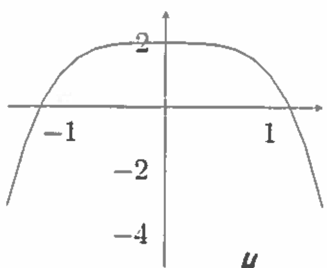
Key

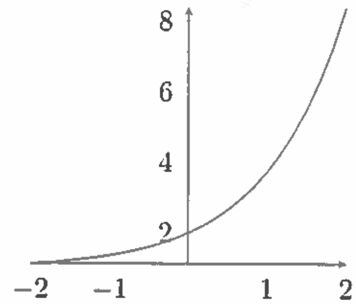
WEEK 1 HANDOUT:
TRANSFORMATIONS OF FUNCTIONS

1.1. Parent Functions

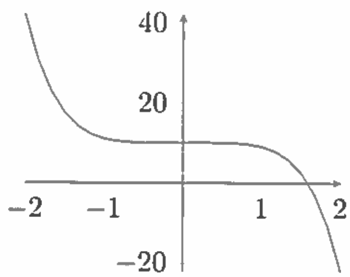
(1) Identify a possible parent function for each of the graphs below.



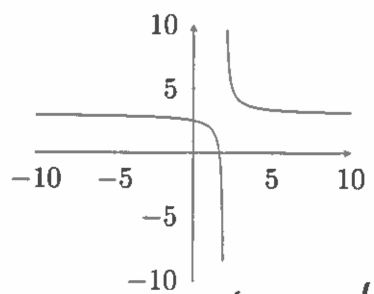
$p(x) = x^4$



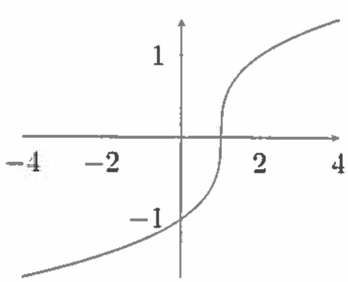
$p(x) = e^x$



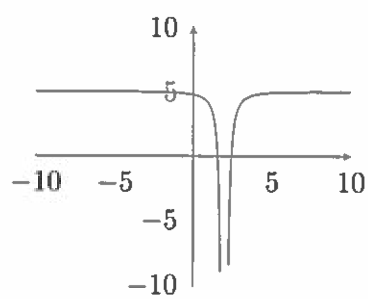
$p(x) = x^3$



$p(x) = \frac{1}{x}$



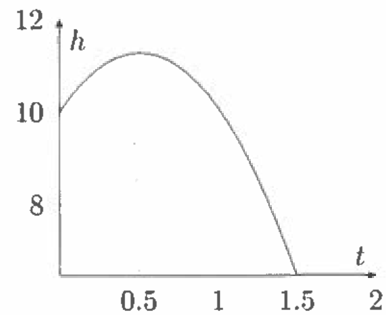
$p(x) = \sqrt[3]{x}$



$p(x) = \frac{1}{x^2}$

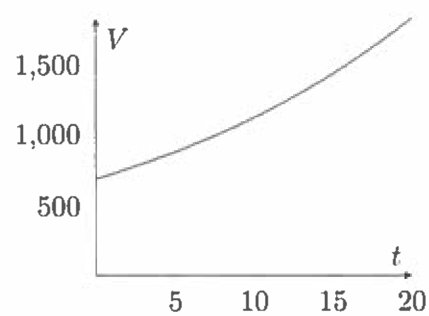
(2) Identify the parent function that best fits each of the following scenarios.

(a) The height of a diver from a swimming pool some number of seconds after leaving the board is modeled by the graph below:



$p(t) = t^2$

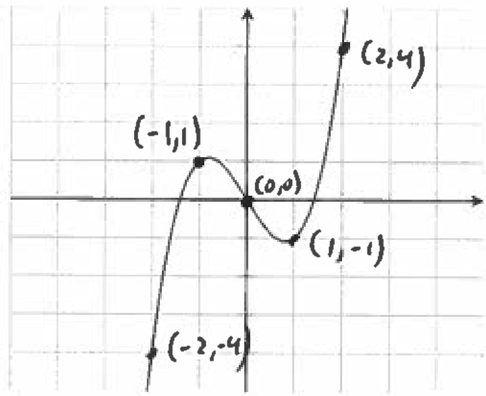
(b) The amount of money in a bank account which accrues compound interest is given by the graph below:



