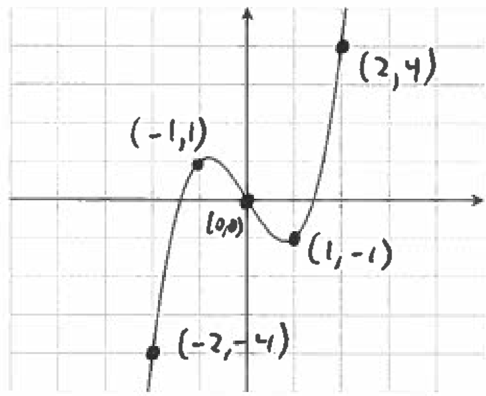
Key

## WEEK 2 HANDOUT: TRANSFORMATIONS OF FUNCTIONS...

### 1.4. Combinations of Transformations

(1) Let $f$ be given by the graph below.

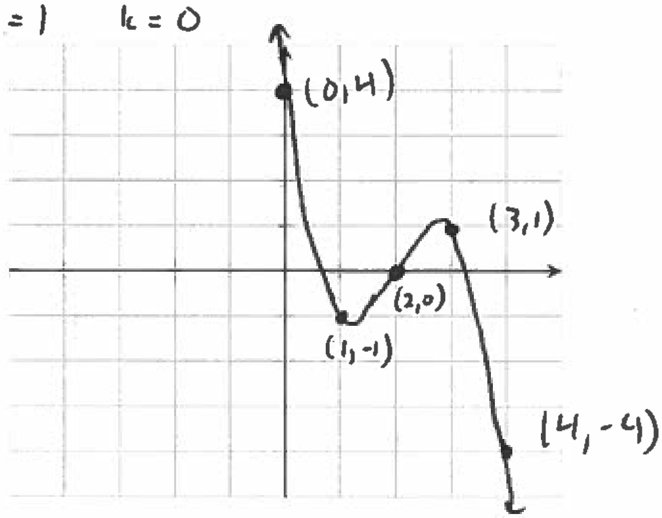
recall transformation form $A f(B(x-h)) + k$




Sketch the following transformations.

(a) $-f(x-2)$ → $A=-1$ $h=2$ $B=1$ $k=0$

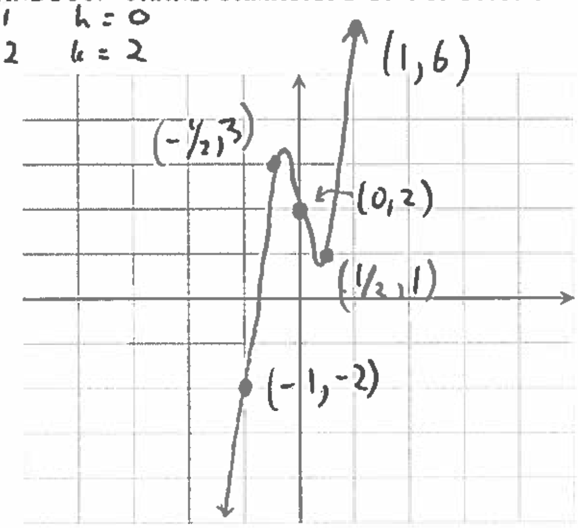
1) vertical reflection
2) shift right 2



(b) $f(2x)+2 \to$ $A=1$ $h=0$, $B=2$ $k=2$

1) horizontal stretch by a factor of $1/2$

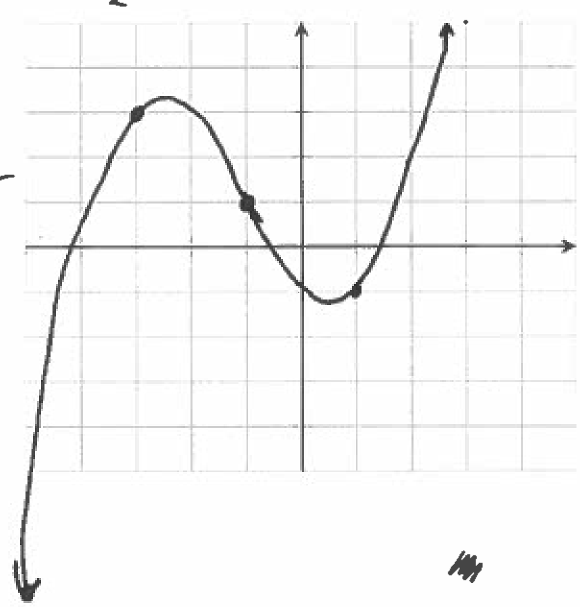
2) shift up 2




$2f\left(\frac{1}{2}(x+1)\right)+1$

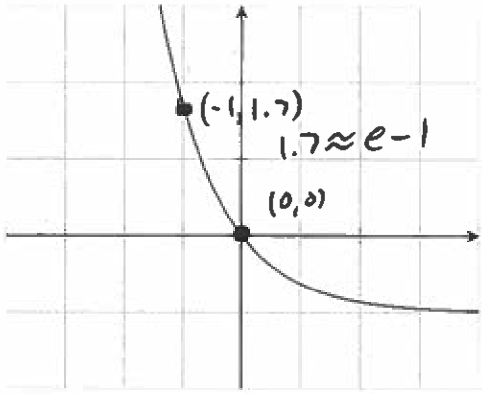
(c) $2f\left(\frac{x+1}{2}\right)+1 \to$ $A=2$ $h=-1$, $B=\frac{1}{2}$ $k=1$

1) vertical stretch by factor of 2
2) horizontal stretch by a factor of 2
3) vertical shift up 1 unit
4) horizontal shift left 1 unit



(2) Find an equation for each of the graphs below.

