# Exam 2

Math 242, Fall 2018

Name: Key

Don't leave anything blank: If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work: If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers: Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself: If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 102 points on this exam. That means you should budget about 0.5 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder: There are to be no devices with internet access nor graphing calculators/devices which can compute derivatives or integrals used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

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1. (8 points) Give an example of two *distinct* functions, $f(x)$ and $g(x)$, so that $f'(x) = g'(x) = 2x$. *Hint:* You shouldn't have any undefined constants in your answer.

$f(x) = x^2$ $\quad g(x) = x^2 + 1$

2. Compute the following indefinite integrals:

(a) (6 points) $\int 3x^2 - 4x^5 + x^{-1}\,dx$ $= x^3 - \frac{4x^6}{6} + \ln(|x|) + C$

(b) (4 points) $\int e^{4x} + 3\,dx$ $= \frac{e^{4x}}{4} + 3x + C$

(c) (10 points) $\int \frac{1}{x\ln(x)}\,dx$ $= \int \frac{1}{u}\,du = \ln(|u|) + C$

$= \ln(|\ln(x)|) + C$

$u = \ln(x)$

$du = \frac{1}{x}dx$

3. Greg breaks chalk at a rate of $\frac{2t+1}{2t^2+2t-1}$ pieces per week, $t$ weeks after the start of the term. Suppose Greg broke 8 pieces of chalk in week 1 (i.e. $t = 1$).

   (a) (12 points) Write a function describing the number of pieces of chalk that Greg breaks in week $t$.

$C(t)$ – number of pieces of chalk Greg breaks in week $t$

$$c'(t) = \frac{2t+1}{2t^2+2t-1}$$

$$c(t) = \int c'(t)\,dt = \int \frac{2t+1}{2t^2+2t-1}\,dt = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\ln(|u|) + K$$

$$= \frac{1}{2}\ln(|2t^2+2t-1|) + K$$

$u = 2t^2+2t-1$

$du = 4t+2\,dt$

$= 2(2t+1)dt$

$\frac{du}{2} = (2t+1)dt$

$$8 = C(1) = \frac{1}{2}\ln(|2+2-1|) + K = \frac{1}{2}\ln(3) + K$$

$$\rightarrow K = 8 - \frac{1}{2}\ln(3) \approx 7.45$$

$$C(t) = \frac{1}{2}\ln(|2t^2+2t-1|) + 8 - \frac{1}{2}\ln(3)$$

   (b) (2 points) How many pieces of chalk will he break in week 8?

$$C(8) = \frac{1}{2}\ln(|128+16-1|) + 8 - \frac{1}{2}\ln(3) \approx 9.9$$

   (c) (4 points) Write a complete sentence interpreting your answer to part (b).

Greg will break about 10 pieces of chalk in week 8.

4. (8 points) Is $y = x^3 - 2x + 1$ a solution to the differential equation $\frac{d^2y}{dx^2} + 2x - 8 = 0$?

$\frac{dy}{dx} = 3x^2 - 2 \rightarrow \frac{d^2y}{dx^2} = 6x$

LHS: $\frac{d^2y}{dx^2} + 2x - 8 = 6x + 2x - 8 = 8x - 8$

RHS: $0$

$y = x^3 - 2x + 1$ is not a solution

5. (16 points) State the fundamental theorem of calculus. (Giving an example of how to use the theorem can earn you half credit, but no more.)

If $F(x)$ is an antiderivative of $f(x)$,

then $\int_a^b f(x)\,dx = F(b) - F(a)$

6. Compute the following definite integrals:

(a) (4 points) $\int_{-2}^{3} x + 2\,dx$ $= \frac{x^2}{2} + 2x \Big|_{-2}^{3}$

$= \left(\frac{3^2}{2} + 2\cdot 3\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)$

$= \frac{9}{2} + 6 - 2 + 4$

$= \frac{25}{2}$

(b) (12 points) $\int_{0}^{8} x\sqrt{x^2+3}\,dx$ $= \int_{0=x}^{8=x} \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\,\frac{u^{3/2}}{(3/2)}\Big|_{x=0}^{x=8}$

$u = x^2 + 3$

$du = 2x\,dx$

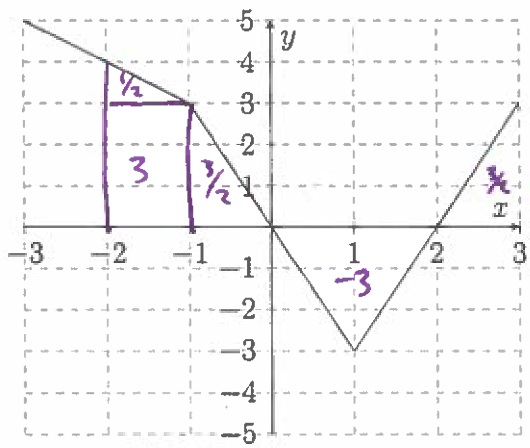
$\frac{du}{2} = x\,dx$

$= \frac{1}{3}(x^2+3)^{3/2}\Big|_{x=0}^{x=8}$

$= \frac{1}{3}\cdot 67^{3/2} - \frac{1}{3}\cdot 3^{3/2}$

$\approx 181.1$

7. Below is the graph of a function, $y = f(x)$.




Compute the following integrals:

(a) (6 points) $\int_{-2}^{2} f(x)\,dx$ $= \frac{1}{2} + 3 + \frac{3}{2} - 3$

$= 2$

(b) (10 points) $\int_{0}^{3} 10f(x) + 1\,dx$ $= 10\int_0^3 f(x)dx + \int_0^3 dx$

$= 10\left(\frac{3}{2} - 3\right) + \left(x \Big|_0^3\right)$

$= 15 - 30 + 3$

$= -12$

Bonus: Compute the following integrals (the first two parts are all or nothing—no partial credit will be awarded):

(a) (4 points) $\int \ln(x)\,dx$

*Hint: what is the derivative of $x\ln(x)$?*

$$\frac{d}{dx}\left(x\ln(x)\right) = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x)+1$$

$$\rightarrow \frac{d}{dx}\left(\underline{x\ln(x)} - \underline{x}\right) = \underline{\ln(x)+1} - \underline{1} = \ln(x)$$

$$\rightarrow \int \ln(x)\,dx = x\ln(x) - x + C$$

(b) (4 points) $\int 2018^x\,dx = \int e^{\ln(2018)x}\,dx = \frac{1}{\ln(2018)}e^{\ln(2018)x} + C$

$$= \frac{2018^x}{\ln(2018)} + C$$

(c) (6 points) $\int \frac{(e^{2x}+3x^2)\ln(e^{2x}+2x^3-1)}{5e^{2x}+10x^3-5}\,dx = \int \frac{(e^{2x}+3x^2)\,u}{5\,(e^{2x}+2x^3-1)}\,dx$

$$u = \ln(e^{2x}+2x^3-1)$$

$$du = \frac{2e^{2x}+6x^2}{e^{2x}+2x^3-1}\,dx = 2\left(\frac{e^{2x}+3x^2}{e^{2x}+2x^3-1}\right)dx$$

$$= \int \frac{u}{2\cdot 5}\,du$$

$$= \frac{1}{10}\,\frac{u^2}{2} + C$$

$$= \frac{1}{20}\left(\ln(e^{2x}+2x^3-1)\right)^2 + C$$