Key

Section 6.4 Lecture Guide

Math 242, Fall 2018

Ex 1 Suppose that the probability density function for a random variable X representing the lifespan of a lightbulb at the store is

$$f(x)=\begin{cases} e^{-x} & x\geqslant 0 \\ 0 & x<0 \end{cases}$$

What is the probability that a randomly chosen lightbulb lasts between 1 and 3 years? What about the probability that it lasts for more than 2 years?

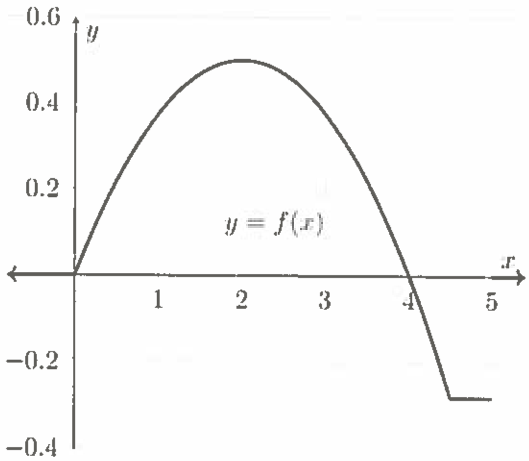
$$P(1\le X\le 3)=\int_1^3 f(x)\,dx=\int_1^3 e^{-x}\,dx=-e^{-x}\Big|_1^3=e^{-1}-e^{-3}\approx\boxed{.32}$$

$$P(X\geqslant 2)=\int_2^\infty f(x)\,dx=\lim_{b\to\infty}\int_2^b f(x)\,dx=\lim_{b\to\infty}\int_2^b e^{-x}\,dx=\lim_{b\to\infty}-e^{-x}\Big|_2^b$$

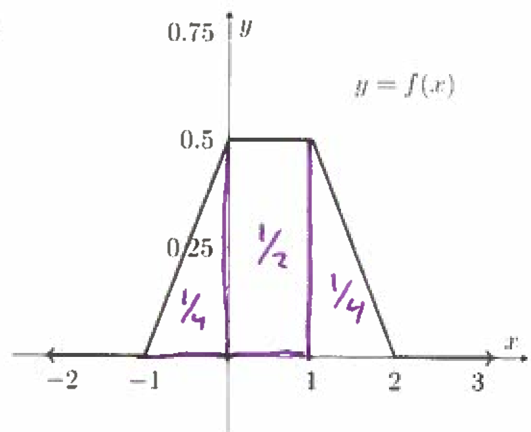
$$=\lim_{b\to\infty} e^{-2}-e^{-b}=e^{-2}\approx\boxed{.14}$$

Ex 2 Could the following two graphs be graphs of a probability density function?

(a)



(b)



on $(4,5)$, $f(x)<0$, so $f(x)$ is not a p.d.f.

for all x, $f(x)\geq 0$

$$\int_{-\infty}^{\infty} f(x)\,dx=\int_{-1}^{2} f(x)\,dx=$$

$$=\frac{1}{4}+\frac{1}{2}+\frac{1}{4}$$

$$=1$$

$\Rightarrow f(x)$ is a p.d.f.

Ex 3 A random number generator (RNG) chooses real numbers between 0 and 100 according to a uniform distribution.

(a) What is the probability density function for this RNG?

$$f(x) = \begin{cases} \frac{1}{100} & 0 \le x \le 100 \\ 0 & \text{else} \end{cases}$$

(b) What is the probability that a randomly chosen number is between 50 and 60?

$$P(50 \le X \le 60) = \int_{50}^{60} f(x)dx = \int_{50}^{60} \frac{1}{100}\,dx = \frac{1}{100}x\Big|_{50}^{60} = \boxed{.1}$$

(c) What is the probability that a randomly chosen number is between 10 and 20?

$$P(10 \le X \le 20) = \int_{10}^{20} f(x)dx = \int_{10}^{20} \frac{1}{100}\,dx = \frac{1}{100}x\Big|_{10}^{20} = \boxed{.1}$$

Ex 4 Let X be a random variable that measures the duration of cell phone class in a particular city and assume that X has an exponential distribution with density function

$$g(t) = \begin{cases} 0.5e^{-0.5t} & t \geqslant 0 \\ 0 & t < 0 \end{cases}$$

where t denotes the duration (in minutes) of a randomly selected call.

(a) Find the probability that a randomly selected call will last between 2 and 3 minutes.

$$P(2 \le X \le 3) = \int_2^3 g(t)dt = \int_2^3 .5e^{-.5t}dt = \frac{.5e^{-.5t}}{-.5}\Big|_2^3$$

$$\approx \boxed{.14}$$

(b) Find the probability that a randomly selected call will last less than 2 minutes.

$$P(X \le 2) = \int_{-\infty}^{2} g(t)\,dt = \underbrace{\int_{-\infty}^{0} g(t)dt}_{=0} + \int_0^2 g(t)dt = \int_0^2 .5e^{-.5t}dt =$$

$$= \frac{.5e^{-.5t}}{-.5}\Big|_0^2 \approx \boxed{.63}$$

Ex 5 Find the expected value of the RNG which is selecting values between 0 and 100. What if it were computing values between 100 and 300?

(a) $E(X) = \int_{-\infty}^{\infty} x \underbrace{f(x)}_{\text{see (3a)}} dx = \int_0^{100} x \cdot \frac{1}{100}\, dx = \frac{1}{100} \frac{x^2}{2}\Big|_0^{100} = \boxed{50}$

(b) now, the p.d.f. is $g(x) = \begin{cases} \frac{1}{200} & 100 \le x \le 300 \\ 0 & \text{else} \end{cases}$

so $E(X) = \int_{-\infty}^{\infty} x\, g(x)\, dx = \int_{100}^{300} \frac{1}{200} x\, dx = \frac{1}{200} \frac{x^2}{2}\Big|_{100}^{300} = \boxed{200}$

Ex 6 What is the expected value of a random variable X with probability density function $\frac{1}{x}$ on the interval $[1, e]$?

the p.d.f. for X is $f(x) = \begin{cases} \frac{1}{x} & 1 \le x \le e \\ 0 & \text{else} \end{cases}$

then $E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_1^e x f(x)\, dx = \int_1^e x \cdot \frac{1}{x}\, dx$

$= \int_1^e 1\, dx = x\Big|_1^e = e - 1 \approx 1.72$