Exam 1

Name: Key

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 100 points on this exam. That means you should budget about 0.5 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder. There are to be no graphing calculators, devices with internet access, or notes of any kind used in conjunction with this test. If you use any such material, you will receive a zero on this assessment. Only scientific calculators are permitted.

1. (16 points) Are the following functions even, odd, neither, or both? Your answer can be one word, but make sure to show your work.

(a) $f(x) = x^2 - x^4$

$f(-x) = (-x)^2 - (-x)^4 = x^2 - x^4 = f(x)$

$\rightarrow$ even

(b) $g(x) = 3e^x$

$g(-x) = 3e^{-x} \neq g(x)$
$\neq -g(x)$

$\rightarrow$ neither

(c) $h(x) = \frac{2x^2-4}{x^3+x}$

$h(-x) = \dfrac{2(-x)^2 - 4}{(-x)^3 + (-x)} = \dfrac{2x^2 - 4}{-x^3 - x} = \dfrac{2x^2 - 4}{-(x^3+x)}$

$= -\dfrac{2x^2 - 4}{x^3 + x} = -h(x)$ $\rightarrow$ odd

(d) $j(x) = \ln(x - 3)$

$j(-x) = \ln(-x-3) \neq j(x)$
$\neq -j(x)$

$\rightarrow$ neither

2. (4 points) Suppose that $g(x) = (f(x))^2$ where f is some odd function. Is $g(x)$ even, odd, neither, or both? Again, your answer can be one word, but make sure to show your work.

$$g(-x) = (f(-x))^2 = (-f(x))^2 = (f(x))^2 = g(x)$$

Therefore, $g(x)$ is <u>even</u>

3. (12 points) Each part of this problem lists a parent function, $p(x)$, and a transformation of that parent function, $f(x)$. List all of the transformations that you would apply to $p(x)$ to obtain $f(x)$, making sure to list them in a correct order.

   (a) $p(x) = x^{-3}$ and $f(x) = (4x)^{-3}$

   ① horizontal stretch by a factor of 1/4

   (b) $p(x) = e^x$ and $f(x) = 4e^x - 2$

   ① vertical stretch by a factor of 4
   ② vertical shift down two units

   (c) $p(x) = x^2$ and $f(x) = x^2 - 6x + 9 = (x-3)^2$

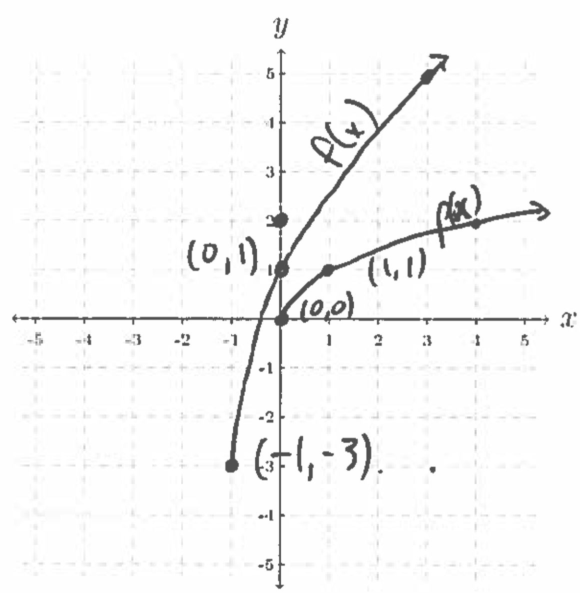
   ① horizontal shift right 3 units

4. (20 points) Consider the parent function $p(x) = \sqrt{x}$ and transformation $f(x) = 2\sqrt{4x+4} - 3$.

(a) List the transformations that you would apply to $p(x)$ in order to obtain a graph of $f(x)$. $f(x) = 2\sqrt{4(x+1)} - 3$ so we would have

① vertical stretch by a factor of 2
② horizontal stretch by a factor of 1/4
③ vertical shift down 3 units
④ horizontal shift left 1 unit

(b) Provide detailed graphs of $p(x)$ and $f(x)$ on the following axes, labeling at least two points on each graph. Your graph of $f(x)$ should be consistent with your answer from part (a). Note that since all four transformations are clearly present in the formula of $f(x)$, your graph of f should include all four transformation types.



(c) What are the domain and image of $f(x)$?

Domain : $[-1, \infty)$
Range/Image : $[-3, \infty)$

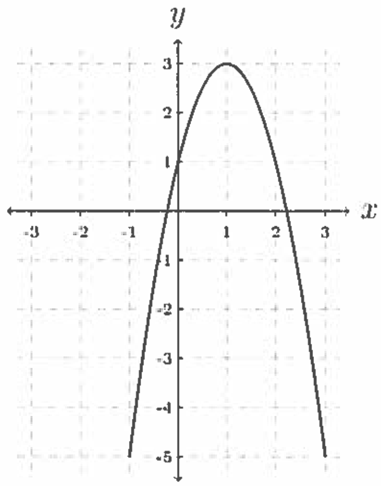
5. (4 points) Describe two possible ways in which $f(x) = e^{x-3}$ is a transformation of $p(x) = e^x$. *Hint:* One way is as a horizontal transformation, the other is a vertical transformation. ① horizontal shift 3 units right

$f(x) = e^{x-3} = e^{x} e^{-3}$

OR

② vertical stretch by a factor of e^{-3}

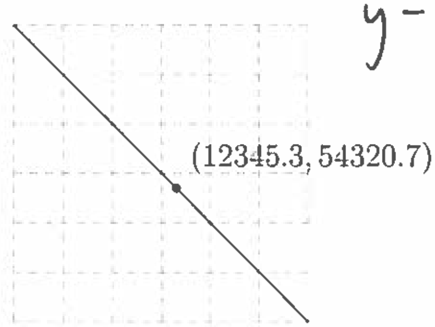
6. (8 points) Find a formula for the function, $f(x)$, which has the following graph, given that the parent function is $p(x) = x^2$.



