Exam 2

Math 112, Spring 2018

Name: Key

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 100 points on this exam. That means you should budget about 0.5 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder. There are to be no graphing calculators or devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment. Only scientific calculators and one 8.5×11 inch piece of paper are permitted.

1. (10 points) **True/False** Circle the word "true" if the statement is always true. Otherwise, circle the word "false."

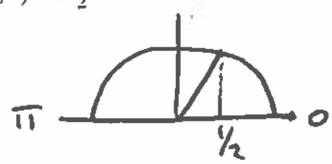
   (a) True or False: the domain of the function $\tan(\theta)$ is the set of all real numbers.

   See review, T/F, problem 3

   (b) True or False there exists an angle θ so that $\tan(\theta) > 0$ and $\sin(\theta) = 0$

   Definition of $\tan\theta = \frac{\sin\theta}{\cos\theta}$

   (c) True or False there are two distinct angles, α and β, between 0 and π so that $\cos(\alpha) = \cos(\beta) = \frac{1}{2}$

   only one angle

   (d) True or False a right triangle can have an obtuse angle.

   Sum of interior angles of a triangle

   (e) True or False there are infinitely many solutions to the equation $\sin(\theta) = 2$

   Definition of sin - there are no solutions

2. (12 points) Compute the following quantities, leaving your answer in exact form.

   (a) $\tan\left(\frac{9\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

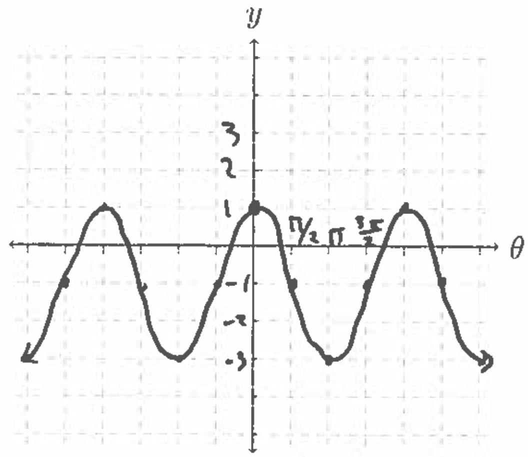
   (b) $\sin\left(\arcsin\left(\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

   (c) $\arccos\left(\cos\left(\frac{5\pi}{3}\right)\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

   see HW problem 2.6.2 a-f

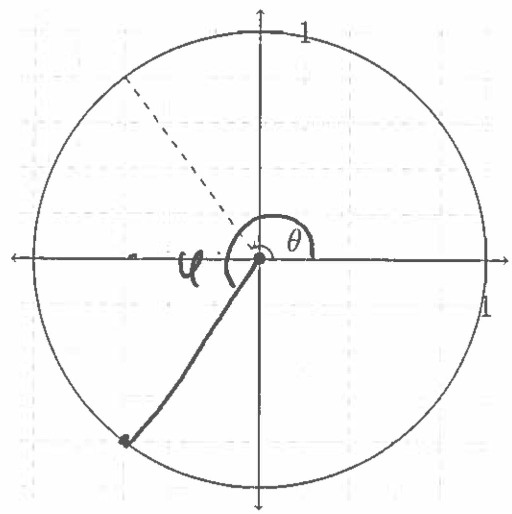
3. (12 points) On the following axes, graph the function $f(\theta) = 2\cos(\theta) - 1$. You should graph at least two periods, so make sure to give both of your axes an appropriate scale (and make sure to write that scale on the graph).

see HW problems 2.4 (1, 2)



4. (8 points) For the following questions, use the graph below. The light grid lines are in increments of 0.2 (one-fifth) and are provided to help you estimate.

see Quick Hit 9



(a) What are the sine and cosine of the angle θ pictured to the right? Give approximate answers to the nearest tenth.

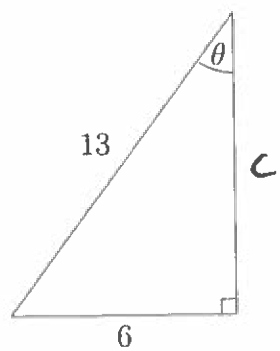
$\cos(\theta) =$ $-.6$ $\sin(\theta) =$ $.8$

(b) Sketch on the unit circle another angle φ so that $\cos(\varphi) = \cos(\theta)$, but φ is not $\theta + 2\pi n$ for any integer n. What is $\sin(\varphi)$?

$\sin \varphi = -.8$

5. (4 points) Compute $\tan(\theta)$, leaving your answer in exact form.

see HW problems
2.5 (2, 3) and review - free
response 6



$6^2 + c^2 = 13^2$

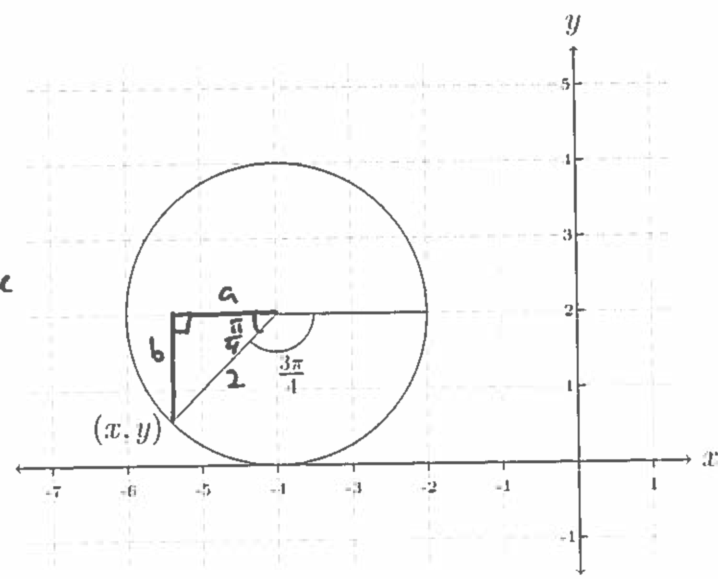
$\rightarrow c^2 = 133$

$\rightarrow c = \sqrt{133}$

$\rightarrow \tan\theta = \frac{6}{c} = \frac{6}{\sqrt{133}}$

6. (12 points) Find the coordinates of the point (x, y), leaving your answer in exact form.

