

Review Quick Hit (2)

1. Find the solution $y(x)$ with $y(0) = 4$ of the differential equation

$$\begin{aligned} \frac{dy}{dx} &= x^2 y \\ \Rightarrow \frac{dy}{y} &= x^2 dx \rightarrow \int \frac{dy}{y} = \int x^2 dx \rightarrow \ln(|y|) = \frac{x^3}{3} + C \\ \Rightarrow |y| &= e^{\frac{x^3}{3} + C} = e^{\frac{x^3}{3}} e^C = k e^{\frac{x^3}{3}} \\ \Rightarrow 4 &= k e^0 = k \\ \Rightarrow |y| &= 4 e^{\frac{x^3}{3}} \end{aligned}$$

2. Find the average value of the function $g(x) = \frac{x^2}{x^3-1}$ on the interval $[2, 4]$

$$\begin{aligned} \frac{1}{4-2} \int_2^4 \frac{x^2}{x^3-1} dx &= \frac{1}{2} \int_{x=2}^{x=4} \frac{1}{u} \frac{du}{3} = \frac{1}{6} \ln(|u|) \Big|_{x=2}^{x=4} \\ &\quad (u = x^3 - 1; \frac{du}{3} = x^2 dx) \\ &= \frac{1}{6} \ln(|x^3 - 1|) \Big|_2^4 \\ &= \frac{1}{6} \ln(63) - \frac{1}{6} \ln(7) \end{aligned}$$

3. Compute $\int_0^\infty x e^{-x^2} dx$

$$\begin{aligned} &= \lim_{b \rightarrow \infty} \int_0^b x e^{-x^2} dx = \lim_{b \rightarrow \infty} \int_{x=0}^{x=b} e^u \frac{du}{-2} = \lim_{b \rightarrow \infty} -\frac{1}{2} e^u \Big|_{x=0}^{x=b} \\ &\quad (u = -x^2; \frac{du}{-2} = x dx) \\ &= \lim_{b \rightarrow \infty} -\frac{1}{2} e^{-x^2} \Big|_0^b \\ &= \lim_{b \rightarrow \infty} -\frac{1}{2} e^{-b^2} + \frac{1}{2} e^0 \\ &= \boxed{\frac{1}{2}} \end{aligned}$$