Ex 1 Exploring a new "dynamic pricing" scheme, a company determines that the rate of change in monthly demand of a product is given by $\frac{dq}{dp} = -12pe^{-0.5p^2}$, where q is thousands of units sold and p is the unit price in dollars. Find a function to represent the total number of units sold each month at a unit price of p dollars if the potential market (number of units sold if the product were free) is 14 thousand units.

$$\frac{dq}{dp} = -12pe^{-.5p^2} \rightarrow \int dq = \int -12pe^{-.5p^2}\,dp \qquad u=-.5p^2 \quad du=-p\,dp$$

$$\rightarrow q(p) = \int 12 e^u\,du = 12e^u + C = 12e^{-.5p^2} + C$$

$$14 = q(0) = 12e^{-.5\cdot 0^2} + C = 12 + C \rightarrow C = 2$$

$$q(p) = 12e^{-.5p^2} + 2$$

Ex 2 A company's production changes at a rate of $\frac{4}{3t+1}$ thousand items per month, t months after the product's public release. Find the total change in production between three months and six months after release.

$$P(6) - P(3) = \int_3^6 P'(t)\,dt = \int_3^6 \frac{4}{3t+1}\,dt$$

$$u = 3t+1$$

$$du = 3\,dt \rightarrow dt = \frac{du}{3}$$

$$= \int_{t=3}^{t=6} \frac{4}{u}\,\frac{du}{3} = \frac{4}{3}\int_{t=3}^{t=6} \frac{1}{u}\,du = \frac{4}{3}\ln(|u|)\Big|_{t=3}^{t=6}$$

$$= \frac{4}{3}\ln(|3t+1|)\Big|_{t=3}^{t=6} = \frac{4}{3}\ln(19) - \frac{4}{3}\ln(10)$$

$\approx .85$ thousand units

Production increased about 850 units between 3 and 6 months after release