Homework 2; 1.4A (1a–h, 2a–c, 3, 4, 9, 11)
2 extra problems

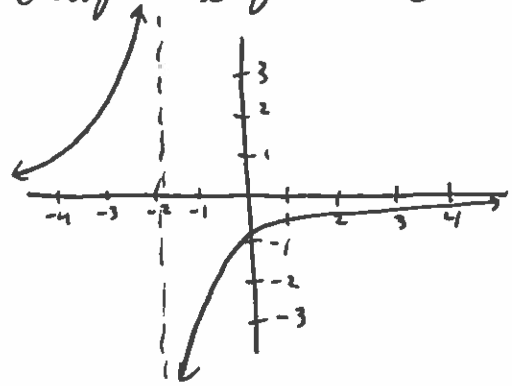
1.4A

1a) (i) $p(x) = \frac{1}{x}$

(ii) $f(x) = \frac{3}{2x+4} = \frac{3}{2(x+2)} \Rightarrow A = 3 \quad h = -2$
$B = 2 \quad k = 0$

(iii) Stretch vertically by a factor of 3
Stretch horizontally by a factor of ½
Shift left two units

(iv)

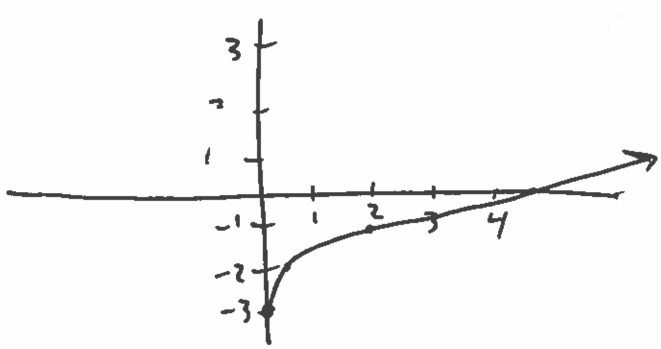


b) (i) $p(x) = \sqrt{x}$

(ii) $f(x) = \sqrt{2x} - 3 \Rightarrow A = 1 \quad h = 0$
$B = 2 \quad k = -3$

(iii) Stretch horizontally by a factor of ½
Shift down 3 units

(iv)

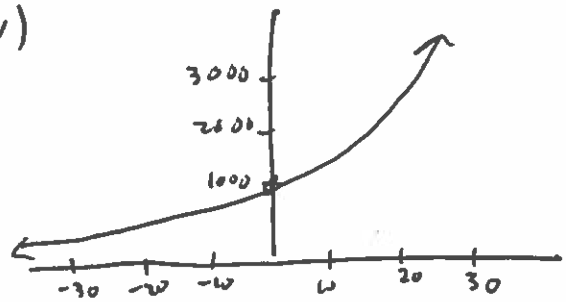


c) (i) $p(x) = e^x$

(ii) $f(x) = 1000e^{.05x} \Rightarrow A = 1000 \quad h = 0$
$B = .05 \quad k = 0$

(iii) Stretch vertically by a factor of 1000
Stretch horizontally by a factor of $\frac{1}{.05} = 20$

1c)(iv)

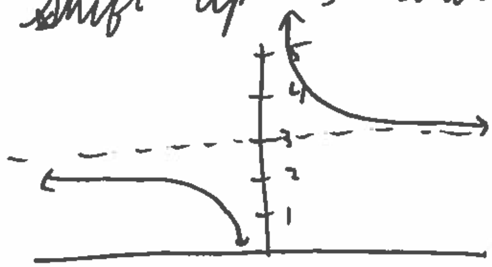


d) (i) $p(x) = \frac{1}{x}$

(ii) $f(x) = \frac{2}{x} + 3 \Rightarrow$ $A = 2$, $B = 1$, $h = 0$, $k = 3$

(iii) Stretch vertically by a factor of 2
Shift up 3 units

(iv)

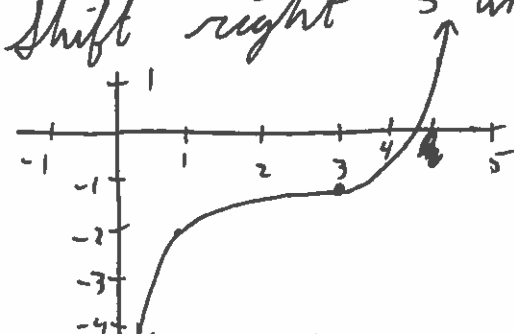


e)(i) $p(x) = x^3$

(ii) $f(x) = \frac{1}{4}(x-3)^3 - 1 \Rightarrow$ $A = \frac{1}{4}$, $B = 1$, $h = 3$, $k = -1$

(iii) Stretch vertically by a factor of $\frac{1}{4}$
Shift ~~w~~ down 1 unit
Shift right 3 units

(iv)

