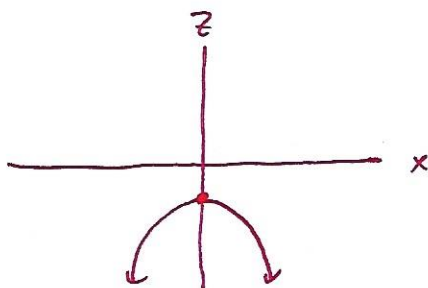**Ex 1** Consider the function $z = g(x, y) = yx^2 + y$. Sketch the following

(a) The trace of $g(x, y)$ at $y = -1$

$$z = g(x, -1) = (-1)x^2 + (-1) = -x^2 - 1$$

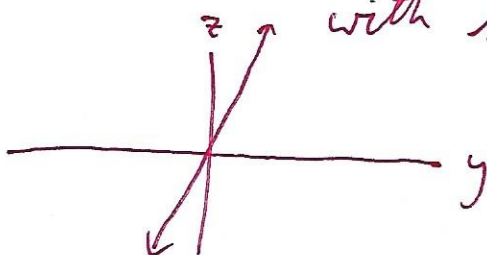$\hookrightarrow$ parabola with max at $x = \dfrac{-0}{2(-1)} = 0$

$$z = -0^2 - 1 = -1$$

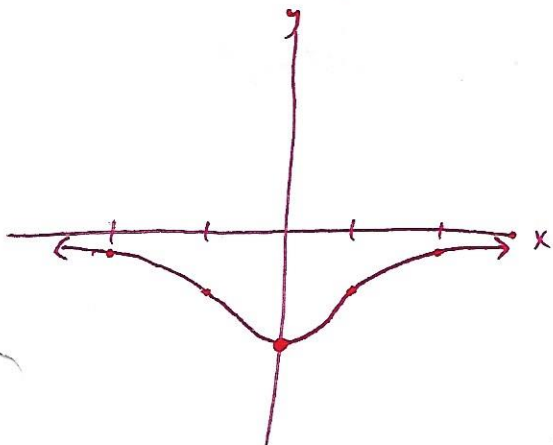(b) The trace of $g(x, y)$ at $x = -1$

$$z = g(-1, y) = y(-1)^2 + y = y + y = 2y$$

$\hookrightarrow$ line passing through origin with slope 2

(c) The level curve of $g(x, y)$ with $z = -1$

$$-1 = g(x, y) = yx^2 + y = y(x^2 + 1) \Rightarrow \dfrac{-1}{x^2 + 1} = y$$

x	y
-2	$-\frac{1}{5}$
-1	$-\frac{1}{2}$
0	-1
1	$-\frac{1}{2}$
2	$-\frac{1}{5}$

Ex 2 LotSW Incorporated produces two top-selling items: replicas of the sword Andúril and replicas of Jedi light sabres. Light sabre replicas cost \$25 each to produce while Andúril replicas cost \$35 each to produce. Light sabres sell for \$100 each while Andúril replicas sell for \$150 each. There is a fixed overhead cost of \$1500 associated with producing these items every month. Find the profit function and determine the profit obtained from producing and selling 50 swords and 75 light sabres.

a : number of Andúril replicas sold

ℓ : number of light sabre replicas sold

revenue: $R(a, \ell) = 100\ell + 150a$

cost : $C(a, \ell) = 25\ell + 35a + 1500$

profit: $P(a, \ell) = R(a, \ell) - C(a, \ell) = (100\ell + 150a) - (25\ell + 35a + 1500)$

The profit from selling 50 swords and 75 light sabres is $P(50, 75) = 9875$ dollars.

Ex 3 Recall that the present value of an investment, B, at interest rate, r, compounded continuously, for t years is given by the formula $P(B, r, t) = Be^{-rt}$. Compute $P(2000, .05, 3)$ and interpret the value in context.

$$P(2000, .05, 3) = 2000 \, e^{(-.05)(3)} \approx 1721.42$$

If you want to have \$2000 in 3 years, you should invest \$1721.42 now in an account with a 5% interest rate compounded continuously