Homework 1 Key

Problems: 1.1A (1, 3, 4a-f)
1.2A (1, 2a-g, 3a-g, 4a-c, 11)
1.3A (1, 2a-g, 3a-g)

1.1

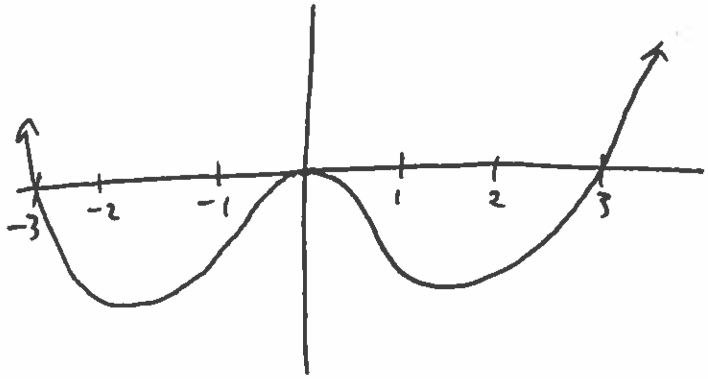
1a) $f(x)$ has reflective symmetry about the y-axis, so it is <u>even</u>

b) $p(x)$ has $180°$ rotational symmetry about the origin, so $p(x)$ is [redacted] <u>odd</u>

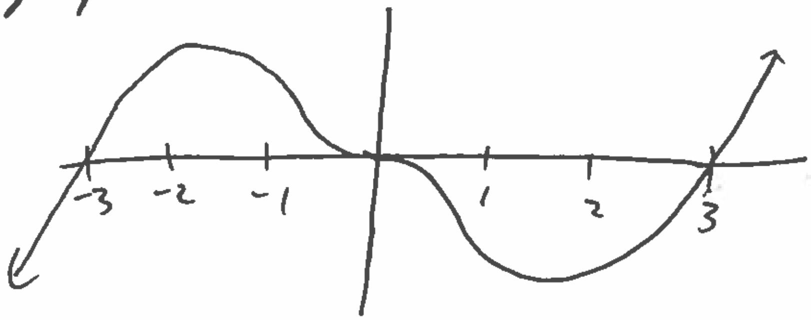
c) $Q(t)$ has neither rotational symmetry, nor reflective symmetry, so it is <u>neither</u> even, nor odd

d) $K(t)$ has $180°$ rotational symmetry about the origin, so it is <u>odd</u>

3a) If g is even (this is a typo in the book), then it has reflective symmetry about the y-axis, so we reflect the given half of the graph to obtain



3b) If g is odd, then it must have $180°$ rotational symmetry about the origin, so we rotate the half graph and obtain



4a) $f(-x) = (-x)^4 - 3(-x)^2 + 2 = x^4 + 3x^2 + 2 = f(x)$

Therefore, $f(x)$ is even

b) $p(-t) = e^{-t}$. $e^{-t} \neq e^{t} = p(t)$ and $e^{-t} \neq -e^{t} = -p(t)$.

Therefore, $p(t)$ is neither even nor odd.

c) $q(-x) = (-x)e^{(-x)^2} = -xe^{x^2} = -q(x)$

Therefore, $q(x)$ is even

d) $R(-u) = \dfrac{(-u)^3 - 4(-u)}{1 + (-u)^8} = \dfrac{-u^3 + 4u}{1 + u^8} = -\left(\dfrac{u^3 - 4u}{1 + u^8}\right) = -R(u)$