$f(x) = -2(x-1)^2 + 3$

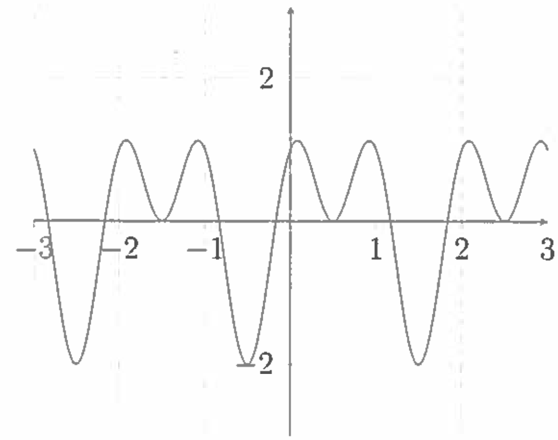
7. (4 points) Find an equation for the following line. The grid lines occur every 1 unit in the x and y directions.

$y - 54320.7 = -(x - 12345.3)$



8. (12 points) Are the following functions periodic? For each part, if the given function is periodic, find the period, midline, and amplitude (or say that such things don't exist and explain why, using complete sentences). If the given function is not periodic, you do not need to say anything about the period, midline, or amplitude.

(a) $S(x)$



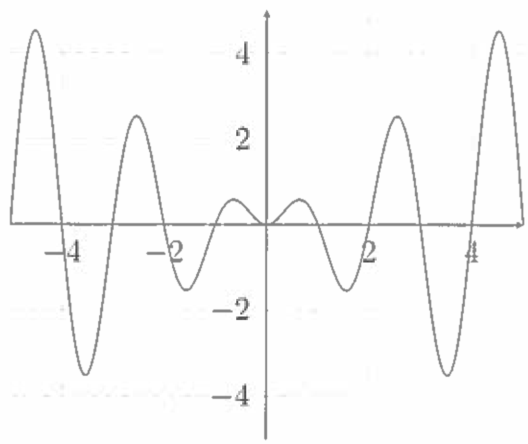
yes, periodic

$p = 2$

midline: $y = \frac{1+(-2)}{2} = -\frac{1}{2}$

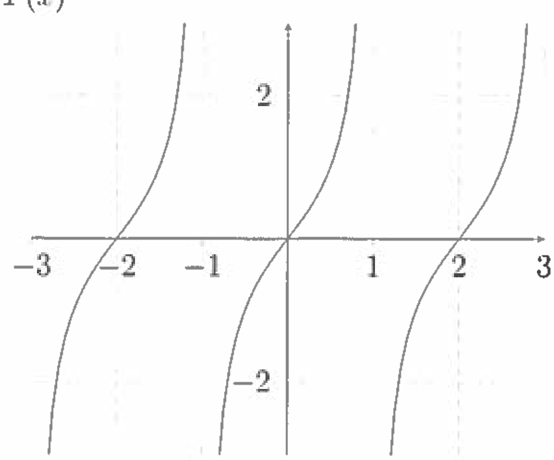
amplitude: $\frac{1-(-2)}{2} = \frac{3}{2}$

(b) $B(x)$



not periodic

(c) $T(x)$



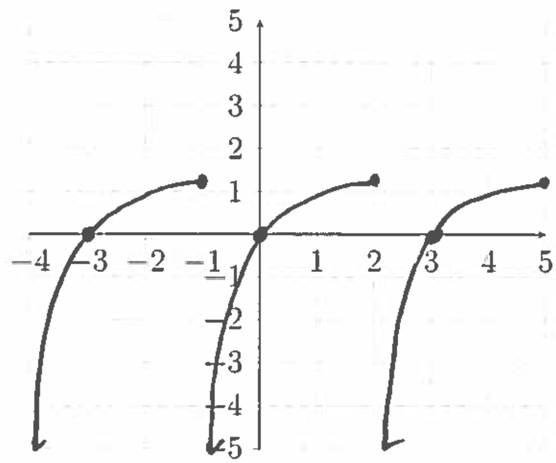
yes, periodic

$p = 2$

There is no midline and no amplitude because $T(x)$ has no maximum and no minimum y-values

9. (20 points) Consider the function $f(x)$ which is periodic with period 3. On the interval $(-4,-1]$, $f(x)$ is defined by the equation $f(x) = \ln(x+4)$.

(a) Graph $f(x)$ on the axes below.



(b) Compute $f(-3)$ $= \ln(-3+4) = \ln(1) = 0$

(c) Compute $f(12)$ $= f(9) = f(6) = f(3) = f(0) = f(-3) = 0$

(d) Find all solutions to $f(x) = 0$

The only solution to $f(x) = 0$ in $(-4,-1]$ is $x = -3$. Since $f(x)$ has period 3, the solutions have the form $-3 + 3n$ for some integer n. (equivalently, solutions have the form $3n$ for some integer n).

Bonus Question: (10 points) Demonstrate why $f(x) = 0$ is the only function which is both even and odd.

Suppose that $f(x)$ is both even and odd.
Then for all x in the domain of f,
$f(-x) = f(x)$ (since f is even), and
$f(-x) = -f(x)$ (since f is odd).
But these two equations imply that
$f(x) = -f(x)$ for all x in the domain of f
Adding $f(x)$ to both sides gives $2f(x) = 0$
and dividing by 2 gives $f(x) = 0$
for all x in the domain of f.