Key

Ex 1 A particular manufacturer's productivity (in millions of units produced) is modeled well by the function $P(K, L) = 0.3(0.4K^{-0.5} + 0.6L^{-0.5})^{-2}$ where K is millions of dollars of capital investment, and L is thousands of worker-hours every month. Find and interpret the value of $P_K(10, 6)$.

$P(K, L) = 0.3(0.4K^{-0.5} + 0.6L^{-0.5})^{-2}$

chain rule

$P_K(K, L) = (0.3)(-2)(0.4K^{-0.5} + 0.6L^{-0.5})^{-3}(0.4(-0.5)K^{-1.5})$

$P_K(10, 6) \approx 0.074$

Production is increasing at a rate of 0.074 million units per million dollars of capital investment when \$10 million of capital and 6 thousand ~~the~~ worker-hours of labor are used each month

Ex 2 Local demand for grapefruit is given by $f(p, n) = 10 + \frac{5}{p+2} + 3e^{0.4n}$ while demand for oranges is $g(p, n) = 7 - \frac{4}{p+6} - 2n$, where each demand is given in thousands of units per month at p dollars per pound for grapefruit and n dollars per pound for oranges. Are grapefruit and oranges substitute, complementary, or neither?

$f_n(p, n) = 3e^{0.4n} \cdot 0.4 = 1.2e^{0.4n} > 0$

$g_p(p, n) = -\dfrac{(p+6) \cdot 0 - 4(1)}{(p+6)^2} = \dfrac{4}{(p+6)^2} > 0$

Since both f_n and g_p are positive for all $p, n \geq 0$, oranges and grapefruit are substitutes

Ex 3 Suppose that, at a particular factory, its output is given by the function $Q(K, L) = 60K^{0.3}L^{0.7}$ thousand units, where K represents capital investment in millions of dollars, and L represents thousands of workers. Suppose in addition, that the factory has \$3 million in capital investment and employs 5 thousand workers. Estimate the increase in output if another million dollars is invested in capital. Estimate the increase in output if another thousand workers are hired. Which would be a more effective way of increasing output?

$Q_K(K,L) = 60 \cdot 0.3 K^{-0.7} L^{0.7}$

$Q_K(3,5) \approx 25.7$ thousand units per million dollars

$Q_L(K,L) = 60 \cdot 0.7 K^{0.3} L^{-0.3}$

$Q_L(3,5) \approx 36.0$ thousand units per thousand worker-hours

Increasing labor is a more effective way of increasing output

Ex 4 Suppose you manage a store that has two brands, A and B, of the same product. Brand A costs x dollars per unit and brand B costs y dollars per unit. You know that demand for brand A is given by $Q(x,y) = 300 - 20x^2 + 30y$ units and you also know that the price of brand A will be $x = 2 + 0.05t$ and the price of brand B will be $y = 2 + 0.1\sqrt{t}$ in t months. At what rate is demand for brand A changing with respect to time in one month?

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial x}\frac{dx}{dt} + \frac{\partial Q}{\partial y}\frac{dy}{dt} = (-40x)(0.05) + (30)(0.1(\tfrac{1}{2})t^{-1/2})$$

when $t = 1$, $x = 2 + 0.05(1) = 2.05$, $y = 2 + 0.1\sqrt{1} = 2.1$

so $$\left.\frac{dQ}{dt}\right|_{t=1} = (-40 \cdot 2.05)(0.05) + (30)(0.1(\tfrac{1}{2})(1^{-1/2}))$$

$$= -2.6 \ \frac{\text{units}}{\text{month}}$$