Therefore, $R(u)$ is odd

e) $c(-t) = e^{(-t)} + e^{-(-t)} = e^{-t} + e^{t} = c(t)$

Therefore, $c(t)$ is even

f) $s(-t) = e^{-t} - e^{-(-t)} = e^{-t} - e^{t} = -(e^{t} - e^{-t}) = -s(t)$

Therefore, $s(t)$ is odd

1.2

1a) $q(x) = x^2 - x + 3$

b) $q(x) = -\sqrt{4-x^2}$

c) $q(x) = \frac{1}{3} \frac{x}{1+x^2}$

d) $q(x) = 3e^x + 2$

e) $q(x) = \sqrt{x+11} - 11$

f) $q(x) = 6\left(\frac{5}{24}x + \frac{1}{3}\right) = \frac{5}{4}x + 2$

g) $q(x) = 2\ln(x) + 11$

2a) $f(x) = -\ln(x)$ is a vertical reflection of $p(x) = \ln(x)$

b) $f(x) = 4 - x^2$ is a vertical reflection, then a translation up 4 units, of $p(x) = x^2$

c) $f(x) = \frac{1}{3}x^3$ is a vertical stretch of $p(x) = x^3$ by a factor of $\frac{1}{3}$

d) $f(x) = e^{x-3} = e^x e^{-3} = \frac{e^x}{e^3}$ is a vertical stretch of $p(x) = e^x$ by a factor of $\frac{1}{e^3}$

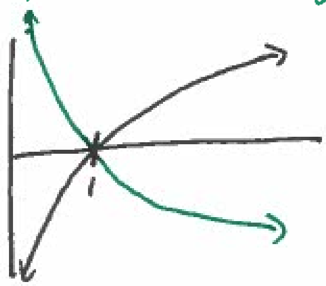
e) $f(x) = \ln\left(\frac{x}{e^2}\right) = \ln(x) - \ln(e^2) = \ln(x) - 2$ is a translation down 2 units of $p(x) = \ln(x)$

f) $f(x) = \ln\left(\sqrt{x^3}\right) = \ln\left(x^{3/2}\right) = \frac{3}{2}\ln(x)$ is a vertical stretch of $p(x) = \ln(x)$ by a factor of $\frac{3}{2}$

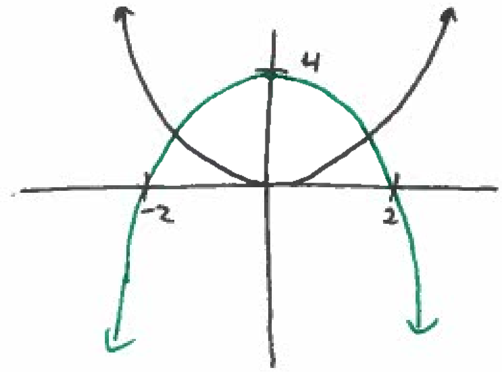
g) $f(x) = \frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$ is a vertical translation of $p(x) = \frac{1}{x}$ up one unit

3) For the following graphs, $p(x)$ is in black and $f(x)$ is in green

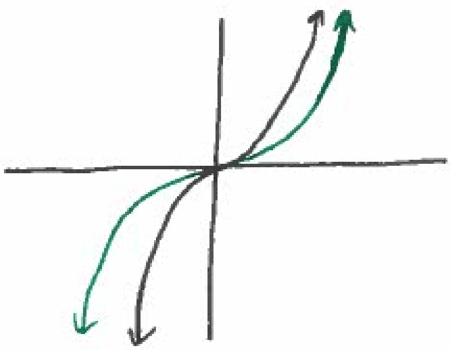
a)



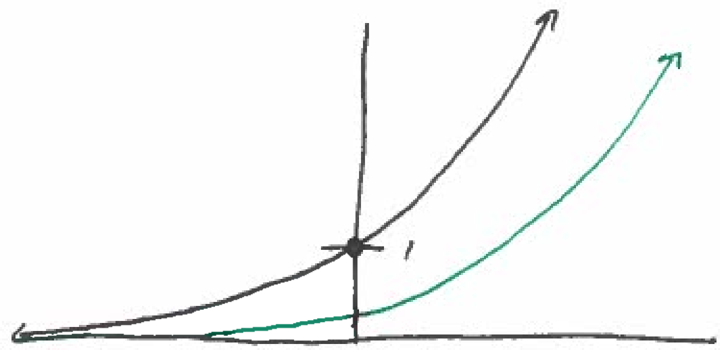
b)