$p(t) = e^t$

1.2. Vertical Transformations

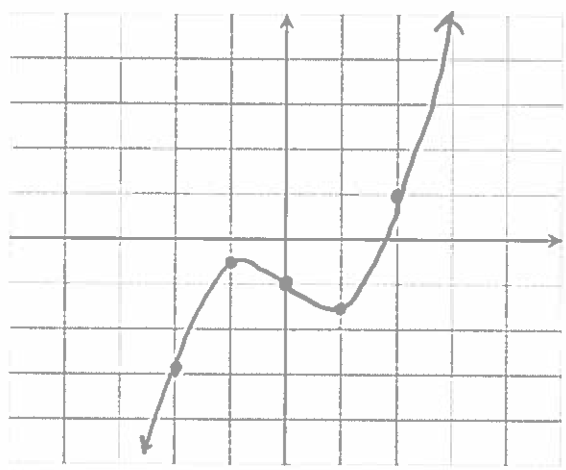
(1) Let f be given by the graph below.



Sketch the following transformations.

(a) $\frac{1}{2}f(x) - 1$

- stretch by a factor of 1/2
- shift down 1

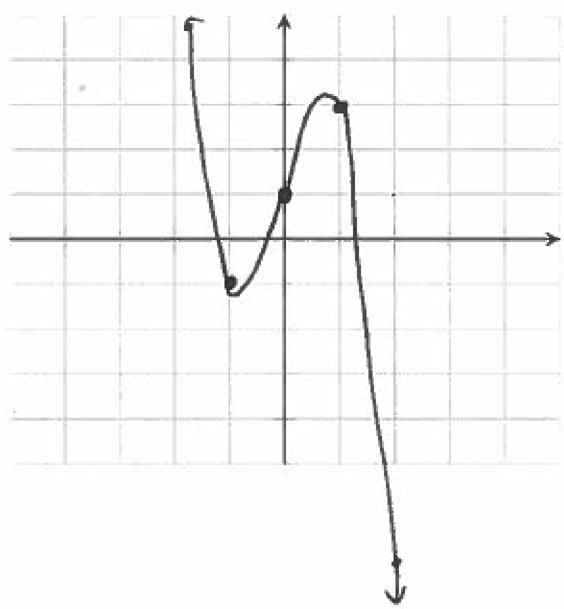


(b) $-(f(x)+2)$

- shift up 2
- then reflect over x-axis

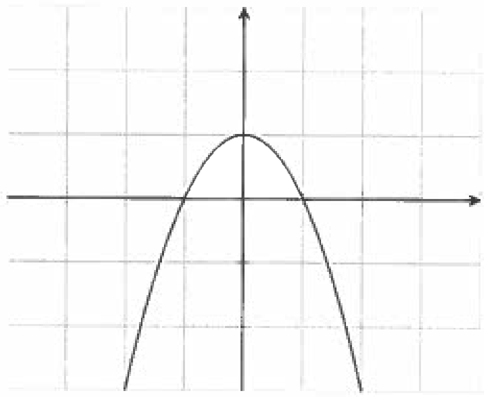
(c) $-2f(x)+1$

- stretch by a factor of -2
- then shift up 1



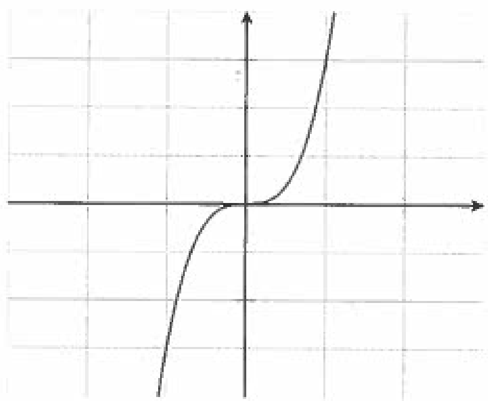
(2) Find an equation for each of the graphs below.

(a)



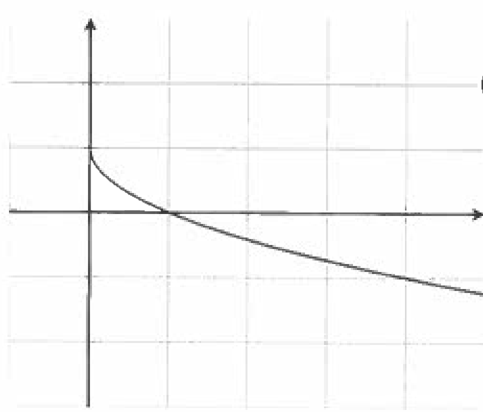
- parent function $p(x)=x^2$
- vertical reflection and shift up 1
- any stretching? moving right 1 from the vertex of this ~~past~~ parabola changes y by 1, as it does with the parent function, so there is no additional stretching
- so $f(x) = -x^2+1$

