Key

Ex 1 Marginal sales for a company t years into the future are estimated to be given by $s(t) = \frac{160}{(t+1)^2}$ thousand units sold per year. Determine the net change in sales...

(a) ...over the next three years.

$$N.C. = \int_0^3 \frac{160}{(t+1)^2}\,dt = \int_{t=0}^{t=3} \frac{160}{u^2}\,du = \left.\frac{-160}{u}\right|_{t=0}^{t=3} = \left.\frac{-160}{t+1}\right|_{t=0}^{t=3}$$

$u = t+1$
$du = dt$

$= 120$ thousand units sold

(b) ...in the long run.

$$N.C. = \int_0^\infty \frac{160}{(t+1)^2}\,dt = \lim_{b\to\infty} \int_0^b \frac{160}{(t+1)^2}\,dt = \lim_{b\to\infty} \left. -\frac{160}{t+1}\right|_0^b$$

$$= \lim_{b\to\infty} \underbrace{-\frac{160}{b+1}}_{\to 0 \text{ as } b\to\infty} - \left(-\frac{160}{1}\right) = 160 \text{ thousand units}$$

Ex 2 When a political campaign puts out an ad, t weeks later, the advertisement is being seen at a rate of $600te^{-2t}$ thousand new viewers per week. How many new viewers should the political campaign expect to see the ad in the long run?

$$\int_0^\infty 600te^{-2t}\,dt = \lim_{b\to\infty} 600\int_0^b te^{-2t}\,dt = \lim_{b\to\infty} 600 \left.\frac{e^{-2t}}{-2}\left(t - \frac{1}{-2}\right)\right|_0^b$$

$$= \lim_{b\to\infty} \left. -300e^{-2t}\left(t+\frac{1}{2}\right)\right|_0^b = \lim_{b\to\infty} -300e^{-2b}\left(b+\frac{1}{2}\right) - \left(-300\cdot\frac{1}{2}\right)$$

$$= \lim_{b\to\infty} \underbrace{-300be^{-2b}}_{\to 0 \text{ as } b\to\infty} \underbrace{-150e^{-2b}}_{\to 0 \text{ as } b\to\infty} + 150 = 150 \text{ thousand new viewers}$$

Ex 3 Uday wishes to endow a scholarship at a local college with a gift that provides a continuous income stream at the rate of $25000 + 1200t$ dollars per year in perpetuity. Assuming the prevailing annual interest rate stays fixed at 5% compounded continuously, what donation is required to finance the endowment?

forever, so $T = \infty$

↳ "lump sum to fund an income stream" implies that we want the present value of the income stream

$$P.V. = \int_0^\infty (25000 + 1200t)e^{-.05t}\,dt$$

$$= \lim_{b\to\infty} \int_0^b 25000e^{-.05t} + 1200te^{-.05t}\,dt$$

$$= \lim_{b\to\infty} \frac{25000e^{-.05t}}{-.05} + \frac{1200e^{-.05t}}{-.05}\left(t - \frac{1}{-.05}\right)\Big|_0^b$$

$$= \lim_{b\to\infty} -500000e^{-.05t} - 240000e^{-.05t}(t+20)\Big|_0^b$$

$$= \lim_{b\to\infty} -500000e^{-.05b} - 240000e^{-.05b}\cdot b - 4800000e^{-.05b} - \left(-500000 - 240000(0+20)\right)$$

$$= 500000 + 480000 = 980000$$

↳ since each term with a "b" in it approaches 0 as $b \to \infty$.