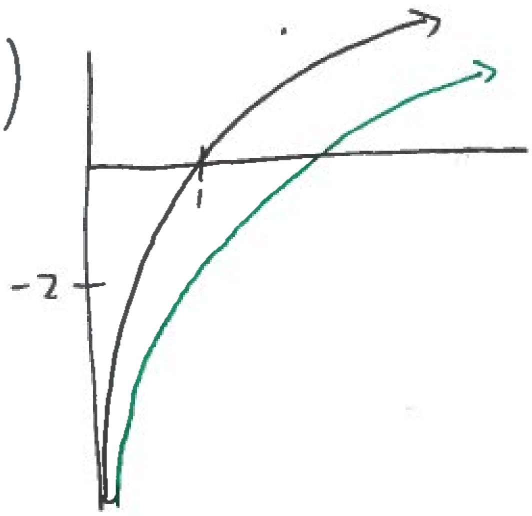
c)



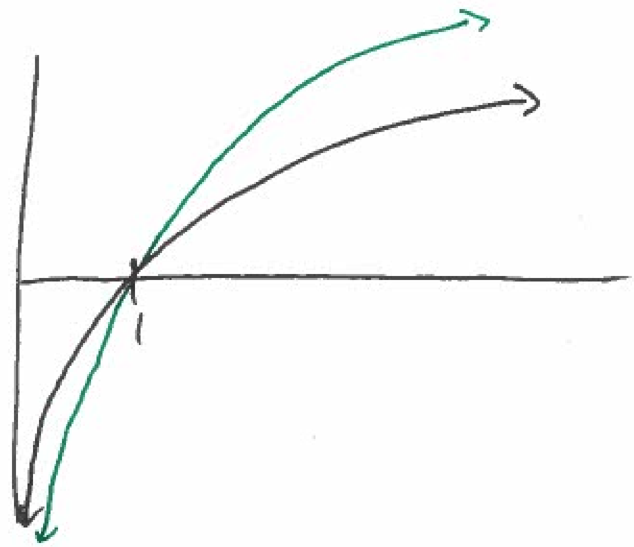
d)



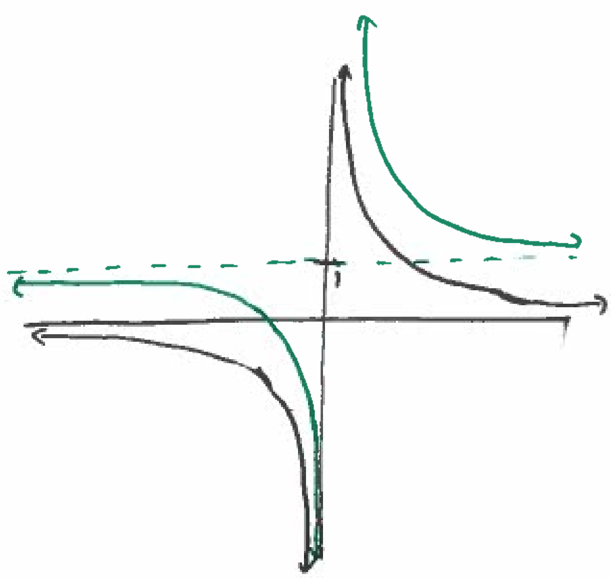
e)



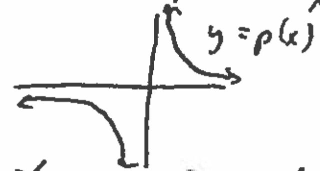
f)