(b)



- parent function $p(x) = x^3$
- no reflection and no shift
- any stretching? moving right 1 from the origin now changes y by 3, whereas with the parent function, moving right 1 changes y by 1. So there is a stretch by a factor of 3
- so $f(x) = 3x^3$

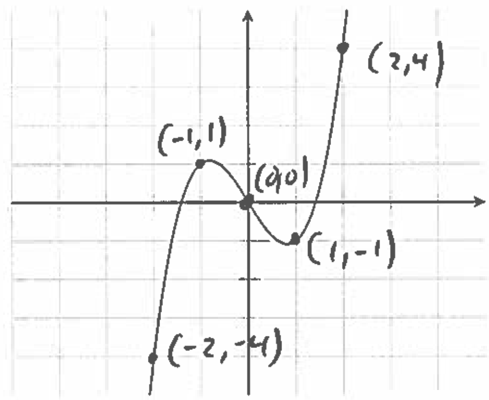
(c)



- parent function $p(x)=\sqrt{x}$
- vertical reflection, and shift up 1
- any stretching? no, by identical reasoning to part a
- so $f(x) = -\sqrt{x} + 1$

1.3. Horizontal Transformations

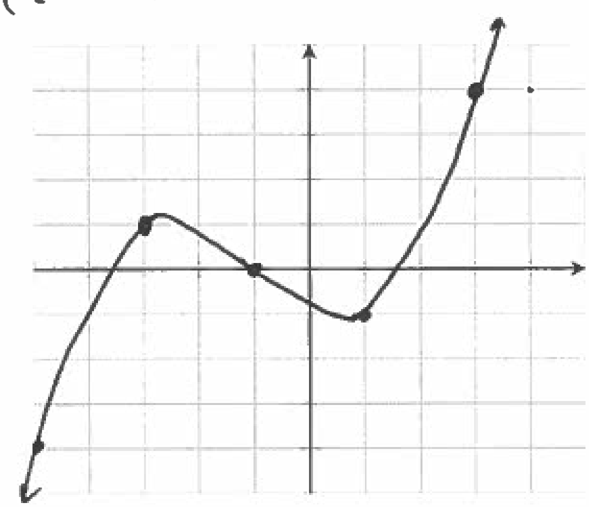
(1) Let f be given by the graph below.



Sketch the following transformations.

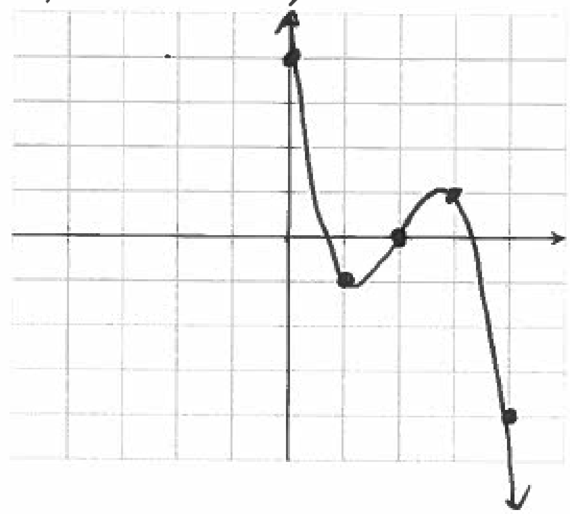
(a) $f\left(\frac{x+1}{2}\right) = f\left(\frac{1}{2}(x+1)\right)$

