Ex 1 Jamaal manages a grocery store that carries two brands of cat food, a local brand obtained at the cost of 30 cents per can and a national brand obtained for 40 cents per can. He estimates that if the local brand is sold for x cents per can and the national brand for y cents per can, then approximately $70 - 5x + 4y$ cans of the local brand and $80 + 6x - 7y$ cans of the national brand will be sold each day. How should Jamaal price each brand to maximize total daily profit from the sale of cat food?

Local: cost: 30 cents per can
price: x cents per can
demand: $70 - 5x + 4y$ cans

National: cost: 40 cents per can
price: y cents per can
demand: $80 + 6x - 7y$ cans

Local profit per can: $x - 30$
Local profit: $(x-30)(70-5x+4y)$
National profit per can: $y - 40$
National profit: $(y-40)(80+6x-7y)$

Total profit $P(x,y) = (x-30)(70-5x+4y) + (y-40)(80+6x-7y)$

1. Crit pts. of P

$P_x(x,y) = (1)(70-5x+4y) + (x-30)(-5) + (y-40)(6)$

$= -20 - 10x + 10y$

$P_y(x,y) = (x-30)(4) + (1)(80+6x-7y) + (y-40)(-7)$

$= 240 + 10x - 14y$

$(1)\quad -20 - 10x + 10y = 0$

$(2)\quad 240 + 10x - 14y = 0$

$(1) \Rightarrow 10x = -20 + 10y$

$(2) \Rightarrow 240 + (-20 + 10y) - 14y = 0$

$\Rightarrow 220 - 4y = 0$

$\Rightarrow 220 = 4y$

$\Rightarrow \frac{220}{4} = y = 55$

$10x = -20 + 10(55) = 530$

$x = 53$

Critical point: $(53, 55)$

Verify that this is a max:

$f_{xx} = -10 \qquad f_{xy} = 10 \qquad f_{yy} = -14$

$D(53,55) = f_{xx} f_{yy} - f_{xy}^2$

$= (-10)(-14) - 10^2$

$= 40 > 0$ and $f_{xx}(53,55) = -10 < 0$

So $(53,55)$ is a max.

Hence, Jamaal should price the local brand at 53 cents per can and the national brand at 55 cents per can in order to maximize profit.