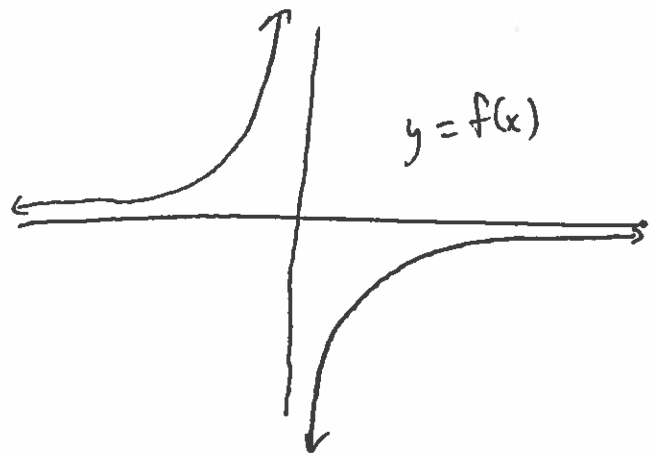
g)



4a) First, we identify the parent function $p(x) = \frac{1}{x^3}$ and sketch a quick graph

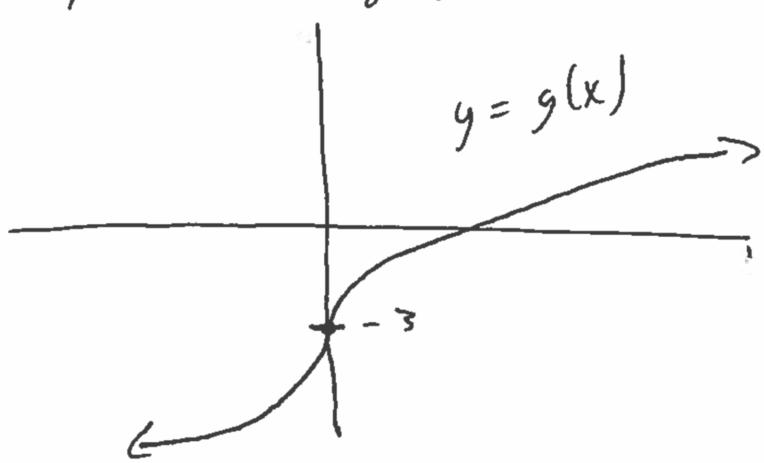


$f(x) = \frac{-4}{x^3}$ is a vertical reflection and stretch by a factor of 4 of $p(x) = \frac{1}{x^3}$, so the graph of f(x) looks something like this



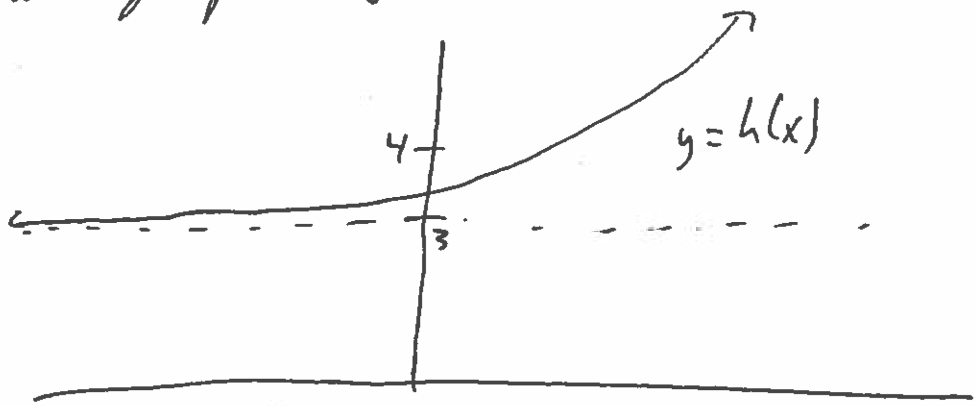
b) again, we identify the parent function $p(x) = x^{1/3}$ and sketch a graph

$g(x) = x^{1/3} - 3$ is a translation of p(x) down three units, so the graph of g(x) looks something like

