canonical projection.
Eq. (3.8.56): $\pi^*\mathcal{V} = \Pi^+(p \circ \pi)$.
and below: $K(SM; \mathbb{C})$
198. Page 292, the first line (and accordingly on page 354 and in the Index on page 515): We use the Stone-Weierstrass theorem

199. Page 294, in the Remark: defines a lifting
Above Eq. (3.9.6): We use Lemma 1.10.3 to see
200. Page 295, below Eq. (3.9.10): Then $\rho(B) \in \text{SO}(2)$
The first line of Eq. (3.9.11): $\text{Tr}(\Delta^\pm(B)) = e^{\mp\sqrt{-1}\theta/2}$
Below: Then $\det(1 - C_r) = (1 - \lambda)(1 - \bar{\lambda})$.
The end of Eq. (3.9.15): $= 2(1 + \sqrt{-1})^{-1}$.
201. Page 296, above Eq. (3.9.22): Fix a point $x_0 \in N$.
202. Page 298, in Eq. (3.9.35): $\mathcal{F}(i(\tilde{T}))$
203. Page 299, in the first line of Eq. (3.9.42) and in Eq. (3.9.44), the indices 1 and 2 should be dropped.
204. Page 302, in the paragraph above the Definition: Let Γ be the Christoffel symbols of the Levi-Civita connection on M ;
205. Page 303, in Lemma 3.10.2 (e), the expression $\eta(A^\pm)$ will only be defined in Eq. (3.10.15).
206. Page 305, below Eq. (3.10.20): since $f|_{\partial M} \in \bigoplus_{\lambda < 0} E(\lambda, A^\pm)$, and
$$\gamma_m \tilde{f}|_{\partial M} \in \gamma_m \{ \bigoplus_{\lambda \leq 0} E(\lambda, A^\mp) \} = \bigoplus_{\lambda \geq 0} E(\lambda, A^\pm)$$
.
207. Page 306, at the end of the first line of the Theorem, ψ is missing in the index.
208. Page 307, in the paragraph above Eq. (3.10.29): invariant homogeneous polynomials of order n
209. Page 308, above Eq. (3.10.38): let $\nabla^W \in \text{Con}^U(W)$,
210. Page 316, the first line of section 3.11.2: Let P be a self-adjoint elliptic pseudo-differential operator
211. Page 317, Eq. (3.11.15) should read
$$Q(\varepsilon) = \varepsilon P_{\nabla_1} + (1 - \varepsilon) P_{\nabla_0}$$
.
212. Page 318, in the first line, ∇_2 should be ∇_0 .
Below Eq. (3.11.22): If we replace ε by $\varepsilon + j$ for $j \in \mathbb{Z}$,
213. Page 319, above Eq. (3.11.26): Then ω_1 is the connection 1-form of θ .
In Eq. (3.11.29), 960 should be 480.
214. Page 320, above Eq. (3.11.39): 2-plane bundle and
215. Page 321, on the right-hand sides of Lemma 3.11.4 (a) and both lines of Eq. (3.11.42) there should be minus signs. In (b) and the last line of Eq. (3.11.49) the i^{n-1} should be replaced by i^n .
Eq. (3.11.43) should read
$$\text{Tr}((g^* dg)^{2n-1}) = c(x) \text{dvol}(x);$$

In Eq. (3.11.47) the commas should be deleted.
216. Page 323, above Eq. (3.11.59): to show the
217. Page 324, below Eq. (3.11.63): Consequently the characteristic class \mathcal{T} which appears in (3.11.60)
218. Page 328, Eq. (4.0.5) should coincide with Eq. (3.10.5)
$$L_{\alpha\beta} := (e^m, \nabla_{e_\alpha} e_\beta) = -\frac{1}{2} \partial_m g_{\alpha\beta}$$
.
219. Page 329, Eq. (4.0.15) should read
$$f[M] := \int_M f \text{dvol}.$$

