Solutions

1. Use integration by substitution to show that $\int (t-2)\sqrt{t^2-4t}\,dt = \frac{1}{3}\left(t^2-4t\right)^{3/2} + C$.

   We cannot accomplish this using only our rules for functions of the form x^n and e^{kx}, so we turn to substitution.
   Let $u = t^2 - 4t$, so that $du = (2t-4)\,dt$. Then

$$\begin{aligned}
\int (t-2)\sqrt{t^2-4t}\,dt &= \int (t-2)\sqrt{u}\,\left(\frac{1}{2t-4}\,dt\right) \\
&= \int \cancel{(t-2)}\sqrt{u}\,\left(\frac{1}{2\cancel{(t-2)}}\,du\right) \\
&= \frac{1}{2}\int \sqrt{u}\,du \\
&= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C \\
&= \boxed{\frac{1}{3}\cdot(t^2-4t)^{3/2} + C}
\end{aligned}$$

2. Solve the initial value problem: $y'(t) = 4y + 10$ with $y(0) = 3.5$.

   We need to solve $y'(t) = 4y + 10$, or written with differentials, $\frac{dy}{dt} = 4y + 10$. So we begin by isolating differentials and then getting variables to match (using only multiplication or division!). Then on the left side, we use the substitution $u = 4y + 10$, so that $du = 4\,dy$, or $\frac{1}{4}\,du = dy$.

$$\begin{aligned} dy &= (4y+10)\,dt \\ \int \frac{1}{4y+10}\,dy &= \int dt \\ \int \frac{1}{u}\left(\frac{1}{4}\,du\right) &= \int dt \\ \frac{1}{4}\ln|u| &= t + C_1 \\ \ln|4y+10| &= 4t + C_2, \text{ where } C_2 = 4C_1 \\ |4y+10| &= e^{4t+C_2} = Ce^{4t}, \text{ where } C = e^{C_2} \end{aligned}$$

   Now we can solve for C using the initial condition $y(0) = 3.5$;

$$\begin{aligned} |4 \cdot 3.5 + 10| &= Ce^{4\cdot 0} \\ 24 &= C \end{aligned}$$

   So then

$$\boxed{|4y+10| = 24e^{4t}}$$

3. Consider $v(t) = 0.1t^3 - 4t^2 + 1200$.

   (a) Compute $\displaystyle\int_2^{14} v(t)\,dt$.

$$\begin{aligned}\int_2^{14} (0.1t^3 - 4t^2 + 1200)\,dt &= \left[0.1 \cdot \frac{t^4}{4} - 4 \cdot \frac{t^3}{3} + 1200t\right]\Big|_2^{14} \\ &= \left[0.1 \cdot \frac{14^4}{4} - 4 \cdot \frac{14^3}{3} + 1200(14)\right] - \left[0.1 \cdot \frac{2^4}{4} - 4 \cdot \frac{2^3}{3} + 1200(2)\right] \\ &= 11,712\end{aligned}$$

   (b) If $v(t)$ represents the rate of change in daily securities traded (thousands of shares per day), t days from now, then what does the value of the calculation from part (a) represent in context?

   The net change in daily securities traded between 2 days and 2 weeks (14 days) from now is 11,712 thousand shares.

4. An account increases in value at a rate of $A'(x)$ dollars per month, x months from now. Find the net change over the next year if $A'(x) = 300xe^{-0.05x^2}$.

   We need the net change from $x = 0$ to $x = 12$. We can find this using the substitution $u = -0.05x^2$, so that $du = -0.1x\,dx$, or $-10\,du = x\,dx$:

$$\begin{aligned}\int_0^{12} 300xe^{-0.05x^2}\,dx &= \int_{x=0}^{x=12} 300e^u\,(-10\,du) \\ &= \left[-3000e^u\right]\Big|_{x=0}^{x=12} \\ &= \left[-3000e^{-0.05x^2}\right]\Big|_0^{12} \\ &= \left[-3000e^{-0.05(12)^2}\right] - \left[-3000e^{-0.05(0)^2}\right] \\ &= 2,997.76 \text{ dollars}\end{aligned}$$