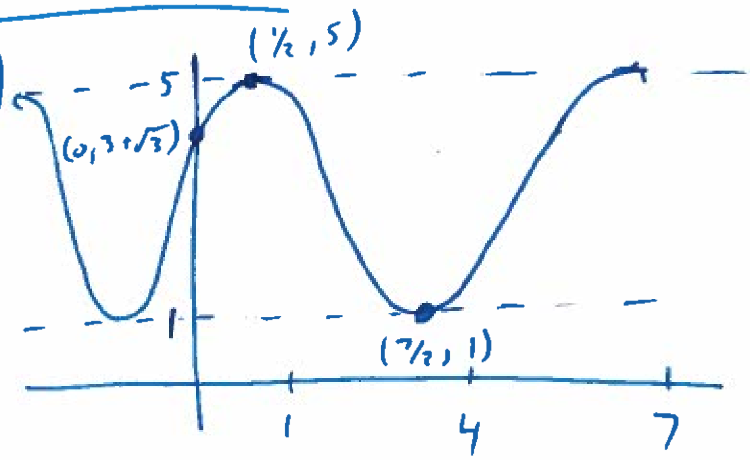
HW 8; 3.4A(1–4, 6, 9)
3.5A(1, 6)
4.1A(1–4, 8)
Extra Problems
Extra Credit: 4.1.C1

Section 3.4

1)



2a) 2000

b) $y = -1500$

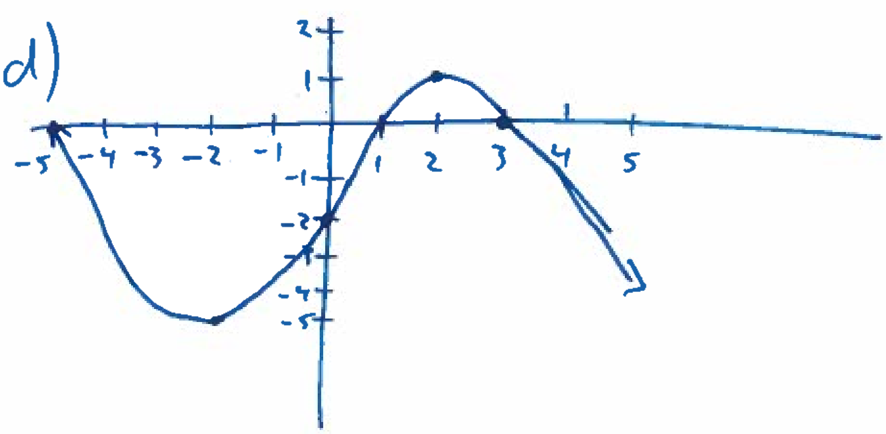
c) period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{(\pi/12)} = \dfrac{12}{\pi}(2\pi) = 24$

3a) 3

b) $y = -2$

c) period $= \dfrac{2\pi}{(\pi/4)} = \dfrac{4}{\pi}(2\pi) = 8$

d)

