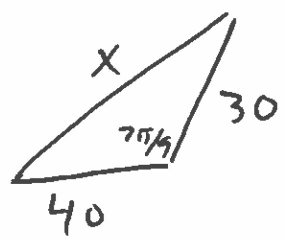
HW 7 Key

3.2 (1-4, 6, 9)

3.3 (3-6, 11)

Section 3.2

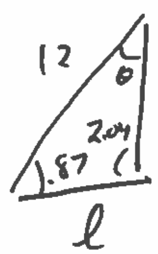
1) 

By the law of cosines,

$$x^2 = 30^2 + 40^2 - 2 \cdot 30 \cdot 40 \cos\left(\frac{7\pi}{9}\right)$$

$$\approx 4338.5$$

so $x \approx 65.87$

2) 

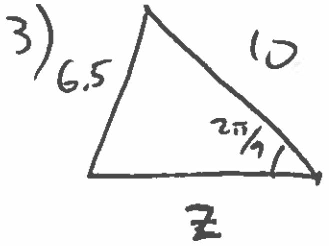
The third angle is

$$\theta = \pi - .87 - 2.04 \approx .23$$

By the law of sines, $\frac{\sin(.23)}{l} = \frac{\sin(2.04)}{12}$

$$\rightarrow l = \frac{12 \sin(.23)}{\sin(2.04)} \approx 3.09$$

3)



Since we only know one angle, and we don't have all 3 sides, we have to use the law of cosines

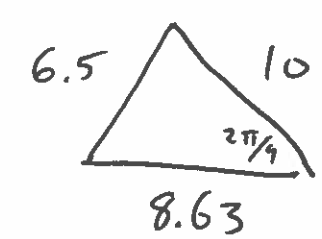
$$6.5^2 = z^2 + 10^2 - 2z \cdot 10\cos\left(\tfrac{2\pi}{9}\right)$$

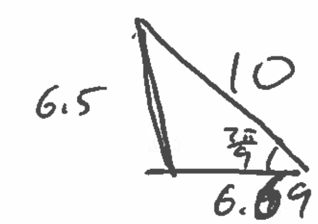
$$\rightarrow\ 0 = z^2 - 15.32z + 57.75$$

$$\rightarrow\ z = \frac{15.32 \pm \sqrt{15.32^2 - 4 \cdot 57.75}}{2}$$

$$\approx 8.63,\ 6.69$$

To see which one is correct, note that our two possible triangles are

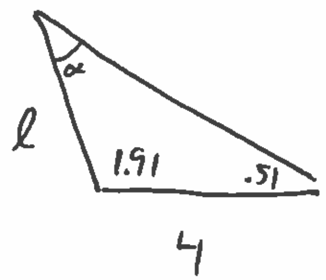




This one is the only one with all acute angles, which we are told to assume, so

$$z = 8.63$$

4)



The final angle is

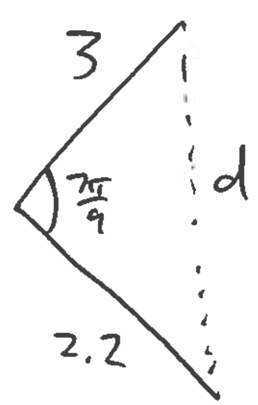
$$\alpha = \pi - 1.91 - .51 \approx .72$$

The law of sines tells us that

$$\frac{\sin(.72)}{4} = \frac{\sin(.51)}{l}$$

$$\rightarrow l = \frac{4\sin(.51)}{\sin(.72)} \approx 2.96$$

6)



By the law of cosines,

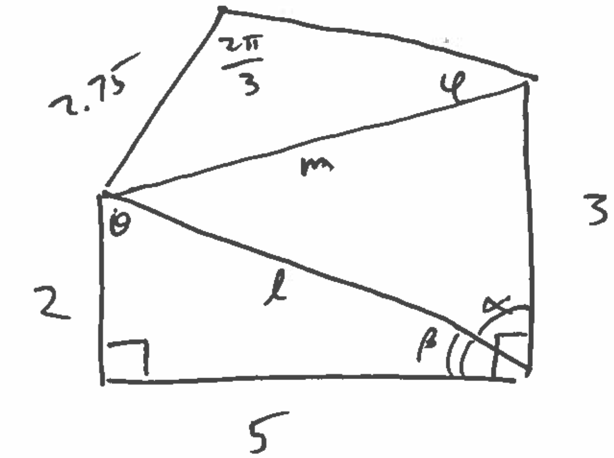
$$d^2 = 3^2 + 2.2^2 - 2\cdot 3\cdot 2.2\cos\left(\tfrac{7\pi}{9}\right)$$

$$\approx 23.95$$

so $d \approx 4.89$

After 10 minutes, they are around 4.89 miles apart

9a)



$2^2 + 5^2 = \ell^2 \rightarrow \quad \ell = \sqrt{29} \approx 5.39$

b) $\cos\theta = \dfrac{2}{\ell} \approx .37$

Since $0 < \theta < \pi$, $\quad \theta = \arccos(.37) \approx 1.19$

c) The angles α and β in the diagram have $\alpha + \beta = \pi/2$

Also, $\beta + \theta + \pi/2 = \pi \rightarrow \beta = \pi - \pi/2 - \theta \approx .38$

Hence, $\alpha = \pi/2 - \beta \approx 1.19$

(Aside: note that $\alpha = \theta$. Why?)