see HW problem
2.4 (3) and
review - free response
number 11



radius : 2

center : (−4, 2)

$\cos\left(\frac{\pi}{4}\right) = \frac{a}{2} \rightarrow a = 2\cos\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

$\sin\left(\frac{\pi}{4}\right) = \frac{b}{2} \rightarrow b = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$

$x = -4 - a = -4 - \sqrt{2}$

$y = 2 - b = 2 - \sqrt{2}$

7. (16 points) Find all solutions, θ, to $3\cos\left(\pi\theta - \frac{\pi}{2}\right) + 1 = \frac{3\sqrt{2}+2}{2}$, leaving your answer in exact form.

see HW problems 3.3(3-6, 11), Quick Hit 9, review guide-free response number 9, any of the problems we went over in class

$$3\cos\left(\pi\theta - \frac{\pi}{2}\right) + 1 = \frac{3\sqrt{2}+2}{2}$$

$$3\cos\left(\pi\theta - \frac{\pi}{2}\right) = \frac{3\sqrt{2}+2}{2} - 1 = \frac{3\sqrt{2}+2}{2} - \frac{2}{2} = \frac{3\sqrt{2}}{2}$$

$$\cos\left(\pi\theta - \frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

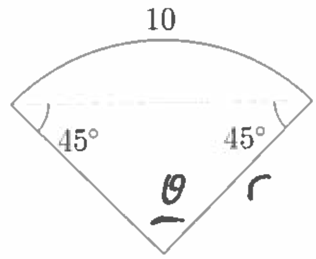
$$\pi\theta - \frac{\pi}{2} = \frac{\pi}{4} + 2\pi n \quad \text{or} \quad \pi\theta - \frac{\pi}{2} = -\frac{\pi}{4} + 2\pi n$$

$$\pi\theta = \frac{3\pi}{4} + 2\pi n \quad \text{or} \quad \pi\theta = \frac{\pi}{4} + 2\pi n$$

$$\boxed{\theta = \frac{3}{4} + 2n \quad \text{or} \quad \theta = \frac{1}{4} + 2n \quad \text{for integers } n}$$

8. (8 points) Find the perimeter of the following figure, leaving your answer in exact form. The arc shown is a portion of a circle, centered at the point O.

see review guide - free response 4, 7



$\theta = 180° - 45° - 45° = 90°$

$\rightarrow \theta = \frac{\pi}{2}$

$r\theta = 10 \rightarrow r \cdot \frac{\pi}{2} = 10$

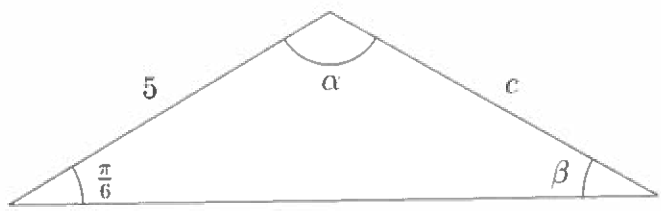
$\rightarrow r = \frac{20}{\pi}$

$\rightarrow$ perimeter $= 10 + 2r = 10 + \frac{40}{\pi}$

9. (18 points) Consider the following triangle.

see HW problems 3.2 (1,4,6,9)

review guide - free response 3, and Quick Hit 8



(a) Compute the side length, c.

(b) Compute the angles α and β. Note that $\alpha > \frac{\pi}{2}$ and $\beta < \frac{\pi}{2}$.

a) $c^2 = 5^2 + (5\sqrt{3})^2 - 2 \cdot 5 \cdot 5\sqrt{3} \cos(\pi/6)$

$= 25 + 25 \cdot 3 - 50\sqrt{3}\,(\sqrt{3}/2)$

$= 25 + 75 - 50 \cdot \frac{3}{2}$

$= 25$

$\rightarrow c = 5$

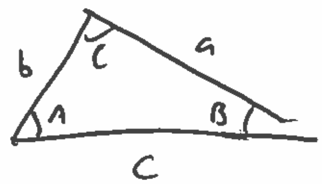
b) $\frac{\sin(\beta)}{5} = \frac{\sin(\pi/6)}{5} \rightarrow \sin(\beta) = \sin(\pi/6) = 1/2$

since $\beta < \pi/2$, $\beta = \arcsin(1/2) = \pi/6$

$\alpha = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$

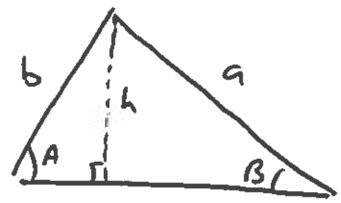
Bonus: (10 points) State and prove the Law of Sines

Theorem: In the following triangle,



we have $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Pf: Draw a height, h



$\sin(A) = \frac{h}{b}$, $\sin(B) = \frac{h}{a}$

Hence, $h = b\sin(A)$ and $h = a\sin(B)$

Then $b\sin(A) = a\sin(B)$ implying that

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Showing that $\frac{\sin(C)}{c} = \frac{\sin(B)}{b}$ is symmetric