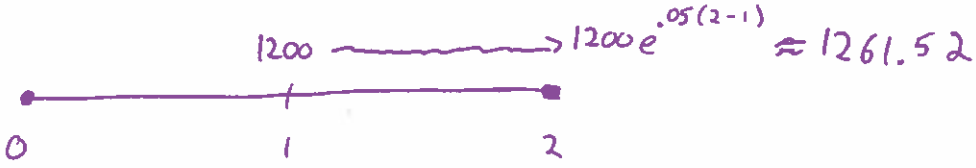
Ex 1 You have a (currently empty) account into which you add a total of \$1200 per year. The account earns interest compounded continuously at a rate of 5%. How much will the account be worth at the end of 2 years if you....

(a) ...add in your money only once per year?

1200



→ total: $1200 + 1261.52 = \underline{2461.52}$

(b) ...add in your money twice per year? 600

$600 \rightsquigarrow 600e^{.05(2-3/2)} \approx 615.19$

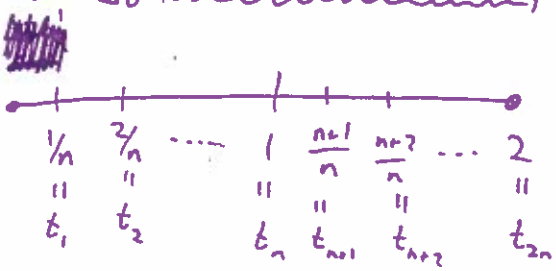
$600 \rightsquigarrow 600e^{.05(2-1)} \approx 630.76$

$600 \rightsquigarrow 600e^{.05(2-1/2)} \approx 646.73$

→ total: $600 + 615.19 + 630.76 + 646.73 = \underline{2492.68}$

(c) ...add in your money n times per year? set $\Delta t = \frac{1}{n}$

$1200\Delta t \rightsquigarrow 1200\Delta t e^{.05(2-t_i)}$ if deposited at time t_i



total: $1200\Delta t e^{.05(2-t_1)} + 1200\Delta t e^{.05(2-t_2)} + \dots + 1200\Delta t e^{.05(2-t_{2n})}$

$= \left(1200e^{.05(2-t_1)} + 1200e^{.05(2-t_2)} + \dots + 1200e^{.05(2-t_{2n})}\right)\Delta t$

(d) ...add in your money continuously?

$$\lim_{n\to\infty}\left(1200e^{.05(2-t_1)}+\cdots+1200e^{.05(2-t_n)}\right)\Delta t=\int_0^2 1200e^{.05(2-t)}\,dt$$

$$=1200\int_0^2 e^{.1-.05t}\,dt=1200\int_0^2 e^{.1}e^{-.05t}\,dt=1200e^{.1}\int_0^2 e^{-.05t}\,dt$$

$$=\frac{1200e^{.1}e^{-.05t}}{-.05}\Big|_0^2\approx 2524.10$$

Ex 2 Beginning in the year 2010 and until 2040, GiantCo invests 10 million dollars per year (continuously) in treasury bonds that mature in 2040 and which pay 3.75% interest annually (compounded continuously). Find the future value of GiantCo's investment.

$T=30$

$r=.0375$

$f(t)=10$

$$FV=e^{.075\cdot 30}\int_0^{30}10e^{-.0375t}\,dt=\frac{10e^{1.125}e^{-0.375t}}{-.0375}\Big|_0^{30}$$

≈ 554.72 million dollars

Ex 3 An 18-year-old is gifted with a sizable trust fund. Her benefactor chose the quantity to be equivalent to continuously investing \$50000, with the assumption that both trust fund and income would be accruing interest at a 6% annual rate compounded continuously, and that the two investments would be equal at age 50. How large is the trust fund?

$T=32$

$r=.06$

$f(t)=50000$

$$\Rightarrow PV=\int_0^{32}50000e^{-.06t}\,dt=\frac{50000e^{-.06t}}{-.06}\Big|_0^{32}$$

$\approx 711,160.86$