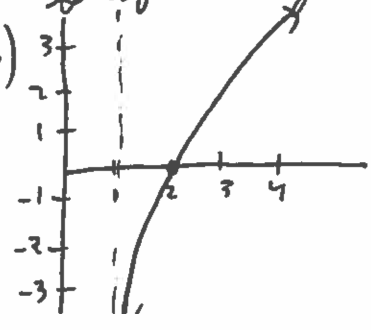


f) (i) $p(x) = \ln(x)$

(ii) $f(x) = \ln(x^3 - 3x^2 + 3x - 1) = \ln((x-1)^3) = 3\ln(x-1) \Rightarrow$ $A = 3$, $B = 1$, $h = 1$, $k = 0$

(iii) Stretch vertically by a factor of 3
Shift right 1 unit

(iv)



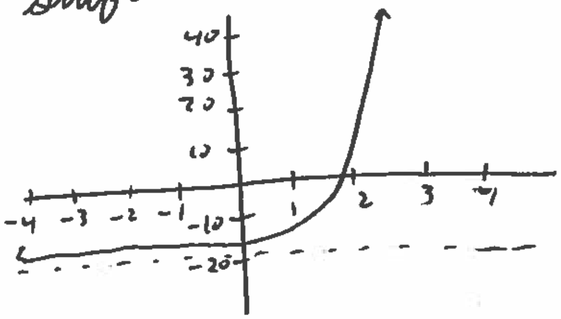
1g) (i) $p(x) = e^x$

(ii) $f(x) = 3(e^x - 6) = 3e^x - 18 \Rightarrow$ $A = 3$, $B = 1$, $h = 0$, $k = -18$

(iii) Stretch vertically by a factor of 3

Shift down 18 units

(iv)

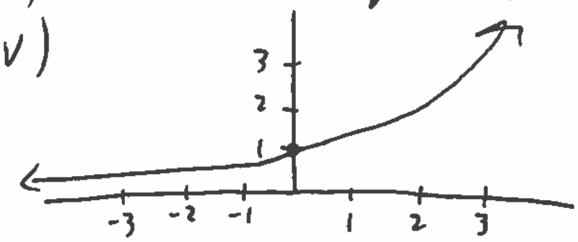


h) (i) $p(x) = e^x$

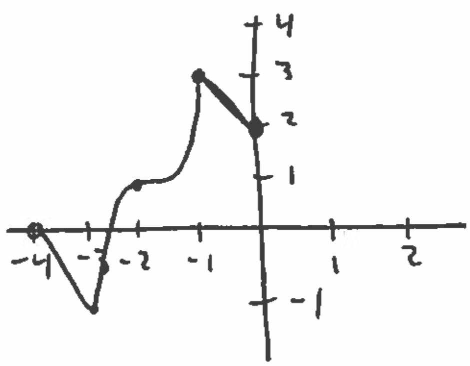
(ii) $f(x) = 1.4^x = \left(e^{\ln(1.4)}\right)^x = e^{\ln(1.4)x} \Rightarrow$ $A = 1$, $B = \ln(1.4)$, $h = 0$, $k = 0$

(iii) Stretch horizontally by a factor of $1/\ln(1.4)$

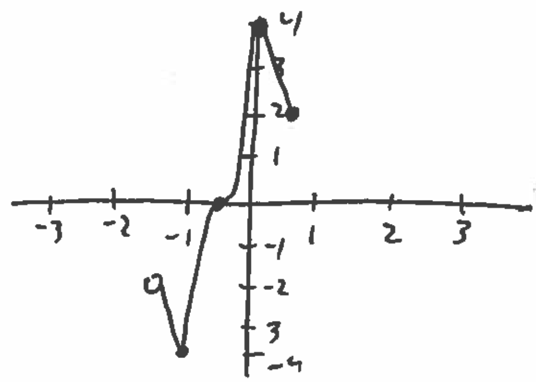
(iv)



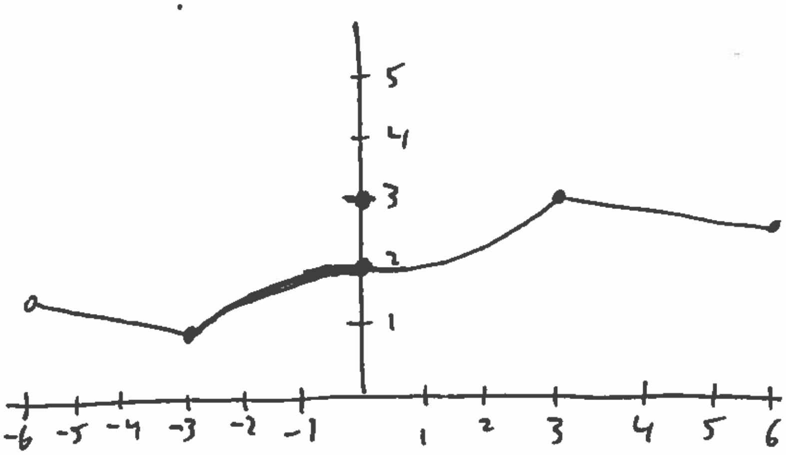
2 a)



