Midterm 2 Review

General Information

The exam will take place on Thursday, May 24 at 3:00 pm in Deady 205 (at the usual time in the usual classroom). You will be permitted to use a *scientific calculator only*, though a calculator will not be necessary to solve many of the problems on the exam. You must bring your own calculator if you wish to use one. You will be permitted to use a piece of 8.5×11 inch sheet of paper with whatever notes you want on it. In addition to using this review guide, you should study the example problems we've done in class, homework problems, and previous quick hit problems. You should be prepared to answer questions about...

Topics

- Triangles
  - Right triangles
    - Using sine, cosine, and tangent
    - The Pythagorean Theorem
  - General triangles
    - Sum of interior angles
    - Law of sines and cosines
- Unit circle
- Trig functions
  - Definitions
  - Geometric meaning (e.g. cosine is x-coordinate, sine is y-coordinate, tangent is slope)
  - Domain, image, period, midline, amplitude
  - Pythagorean Identity
  - Special angles
  - Graphs (and vertical transformations)
- Inverse trig functions
  - When can they be used? When can't they?
- Radians
- Arc length
- Solving trigonometric equations

Key

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Practice Problems

True/False Questions: For the following questions, circle the word "true" if the statement is always true. Otherwise, circle the word "false."

1. True or False: There is an angle θ in the second quadrant so that $\tan(\theta) > 0$.

2. True or False: If $\sin(\theta) < 0$ then it must be the case that $\tan(\theta) < 0$.

3. True or False: The domain of tangent is all real numbers.

4. True or False: The image of tangent is all real numbers

5. True or False: There exists a triangle with angles $A = 40°$ and $B = 71°$ and opposite side lengths $a = 7$ and $b = 6$, respectively.

6. True or False: Let S be the arclength cut out by $\theta = 60°$ on a circle of radius of 2. Then, $S = 120$.

7. True or False: All solutions to the equation $\sin(\theta) = 1/2$ are given by $\pi/6 + 2\pi n$, for all integers n.

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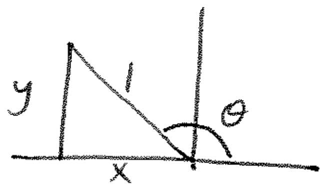
Short Answer Questions:

1. Suppose $\sin(\theta) = y$ for some angle θ in the second quadrant. Find $\tan(\theta)$ in terms of y.



$x^2 + y^2 = 1$

$\rightarrow x^2 = 1 - y^2$

$\rightarrow x = -\sqrt{1-y^2}$

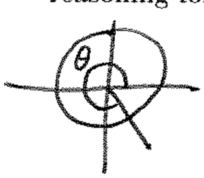
$\rightarrow \tan(\theta) = \frac{y}{x} = \boxed{\frac{y}{-\sqrt{1-y^2}}}$

2. If a line passes through the points $(2,5)$ and $(3,7)$, what angle does it make with the x-axis? slope is $\frac{7-5}{3-2} = 2$

since tangent corresponds to slope, we have the angle $\theta = \arctan(2) \approx 1.11$

3. Explain why $\sin^{-1}(2)$ is undefined. ("Because my calculator doesn't accept it" is not an explanation.) $\arcsin(2)$ would be a solution to the equation $\sin(\theta) = 2$. But this equation has no solutions because $-1 \leq \sin(\theta) \leq 1$ for all θ. Hence, $\arcsin(2)$ does not exist

4. What is the sign of tangent (i.e. positive or negative) in the fourth quadrant? Provide reasoning for your answer.



Lines from the origin into the fourth quadrant have negative ~~so~~ slope, so angles there have negative tangent

5. What is a radian?

A radian is a unit of measurement of angles. One radian corresponds to the angle which cuts out an arc length of a circle equal to the radius of the circle

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Free Response

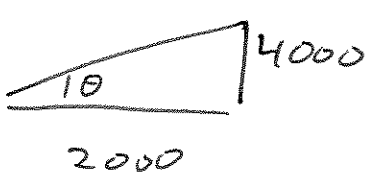
1. Find the *exact* value for each of the following.

   (a) $\tan(2\pi/3)$ $= -\sqrt{3}$

   (b) $\cos(7\pi/4)$ $= +\frac{\sqrt{2}}{2}$

   (c) $\sin(7\pi/6)$ $= -\frac{1}{2}$

2. An airplane takes off at a constant angle of inclination. If the airplane is 4,000 meters high at the moment it has passed over 2,000 meters of flat ground, what must have been the angle of inclination (i.e. the angle that its trajectory makes with the ground)? Sketch a picture of this scenario.