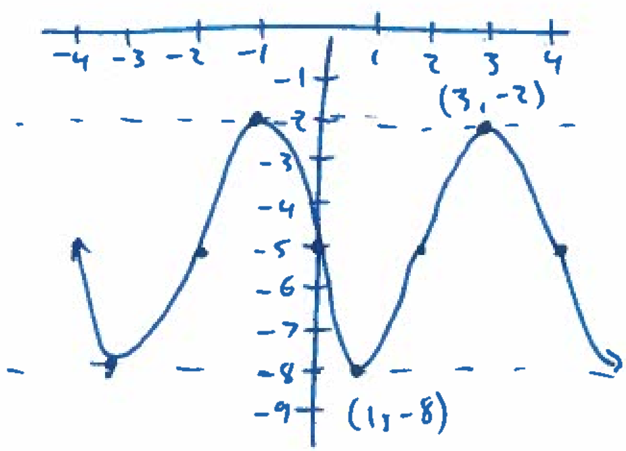


4a) 3

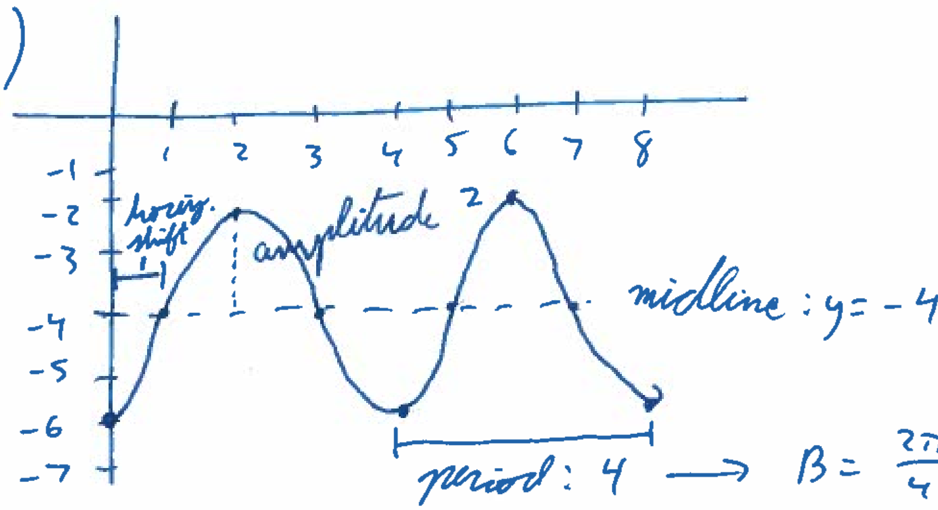
b) $y = -5$

c) period $= \dfrac{2\pi}{(\pi/2)} = 4$

d)



6)



$\longrightarrow B = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

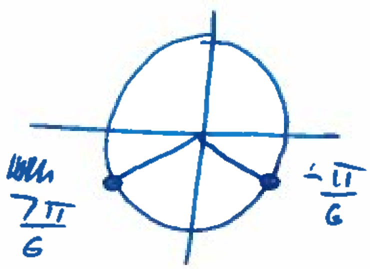
$$p(x) = 2\sin\left(\frac{\pi}{2}(x-1)\right) - 4$$

9) We get $f(x) = 26\sin\left(\frac{\pi}{4}(x-h)\right) + 7$ for some h from the first three piece of information

$f(\frac{8}{3}) = -6$ tells us $-6 = 26\sin\left(\frac{\pi}{4}(\frac{8}{3}-h)\right) + 7$,

so $-13 = 26\sin\left(\frac{\pi}{4}(\frac{8}{3}-h)\right)$

$$-\frac{1}{2} = \sin\left(\frac{\pi}{4}\left(\frac{8}{3}-h\right)\right)$$



Since $\sin(\theta)$ is decreasing at $\theta = \frac{7\pi}{6}$ and increasing at $-\frac{\pi}{6}$ and since $f(x)$ is decreasing at $x = \frac{8}{3}$, we must have

$$\frac{\pi}{4}\left(\frac{8}{3}-h\right) = \frac{7\pi}{6} \to \frac{8}{3} - h = \frac{14}{3} \to \frac{8}{3} = \frac{14}{3} + h$$

$$\to -2 = h$$

So $f(x) = 26\sin\left(\frac{\pi}{4}(x+2)\right) + 7$

Section 3.5

1a) $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{1}{3}\right)^2 + \sin^2\theta = 1$$

$$\sin^2\theta = \frac{8}{9}$$

since $\sin\theta < 0$, $\sin\theta = \frac{-\sqrt{8}}{3} = \frac{-2\sqrt{2}}{3}$

b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{2\sqrt{2}}{3}\right)}{(1/3)} = -2\sqrt{2}$

c) $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{2}}$

d) $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{(1/3)} = 3$

e) $\csc\theta = \frac{1}{\sin\theta} = \frac{-3}{2\sqrt{2}}$

6) $\csc(\theta) = -\frac{2}{\sqrt{3}} \rightarrow \frac{1}{\sin(\theta)} = -\frac{2}{\sqrt{3}} \rightarrow \sin\theta = \frac{-\sqrt{3}}{2}$

