

Ex 1 Suppose you have two investments: Plan 1 increases in value at a rate of $500e^{0.01t}$ dollars per day. Plan 2 increases at a rate of $100e^{0.03t}$ dollars per day. Find the net excess profit between the two plans from now until the plans are growing at the same instantaneous rate.

$$P_1'(t) = 500e^{0.01t}$$

$$P_2'(t) = 100e^{0.03t}$$

Plans grow at same rate when $P_1'(t) = P_2'(t)$,

$$\text{ie. } 500e^{0.01t} = 100e^{0.03t} \rightarrow \frac{500}{100} = \frac{e^{0.03t}}{e^{0.01t}} \rightarrow 5 = e^{0.02t}$$

$$\rightarrow \ln(5) = 0.02t \rightarrow t = \frac{\ln(5)}{0.02} = 50 \ln(5) \approx 80.5$$

$$NEP = \int_0^{80.5} P_1'(t) - P_2'(t) dt = \int_0^{80.5} 500e^{0.01t} - 100e^{0.03t} dt$$

$$= \left. \frac{500e^{0.01t}}{0.01} - \frac{100e^{0.03t}}{0.03} \right|_0^{80.5}$$

$$\approx 27,868.93 \text{ dollars}$$

Ex 2 Find the Gini Index for the United States, with $L(x) = 1.4x^3 - 0.86x^2 + 0.41x$. By this metric, does the US have more or less income inequality than Venezuela, which the CIA considers to have a Gini Index of 0.39? (Note: these numbers are not accurate: they are made up to suit the problem).

$$\begin{aligned}
 GI &= \int_0^1 x - (1.4x^3 - 0.86x^2 + 0.41x) dx \\
 &= \int_0^1 -1.4x^3 + 0.86x^2 + .59x dx \\
 &= \left. -\frac{1.4x^4}{4} + \frac{0.86x^3}{3} + \frac{.59x^2}{2} \right|_0^1 \\
 &\approx 0.456
 \end{aligned}$$

Compared to Venezuela, the US has more income inequality (i.e. less equality).

Ex 3 The quarterly revenue for the Amazon Web Service over the past several years can be predicted roughly by

$$Q(t) = 32.4t^2 + 11.6t + 50$$

million dollars, t years after the beginning of 2009. What was the average quarterly revenue for AWS between the beginning of years 2012 and 2015 according to the model?

$$2012 \rightsquigarrow t=3$$

$$2015 \rightsquigarrow t=6$$

$$\begin{aligned}
 AV &= \frac{1}{6-3} \int_3^6 32.4t^2 + 11.6t + 50 dt \\
 &= \frac{1}{3} \left(\frac{32.4t^3}{3} + \frac{11.6t^2}{2} + 50t \right) \Bigg|_{t=3}^{t=6} \\
 &\approx 781.76 \text{ million dollars}
 \end{aligned}$$