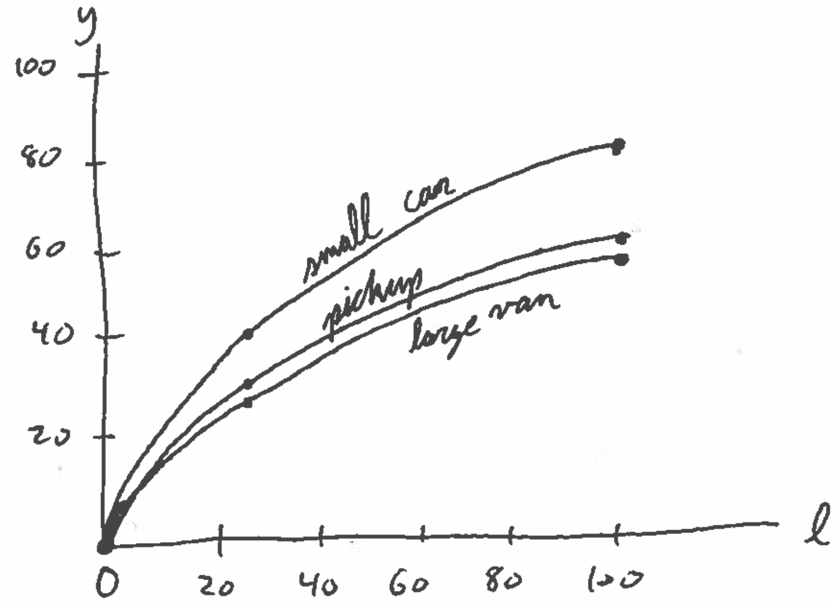


c) The parent function is $p(x) = e^x$, which has graph 

$h(x) = \frac{1}{2}e^x + 3$ is a vertical ~~[illegible]~~ stretch by a factor of $\frac{1}{2}$, then a translation up 3 units, of $p(x)$, so the graph of $h(x)$ looks something like



11)



1.3

1a) $q(x) = (x-3)^3 - 3(x-3) = x^2 - 9x + 18$

b) $q(x) = \sqrt{x+4} + 3$

c) $q(x) = \ln\left(\frac{x}{2}\right)$

d) $q(x) = e^{-x}$

f) $q(x) = 12(3x) + 7 = 36x + 7$

g) $q(x) = \sqrt{-x}$

2a) $f(x) = \sqrt{x+1}$ is a horizontal shift of $p(x) = \sqrt{x}$ to the left by one unit

b) $f(x) = \sqrt{2x}$ is a horizontal stretch of $p(x) = \sqrt{x}$ by a factor of $\frac{1}{2}$

c) $f(x) = (x+3)^2$ is a horizontal shift of $p(x) = x^2$ to the left 3 units

d) $f(x) = 4x^2 - 16x + 16 = (2x-4)^2$ is a horizontal shift by 4 units right, then a horizontal stretch by a factor of $\frac{1}{2}$, of $p(x) = x^2$

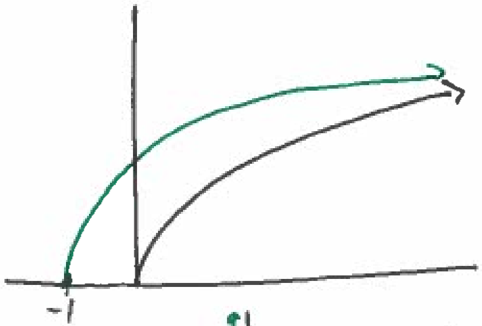
e) $f(x) = \frac{1}{x^2-6x+9} = \frac{1}{(x-3)^2}$ is a horizontal shift of $p(x) = \frac{1}{x^2}$ to the right 3 units

f) $f(x) = \ln(x-3)$ is a horizontal shift of $p(x) = \ln(x)$ to the right 3 units