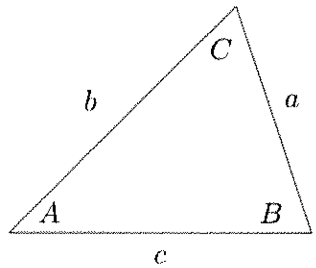


$\tan\theta = \frac{4000}{2000} = 2$

Since $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,

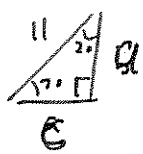
$\theta = \arctan(2) \approx 1.11$

3. Consider the labeling of the triangle below:



Using the given information, find all missing sides and angles. If there is more than one possibility for a triangle, find both. If such a triangle does not exist, state this and provide a reason why.

(a) $A = 70°$, $b = 11$, $C = 20°$

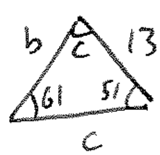


$B = 180 - 70 - 20 = 90$

$\sin 70 = \frac{a}{11} \to a = 11\sin(70) \approx 10.34$

$\cos 70 = \frac{c}{11} \to c = 11\cos(70) \approx 3.76$

(b) $A = 61°$, $B = 51°$, $a = 13$



$\frac{\sin(61)}{13} = \frac{\sin(51)}{b} \to b = \frac{13\sin(51)}{\sin(61)} \approx 11.55$

$C = 180 - 61 - 51 = 68°$

$\frac{\sin(61)}{13} = \frac{\sin(68)}{c} \to c = \frac{13\sin(68)}{\sin(61)} \approx 13.78$

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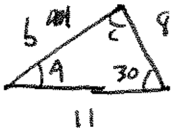
(c) $a = 8$, $B = 30°$, $c = 11$



$b^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cdot \cos(30) \approx 32.6$

$b \approx 5.71$

$\frac{\sin(A)}{8} = \frac{\sin(30)}{5.71} \rightarrow \sin(A) = \frac{8\sin(30)}{5.71} \approx .70$

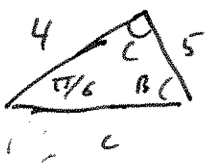
$\rightarrow A = \arcsin(.70) \approx 44.5°$

$\rightarrow C = 180 - 30 - 44.5 \approx 105.5°$

(d) $a = 2$, $b = 3$, $c = 9$

No such triangle is possible! We haven't covered this in class, so you won't be expected to know how to do this on the test, but you should be able to figure out why

(e) $A = \pi/6$, $b = 4$, $a = 5$



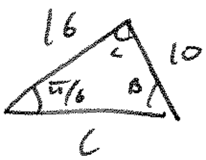
$5^2 = c^2 + 4^2 - 2 \cdot c \cdot 4\cos(\pi/6) \rightarrow 0 = c^2 - 6.93c - 9$

$\rightarrow c = \frac{6.93 \pm \sqrt{6.93^2 - (4)(9)}}{2} \approx 8.05, \cancel{-1.12}$

$\frac{\sin(B)}{4} = \frac{\sin(\pi/6)}{5} \rightarrow \sin(B) = \frac{4\sin(\pi/6)}{5} \approx .4 \rightarrow B = \arcsin(.4) \approx .41$

$\rightarrow C = \pi - \pi/6 - .41 \approx 2.21$

(f) $A = \pi/6$, $a = 10$, $b = 16$.



$10^2 = c^2 + 16^2 - 2c \cdot 16\cos(\pi/6) \rightarrow 0 = c^2 - 27.7c + 156$

$\rightarrow c = \frac{27.7 \pm \sqrt{27.7^2 - 4 \cdot 156}}{2} \approx 19.84, 7.86$

if $c = 19.84$, $\frac{\sin(B)}{16} = \frac{\sin(\pi/6)}{10} \rightarrow \sin(B) = \frac{16\sin(\pi/6)}{10} \approx .8 \rightarrow B = \arcsin(.8) \approx .92$

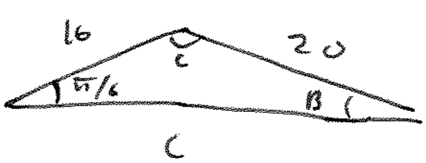
$\rightarrow C = \pi - \pi/6 - .92 \approx 1.69$

if $c = 7.86$, $\frac{\sin(C)}{7.86} = \frac{\sin(\pi/6)}{10} \rightarrow \sin(C) \approx .39$

$\rightarrow C = \arcsin(.39) \approx .4$

$\rightarrow B = \pi - \pi/6 - .4 \approx 2.22$

(g) $A = \pi/6$, $a = 20$, $b = 16$.