220. Page 331: **Remark:** Let \mathcal{L} be a cocompact discrete subgroup of $\mathbb{R}^m \dots$
221. Page 333, in the first line of Eq. (4.1.26): $\theta_{;kk}$
222. Page 336, in Lemma 4.1.5 (c): $e^{-2\epsilon f} F$
223. Page 337, in Eq. (4.1.40): $(4\pi)^{-m/2}$
Below Eq. (4.1.44): The invariants ρ^2 , R^2 , and Ω^2 are additive.
224. Page 338, Eq. (4.1.46) should begin $0 = \text{Tr} F\{-2\alpha_3$
The middle term in the last line of Eq. (4.1.49) should be $\alpha_{10} R_{ijkl} R_{ijlk}$.
225. Page 341, in the third line of Eq. (4.1.72): $\{(\partial_x^2 \theta_\nu, \theta_\nu -$
226. Page 345, the right-hand sides of the equations for α_8 and α_9 should start with $-4f$ as the others.
227. Page 346, the last line of Eq. (4.2.3) should read
$$-c(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}).$$
228. Page 347, above Eq. (4.2.6): We use Theorem (4.1.7) to decompose:
In the first two lines of Eq. (4.2.6): $(4\pi)^{-m/2}$
In the first line of Eq. (4.2.7), $[M]$ can be deleted.
The constant factor is omitted from Eq. (4.2.10) because it is not relevant for the Theorem.
229. Page 349, two lines below Eq. (4.2.14): $\text{Tr}(\rho)$
230. Page 353, above Theorem 4.2.12: Let $r^2 = |x|^2 = x_0^2 + \dots + x_m^2$.
231. Page 354, in the second line of Eq. (4.2.45), \cup_j should be \oplus_j .
Above Eq. (4.2.47): Stone-Weierstrass theorem
232. Page 354, Eq. (4.2.28) should end $r^{-2}\Delta_{5m}$ as in Eq. (4.7.30).
233. Page 355, in Eq. (4.2.51), ν should be j .
234. Page 357, below Eq. (4.2.68): be a lens space.
235. Page 358: **Remark:** Two
236. Page 361, in Eq. (4.3.21), τ should be τ^2 .
237. Page 362, Eq. (4.3.27) should read
$$e_2(x, \Delta_1) = \delta^{-1}(\tau - \delta\rho).$$
238. Page 362, the second line of Eq. (4.3.29) should end $\text{Tr}_{L^2}(F\pi_\delta e^{-bt\Delta_1})$.
239. Page 362, Eq. (3.4.30) should read
$$\left\{ (\phi_\mu^2 d\phi_\nu^1 + \phi_\nu^1 d\phi_\mu^2)(\lambda_\nu^1 + \lambda_\mu^2)^{-1/2}, e^{-t(\lambda_\nu^1 + \lambda_\mu^2)} \right\}_{\lambda_\nu^1 + \lambda_\mu^2 \neq 0}$$
240. Page 364, below Eq. (4.3.44): We adopt the notation of §1.7; we integrate...
(There is no figure 1.7.1.)
241. Page 368, in Theorem 4.4.1 (d): Tr
Remark: Note
In Lemma 4.4.2 (d): Tr
242. Page 369, in Eq. (4.4.13), f is missing at the right of the middle term.
Below: Then a^k is of order at most 1 ...
In Eq. (4.4.17), δ is missing.
243. Page 370, in the first line: and collect coefficients to prove (a).
Above Eq. (4.4.28): Since $\partial_\epsilon P(\epsilon) = -fI_\nu$,
244. Page 371, below Eq. (4.4.35): We use Theorem 4.1.7 to complete the proof ...
245. Page 372, in the last line of Eq. (4.4.37): $\text{Tr}\{f(\tau + \dots$
Eq. (4.4.44) should end with $2\delta_{\sigma\mu}\gamma_\nu$.

246. Page 374, Eq. (4.4.62) should start

$$P \square Q := \begin{pmatrix} 1 \otimes Q & P^* \otimes 1 \\ P \otimes 1 & -1 \otimes Q \end{pmatrix} \in \dots$$

247. Page 377, Theorem 4.5.1 (d), the first and third line of (e) and Theorem 4.5.2 (c):
Tr

248. Page 381, in the third line: near ∂M

249. Page 385, at the end of Lemma 4.6.3: *with respect to the complement of $(0, \infty)$.*
At the end of the Remark: it is not self-adjoint for $a \neq b$ if

250. Page 394, in Theorem 4.6.11 (c), in the third line the first bracket is not closed
and can be dropped together with the + sign.

251. Page 398, in 4.7.2, the line below, and below Eq. (4.7.56): Hurwitz
Three lines above section 4.7.3: The integrand f is holomorphic in u for $u \in \mathbb{C} -$
 $[1, \infty)$ and $\text{Re}(t) > 0$.

252. Page 401, in the line below Eq (4.7.46): $H(m + 1, j, V)$

253. Page 402, in Eq. (4.7.51) the second lower index 2 of Eq. (4.7.50) is missing at
four places.

254. Page 405, the first line of Eq. (4.7.70) should end $e^{-(j+k-1)t}$.

255. Page 406, above Eq. (4.7.73): If $I = \{i_1, \dots, i_k\}$ is a multi-index,

256. Page 408, in the first line of Lemma 4.7.11: det

257. Page 409, above Eq. (4.7.99): $h^\epsilon(0, g, u)$. In Eq. (4.7.99) e^{-t} should be deleted
at two places. Eq. (4.7.100) should end: $2 \det(g) \det(g - I)^{-1}$.

258. Page 410, the last sentence: The lemma now follows from Lemma 4.7.11.

259. Page 411, in Lemma 4.7.13 (b) and in Eq. (4.7.109) the summation index k is
fixed as the complex dimension and should be replaced by j .

260. Page 413, in Example 4.7.2: the cyclic group of order n .

261. Page 419, in the third line: §5.1 is a brief introduction.

262. Page 421, in the middle of the first paragraph: in section 5.2.8