- stretch by a factor of 2
- shift left 1



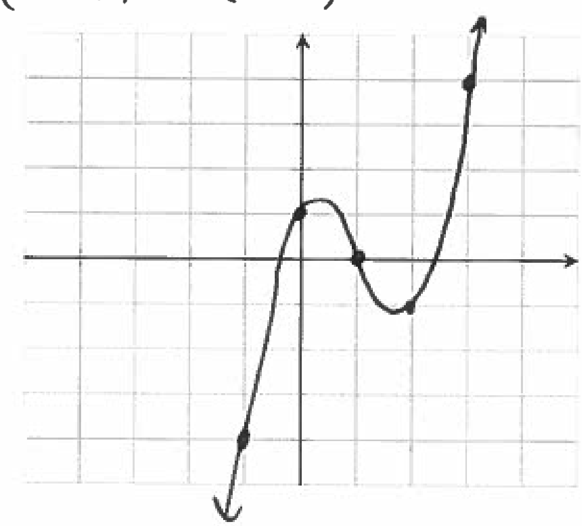
(b) $f(2-x) = f(-x+2) = f(-(x-2))$

- horizontal reflection
- then ~~[illegible]~~ shift right 2



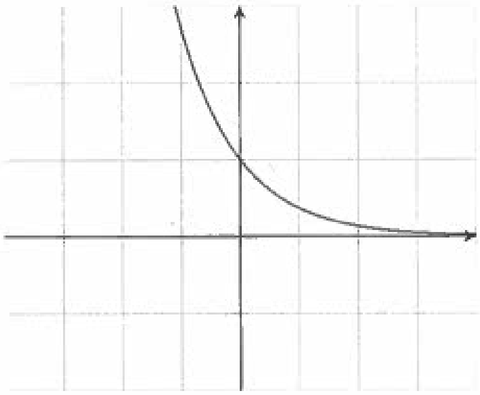
(c) $f(-(1-x)) = f(-1+x) = f(x-1)$

- shift right 1



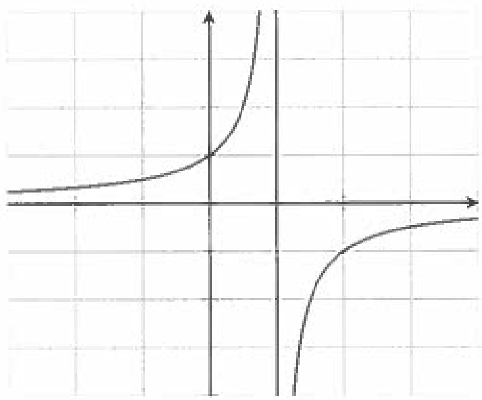
(2) Find an equation for each of the graphs below.

(a)



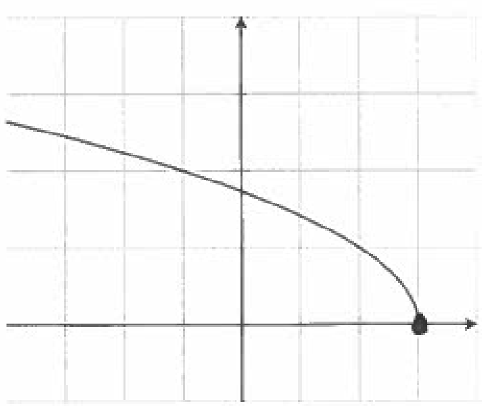
- parent function: $p(x) = e^x$
- horizontal reflection, no shift
- any other stretching?
  no, because $f(-1) \approx p(1)$
- so $f(x) = e^{-x}$

(b)



- parent function: $p(x) = \frac{1}{x}$
- horizontal reflection, then shift right 1
- any other stretching? can't really tell from the picture.
- so $f(x) = \frac{1}{-(x-1)}$

(c)



- parent function: $p(x) = \sqrt{x}$
- horizontal reflection, then shift right 3
- any stretching? moving left 1 unit from the "base point" of the graph changes y by 1, as does moving right one unit from the base point of the graph of the parent function. So there is no additional stretching.

$f(x) = \sqrt{-(x-3)}$

Application

Plutonium-239 is a chemical used in the production of nuclear weapons. As of 2009, the US inventory of plutonium was 95.4 metric tonnes (MT) [1]. Radioactive decay, which is "the process by which the nucleus of an unstable atom loses energy by emitting radiation" [2], causes our inventory to decrease. The way that we measure decay is by half-life, which is the "time required for a quantity to reduce to half its initial value" [3]. Plutonium-239 has a half-life of 24,110 years. Assuming that this is the only variable contributing to the change in inventory, we have the following model for US inventory of plutonium, t years after 2009

$$A(t) = 95.4\left(\frac{1}{2}\right)^{t/24110},$$

where A is the US inventory (in MT).

(1) There are many other variables that contribute to the change in US inventory of plutonium. Between 1994 and 2009, 11.2 MT of the plutonium inventory was expended either in wartime and tests or discarded to waste, which accounts for a yearly decrease of approximately 0.73%. Use this information to update our model.

don't worry about this one

(2) Say we want a long-term picture of the plutonium inventory based only on radioactive decay. Use horizontal graph transformations of our updated model to write the US-inventory of plutonium in 10,000 year intervals.

$$B(t) = A(10000t) = 95.4\left(\frac{1}{2}\right)^{10000t/24110}$$

[1] International Panel on Fissile Materials
[2] Wiki
[3] Wiki