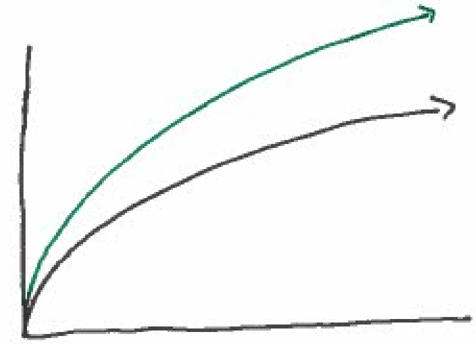
g) $f(x) = e^{-.5x}$ is a horizontal reflection and stretch of $p(x) = e^x$ by a factor of 2

3) For the following graphs, p(x) is in black and f(x) is in green

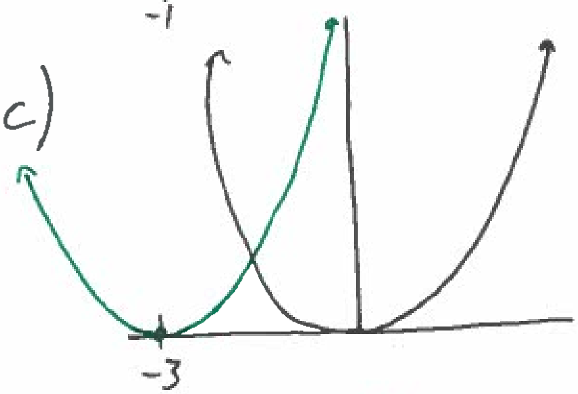
a)



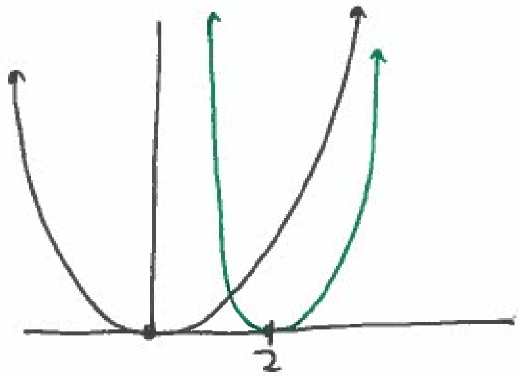
b)



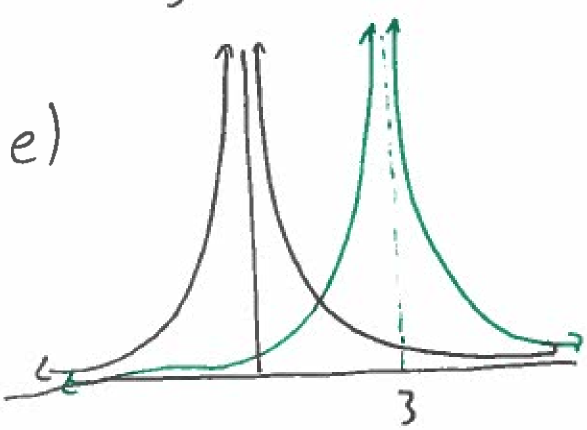
c)



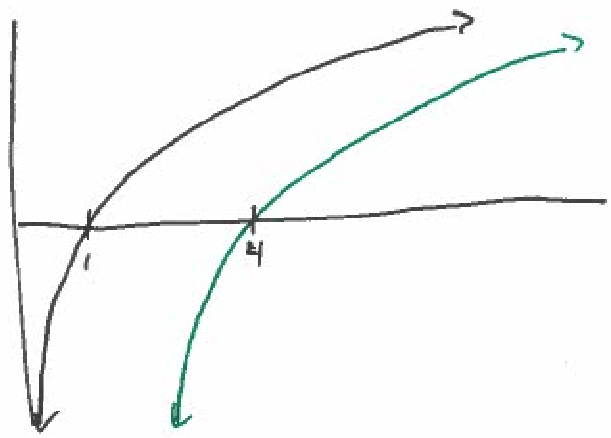
d)



e)



f)



g)