$20^2 = c^2 + 16^2 - 2 \cdot 16 \cdot c \cdot \cos(\pi/6)$

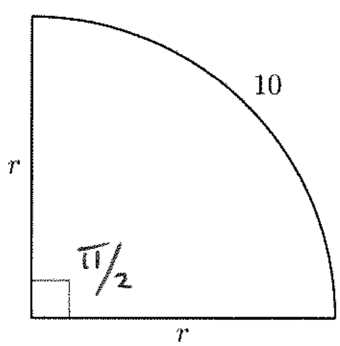
$\rightarrow 0 = c^2 - 27.7c - 144$

$\rightarrow c = \frac{27.7 \pm \sqrt{27.7^2 - 4(-144)}}{2} \approx 32.18, \cancel{-4.48}$

$\frac{\sin(B)}{16} = \frac{\sin(\pi/6)}{20} \rightarrow \sin(B) = \frac{16\sin(\pi/6)}{20} \approx .4 \rightarrow B = \arcsin(.4) \approx .41$

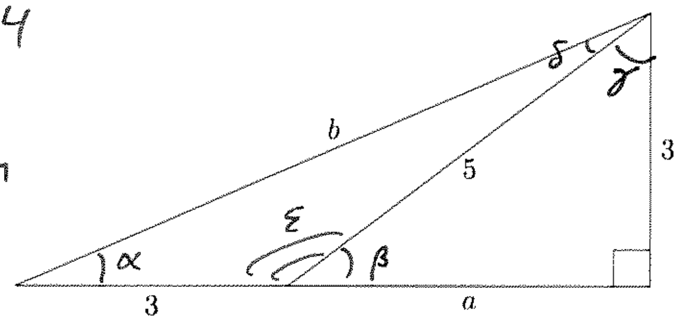
$C = \pi - \pi/6 - .41 \approx 2.21$

4. Find the *exact* value of r.



$10 = \frac{\pi}{2} \cdot r \rightarrow r = \frac{20}{\pi}$

5. Find a, b and all five of the missing angles in the figure below.



$a^2 + 3^2 = 5^2 \rightarrow a^2 = 16 \rightarrow a = 4$

$7^2 + 3^2 = b^2 \rightarrow b = \sqrt{58}$

$58 =$

4

7

$\sin(\alpha) = \frac{3}{\sqrt{58}} \rightarrow \alpha = \arcsin\left(\frac{3}{\sqrt{58}}\right) \approx .40$

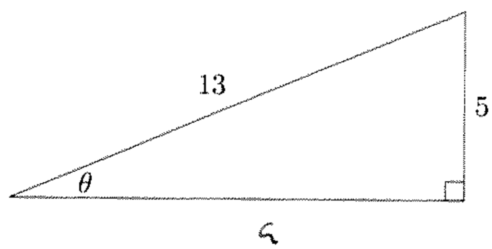
$\sin(\beta) = \frac{3}{5} \rightarrow \beta = \arcsin\left(3/5\right) \approx .64$

$\varepsilon = \pi - \beta \approx 2.50$

$\delta = \pi - \alpha - \varepsilon \approx .24$

$\gamma = \pi - \pi/2 - \beta \approx .93$

6. Using the picture below, find the *exact* value of $\tan(\theta)$.



$5^2 + a^2 = 13^2$

$\to a^2 = 144 \to a = 12$

$\tan\theta = \frac{5}{12}$

7. Find the *exact* perimeter of the tear-drop shown below. (Assume both triangles are right triangles.)

$6^2 + c^2 = 12^2 \to c^2 = 108$

$\to c = \sqrt{108}$

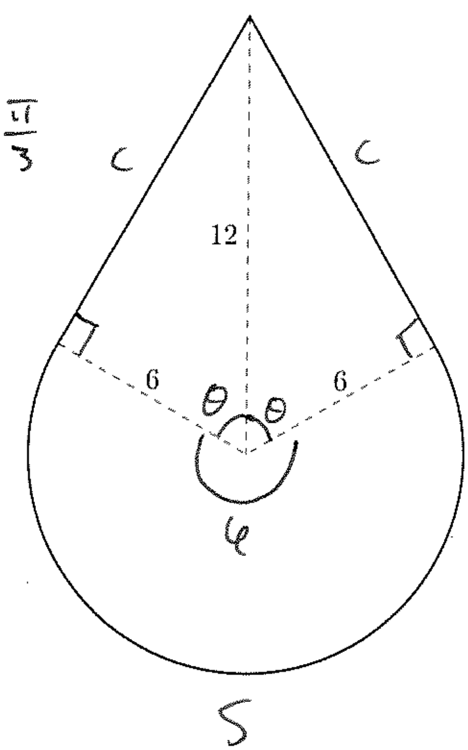
$\cos\theta = \frac{6}{12} = \frac{1}{2} \to \theta = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\varphi = 2\pi - 2\theta = \frac{4\pi}{3}$