$$\rightarrow \theta = \frac{4\pi}{3} + 2\pi n$$

$$\theta = \frac{5\pi}{3} + 2\pi n$$

for integers n

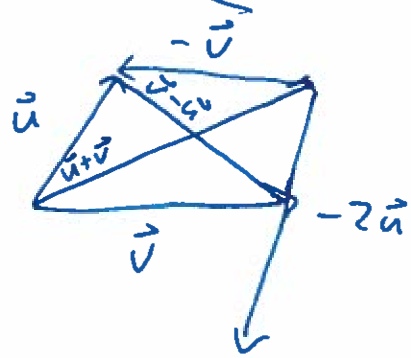
Section 4.1

1) False

not the same vector

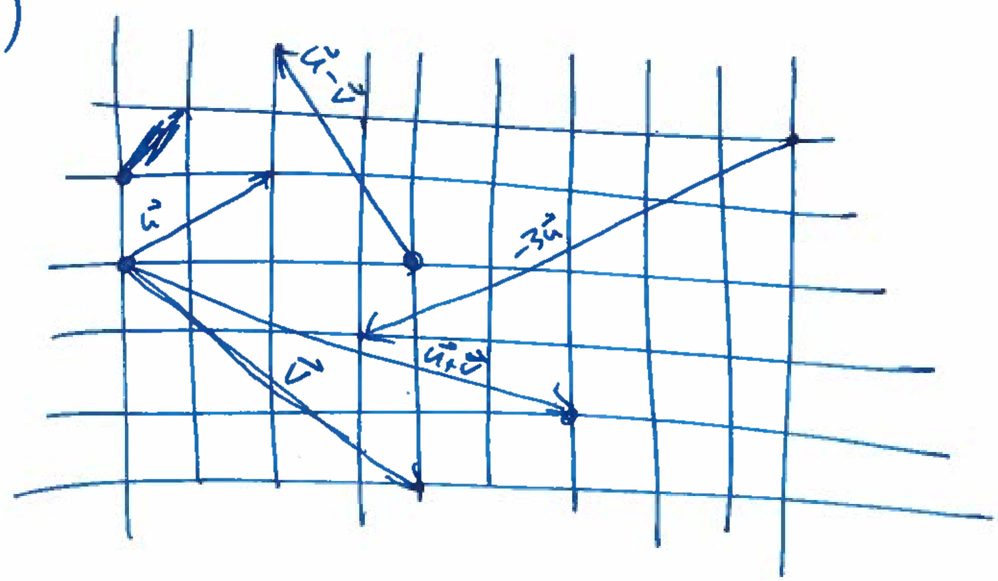
2) True

3)

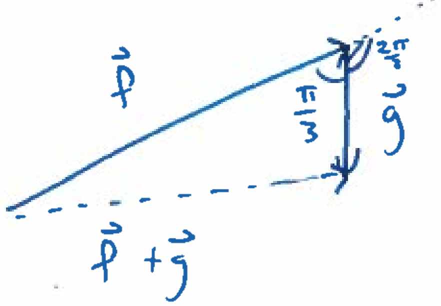


so D: $\vec{u}-\vec{v}$ is not drawn

4)



8)



$$\|\vec{f}+\vec{g}\|^2 = \|\vec{f}\|^2 + \|\vec{g}\|^2 - 2\|\vec{f}\|\,\|\vec{g}\|\cos\left(\frac{\pi}{3}\right)$$
$$= 1225$$
$$\rightarrow \|\vec{f}+\vec{g}\| = 35$$

Extra Problems

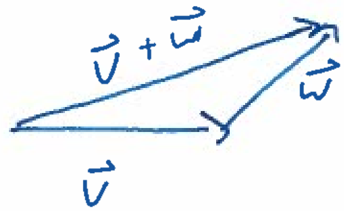
1) $$\cos^2\theta\sin^2\theta = \left(\frac{1+\cos(2\theta)}{2}\right)\left(\frac{1-\cos(2\theta)}{2}\right) = \frac{1}{4}\left(1-\cos^2(2\theta)\right)$$
$$= \frac{1}{4}\left(1-\left(\frac{1+\cos(4\theta)}{2}\right)\right)$$

2) $$\sin^2\left(\frac{\pi}{12}\right) = \frac{1-\cos\left(\frac{\pi}{6}\right)}{2} = \frac{1-\sqrt{3}/2}{2} = \frac{2-\sqrt{3}}{4}$$

$\rightarrow \sin\left(\frac{\pi}{12}\right) = +\sqrt{\frac{2-\sqrt{3}}{4}}$ where we take the positive square root because $\frac{\pi}{12}$ is in the first quadrant

Extra Credit

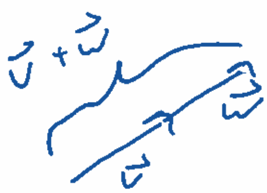
a)



Because the sum of the lengths of any two sides of a triangle must be at least as large as the third,

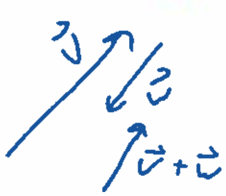
$$\|\vec{v}\| + \|\vec{w}\| \geq \|\vec{v}+\vec{w}\|$$

b) For a proper triangle, the above inequality is strict. The only way we can have equality is if we don't get a triangle, i.e.



if $\vec{v}$ and $\vec{w}$ point in the same direction

c) The smallest $\vec{v}+\vec{w}$ can be is if $\vec{v}$ and $\vec{w}$ lie in opposite directions, so we have



$$\|\vec{v}+\vec{w}\| \geq \big|\, \|\vec{v}\| - \|\vec{w}\| \,\big|$$