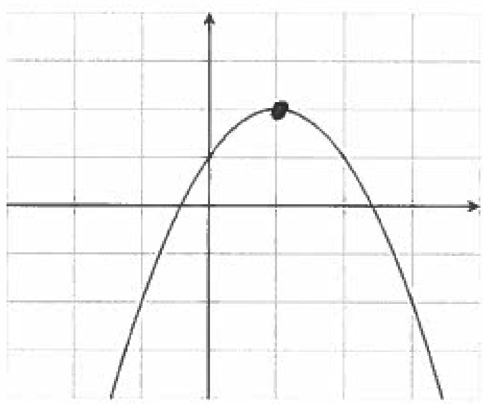
(a)




parent function : $e^x = p(x)$

- we certainly have a horizontal reflection and a shift down 1 unit
- probably no stretching since going left 1 unit from (0,0) means going up $\approx e-1$ on this graph, and going right 1 unit from (0, 1) on the graph of $e^x$ means going up $e-1$.
- so we have $A=1$, $B=1$, $h=0$, $k=-1$
  $\rightarrow f(x) = e^{-x} - 1$

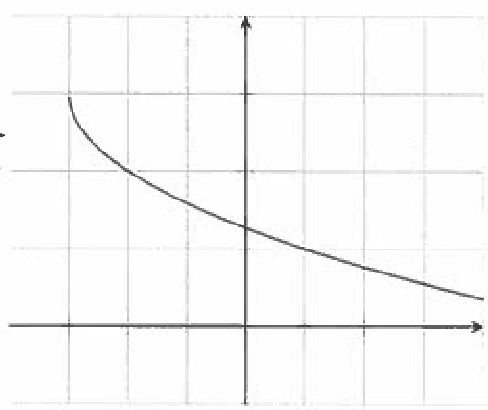
(b)



parent function $p(x) = x^2$

- we have a vertical reflection and a shift up 2 and right 1
- there shouldn't be any other stretching since going right one unit from the vertex means going down one unit here, and going right one ~~in~~ unit ~~the~~ from the vertex of ~~a~~ the parent function means going ~~[illegible]~~ up one unit.
- so $A=-1$, $B=1$, $h=1$, $k=2$
  $\rightarrow f(x) = -(x-1)^2 + 2$

(c)



parent function $p(x) = \sqrt{x}$

- we have vertical reflection and shift up 3 and left 3
- there is no other stretching for similar reasons to (a) and (b)
- so $A=-1$, $B=1$, $h=3$, $k=3$ $\rightarrow f(x) = -\sqrt{x-3} + 3$

## Application

(1) Recall our long-term model for the US inventory of plutonium,

$$A(T) = 95.4\left(\frac{1}{2}\right)^{.41T},$$

typo: T represents hundreds of thousands of years after 2006

where $A$ is the amount of plutonium in MT and $T$ is measured in hundreds of thousands of years. Say that we want to figure out how long it will take for the plutonium to completely decay. Since this process is so slow, we should jump far ahead into the future. Rewrite our model to begin one million years after 2006.

One million years = 10 hundred thousand years, so we should consider

$$B(T) = A(T+10) = 95.4\left(\tfrac{1}{2}\right)^{.41(T+10)}$$

(2) As our new model measures decay after many years, it will be a good idea to use a smaller unit of measurement than metrics tonnes to increase accuracy. Rewrite our new model to measure in kilograms, recalling that 1 metric ton is precisely 1,000 kilograms.

$B(T)$ has output units of MT and since there are $1000\,\frac{kg}{MT}$, we should consider

$$C(T) = 1000\,B(T) = 9540\left(\tfrac{1}{2}\right)^{.41(T+10)}$$

(3) Accounting only for decay, when will there be exactly one kilogram of plutonium left in our inventory?

There will be 1 kg left when

$$1 = C(T) = 9540\left(\tfrac{1}{2}\right)^{.41(T+10)}$$

$$\rightarrow \frac{1}{9540} = \left(\tfrac{1}{2}\right)^{.41(T+10)} \rightarrow \log_{1/2}\left(\frac{1}{9540}\right) = .41(T+10)$$

$$\rightarrow \frac{1}{.41}\log_{1/2}\left(\frac{1}{9540}\right) = T+10 \rightarrow T = \frac{1}{.41}\log_{1/2}\left(\frac{1}{9540}\right) - 10$$

$$\approx$$