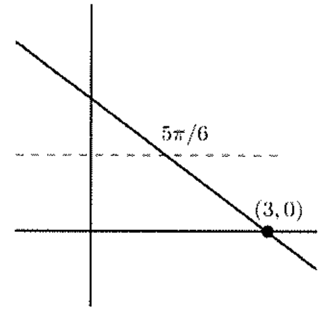
$s = 6 \cdot \frac{4\pi}{3} = 8\pi$

perimeter $= 2c + s$

$= 2\sqrt{108} + 8\pi$



8. Find the equation of the line drawn below. Write all coefficients in exact form.



slope $= \tan(5\pi/6) = \dfrac{-\sqrt{3}}{3}$

$\rightarrow y - 0 = \dfrac{-\sqrt{3}}{3}(x-3)$

9. Find all solutions to the following equations.

(a) [redacted]$\sin(\theta) - 1 = \dfrac{\sqrt{2}-2}{2}$ $\rightarrow$ [redacted]$\sin\theta = \dfrac{\sqrt{2}-2}{2} + 1 = \dfrac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2\pi n$

or

$\theta = \frac{3\pi}{4} + 2\pi n$

for integers n

(b) [redacted]$\cos(\theta) - 1 = \dfrac{\sqrt{2}-2}{2}$ $\rightarrow \cos\theta = \dfrac{\sqrt{2}-2}{2} + 1 = \dfrac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2\pi n$

or

$\theta = \frac{-\pi}{4} + 2\pi n$

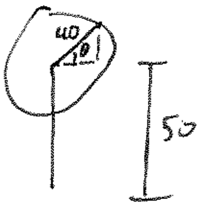
for integers n

(c) $\sqrt{3}\tan(2x-3) = -1$ $\rightarrow \tan(2x-3) = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$

$2x - 3 = -\frac{\pi}{6} + \pi n \rightarrow 2x = 3 - \frac{\pi}{6} + \pi n \rightarrow x = \frac{3}{2} - \frac{\pi}{12} + \frac{\pi}{3}n$

for integers n

10. Rob is on a ferris wheel with a diameter of 80 feet and center 50 feet from the ground. If he goes around the ferris wheel four times, at what angles (with respect to the horizontal, as usual) will he be 25 feet from the ground?



radius = 40 ft, so height given by

$H(\theta) = 40\sin\theta + 50$

$H(\theta) = 25 \to 40\sin\theta + 50 = 25 \to 40\sin\theta = -25 \to \sin\theta = \frac{\cancel{65}-25}{40}$

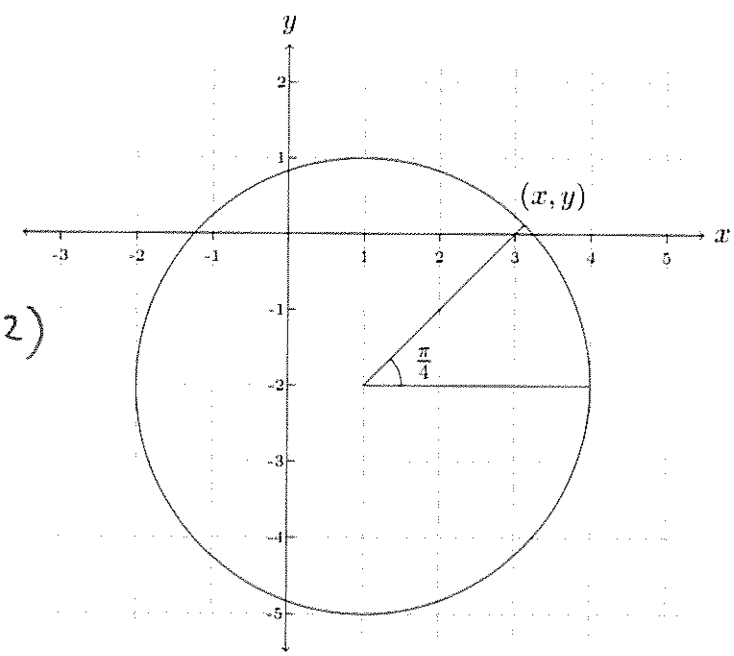
The principal solutions to this are $\theta_1 = \arcsin(-25/40) \approx -.68$

and $\theta_2 = \pi - \theta_1 \approx 3.81$

If Rob goes around 4 times the corresponding angles are

$-.68,\ -.68+2\pi,\ -.68+4\pi,\ -.68+6\pi,\ 3.81,\ 3.81+2\pi,\ 3.81+4\pi,\ 3.81+6\pi$

11. Find the coordinates of the point (x, y), leaving your answer in exact form.



~~dia~~ radius : 3

center : (1, -2)

$x = 3\cos(\pi/4) + 1 = \frac{3\sqrt{2}}{2} + 1$

$y = 3\sin(\pi/4) - 2 = \frac{3\sqrt{2}}{2} - 2$