Then by the law of cosines,

$$m^2 = \ell^2 + 3^2 - 2 \cdot 3 \cdot \ell \cos(\alpha)$$
$$= 26$$

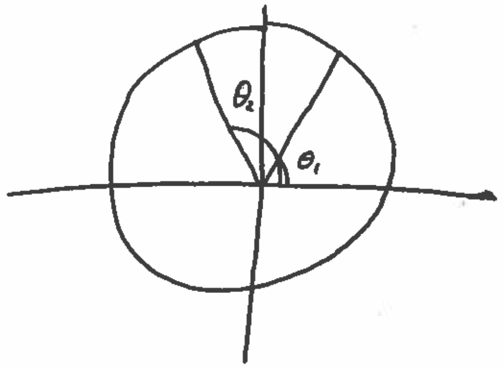
so $m = \sqrt{26} \approx 5.10$

d) By the law of sines, $\dfrac{\sin(\varphi)}{2.75} = \dfrac{\sin\left(\frac{2\pi}{3}\right)}{m}$

$\rightarrow \sin(\varphi) \approx .47 \rightarrow \varphi \approx \arcsin(.47) \approx .49$

This last step is possible since a triangle can have at most one obtuse angle, so φ must be acute

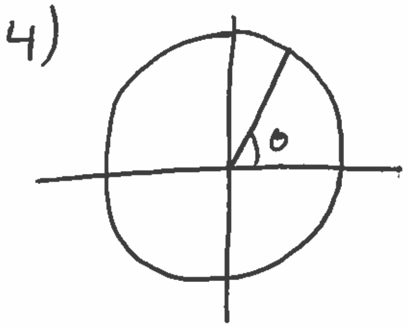
Section 3.3

3) 

There are two points on the unit circle with y-coordinate $\frac{\sqrt{3}}{2}$. The corresponding angles are $\theta_1 = \frac{\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$

Since sin is 2π-periodic, the solutions all have the form

$$\frac{\pi}{3} + 2\pi n$$
or $\qquad$ for some integer n
$$\frac{2\pi}{3} + 2\pi n$$

4) 

There is one angle θ between $-\pi/2$ and $\pi/2$ with $\tan(\theta) = \sqrt{3}$, namely, $\pi/3$

Since tan is π-periodic, all solutions have the form $\pi/3 + \pi n$ for integers n. To see which solutions are between -2π and 2π, we list the solutions .... $-8\pi/3$, $-5\pi/3$, $-2\pi/3$, $\pi/3$, $4\pi/3$, $\frac{7\pi}{3}$ and so on.

The solutions between -2π and 2π are $-5\pi/3$, $-2\pi/3$, $\pi/3$, $4\pi/3$

5) $4\sin(\theta)+3=1 \rightarrow 4\sin\theta=-2 \rightarrow \sin\theta=-\frac{1}{2}$

There are two points on the unit circle with y-coordinate $-1/2$, namely $7\pi/6$ and $11\pi/6$

Since sin is 2π-periodic all solutions have the form

$$7\pi/6 + 2\pi n \quad \text{or} \quad 11\pi/6 + 2\pi n \quad \text{for some integer } n$$

6) $\frac{2\cos(\theta)-1}{3}=-1 \rightarrow 2\cos(\theta)-1=-3 \rightarrow 2\cos(\theta)=-2$

$\rightarrow \cos(\theta)=-1$

There is only one point on the unit circle with x-coordinate -1. Its corresponding angle is π.

Since cos is 2π-periodic, all solutions have the form $\pi + 2\pi n$ for some integer n

11) $500\sin\left(\frac{\pi}{5}(t-3)\right)+8000=8400 \rightarrow 500\sin\left(\frac{\pi}{5}(t-3)\right)=400$

$\rightarrow \sin\left(\frac{\pi}{5}(t-3)\right)=\frac{4}{5}$. Our two principal solutions to this are $\theta_1=\arcsin\left(\frac{4}{5}\right)\approx .93$ and $\theta_2=\pi-\theta_1\approx 2.21$

So we have
$$\frac{\pi}{5}(t-3)=.93+2\pi n \rightarrow t=4.48+10n$$
$$\frac{\pi}{5}(t-3)=2.21+2\pi n \rightarrow t=6.52+10n$$

The t values between 0 and 20 are $t=4.48, 14.48, 6.52, 16.52$, which corresponds to the years 2004.48, 2014.48, 2006.52, 2016.52