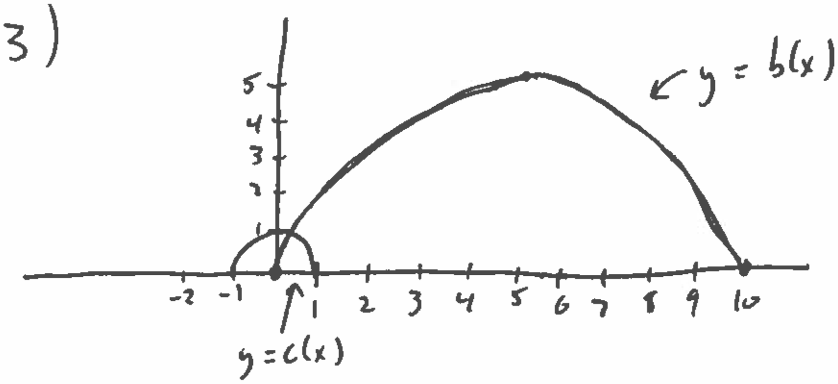
b)



c)



3)



- we want to stretch $c(x)$ vertically and horizontally by a factor of 5
- we want to shift $c(x)$ to the right 5 units
- so $b(x) = 5c\left(\frac{1}{5}(x-5)\right) = 5\sqrt{1-\left(\frac{1}{5}(x-5)\right)^2} = 5\sqrt{1-\left(\frac{1}{25}(x^2-10x+2\right.}$

$$= \sqrt{25\left(1-\frac{1}{25}(x^2-10x+25)\right)} = \sqrt{25-(x^2-10x+25)} = \sqrt{10x-x^2}$$

4) - we need a horizontal stretch by a factor of $1/2$, and a vertical stretch by a factor of $-1/2$
- after stretching, we need to shift up 1 unit and right 1.5 units
- so $f(x) = -\frac{1}{2}\,P\left(2(x-1.5)\right) + 1$

9a) $h(x) = P(3(x+5))$ is a horizontal stretch by a factor of $\frac{1}{3}$, then shift left by 5 units
. Since horizontal transformations affect x-values, the domain will be altered, but the range / image will be untouched
- the domain of P is $[-6, 24]$, so the domain of P will be from $\frac{1}{3}(-6)-5 = -7$ to $\frac{1}{3}(24)-5 = 3$
- domain: $[-7, 3]$, range: $[-4, 9)$

9b) $v(x) = 2P(x) + 4$ is a vertical stretch by a factor of 2, then a shift up by 4 units

- vertical transformations affect y-values only, so the range of v will be altered, but the domain will be untouched
- the range of P is $[-4, 9]$, so the range of v is from $2(-4)+4 = -4$ to $2(9)+4 = 22$
- domain: $[-6, 24]$, range: $[-4, 22]$

c) $T(x) = 2P(3(x+5))+4$ is a combination of the transformations in parts a and b, with the same alterations to the domain and range. So...

- domain: $[-7, 3]$, range: $[-4, 22]$

11a) $Q(t) = P(t) + 500$

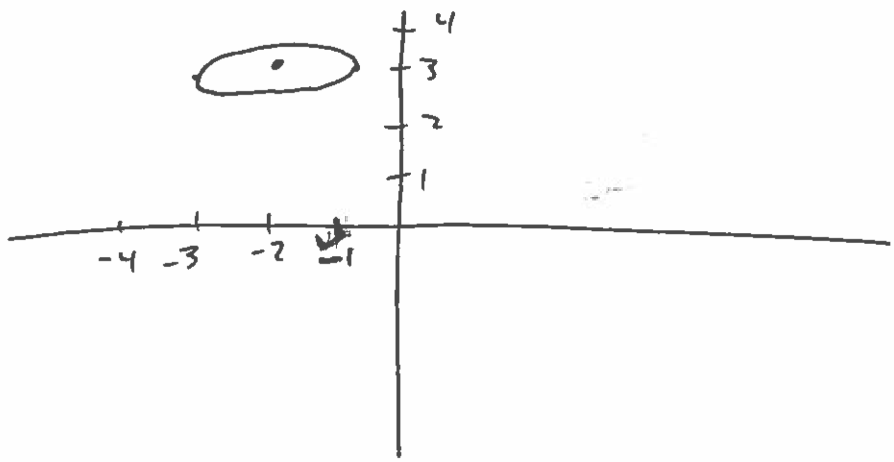
b) $R(t) = 1.4P(t)$

c) $Q(t) = P(t+15)$

d) The new plan produces toys half as fast as the normal production plan would dictate

Extra Problems

1) We have a horizontal reflection (which has no effect!), vertical stretch by a factor of $1/3$, shift left 2 and up 3. So the final graph looks like



2) The line has slope $\dfrac{5-(-2)}{4-1} = \dfrac{7}{3}$

So point slope form gives us two forms of the equation right away:

$$y - 5 = \frac{7}{3}(x-4) \quad \text{and} \quad y + 2 = \frac{7}{3}(x-1)$$

To get a third form, we need a third point. Plugging $x = 0$ into either of the above formulas gives $y = \frac{-13}{3}$, so we have $y + \frac{13}{3} = \frac{7}{3}(x-0)$ as well