Math 242
Fall 2018

Name: Key

Review Quick Hit (1)

1. Compute the following integrals

(a) $\int 2x^{-4} + e^{4x} + 2x^5 \, dx$ $= \frac{2x^{-3}}{-3} + \frac{e^{4x}}{4} + \frac{2x^6}{6} + C$

(b) $\int \frac{t+1}{t^2} \, dt$ $= \int \frac{t}{t^2} + \frac{1}{t^2} dt = \int \frac{1}{t} + \frac{1}{t^2} dt = \ln(|t|) + \frac{t^{-1}}{-1} + C$

(c) $\int (x^{-3} - x)\sqrt{x^2 + x^{-2}} \, dx$ $= \int \sqrt{u} \frac{du}{-2} = -\frac{1}{2} \frac{u^{3/2}}{(3/2)} + C = \frac{-(x^2+x^{-2})^{3/2}}{3} + C$

$u = x^2 + x^{-2}$

$du = 2x - 2x^{-3} dx = -2(x^{-3} - x)dx$

$\frac{du}{-2} = (x^{-3} - x)dx$

2. State the Fundamental Theorem of Calculus.

If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

3. Is $y = \ln(x) + e^x + 2018$ a solution to the following differential equation?

$$\frac{d^2y}{dx^2} + \frac{1}{x^2} = \frac{dy}{dx} - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} + e^x \qquad \frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$$

$$\left.\begin{array}{l} \text{LHS: } \frac{d^2y}{dx^2} + \frac{1}{x^2} = \left(-\frac{1}{x^2} + e^x\right) + \frac{1}{x^2} = e^x \\ \text{RHS: } \frac{dy}{dx} - \frac{1}{x} = \left(\frac{1}{x} + e^x\right) - \frac{1}{x} = e^x \end{array}\right\} \text{the same}$$

Yes, $y = \ln(x) + e^x + 2018$ is a solution