

**Definition 5.1** A combination is an order ignored selection of objects from a larger group of objects.

We know that the number of different permutations of  $r$  different objects is

$$P_r^r = r!.$$

The  $P_r^n$  can be thought as the multiplication of two numbers:

(1) number of ways to select  $r$  different objects from  $n$  different objects. We denote this number by  $C_r^n$

(2) number of ways to get different arrangements for each selected  $r$  objects. This is just  $P_r^r$ .

Therefore, by  $mn$  rule we have

$$P_r^n = C_r^n \times P_r^r$$

or

$$C_r^n = P_r^n / P_r^r = \frac{n!}{r!(n-r)!}$$

Counting Rule for Combination The number of different combinations of  $n$  different objects that can be formed, taking them  $r$  at a time, is

$$C_r^n = \frac{n!}{r!(n-r)!}$$

**Example 5.1** A student prepares for a quiz by studying a list of ten problems. She only can solve six of them. For the quiz, the instructor selects five questions at random from the list of ten. What is the

probability of event  $A$  that the student can solve all five problems on the exam?

**Solution:** The number of simple events in the sample space is

$$C_5^{10} = \frac{10!}{5!(10-5)!}$$

The event  $A$  that the student can solve all five problems can be described in the way that the five problems are selected from the six problems she learned. Thus, the number of simple events in the event  $A$  is

$$C_5^6 = \frac{6!}{5!(6-5)!}$$

and

$$\mathbb{P}(A) = \frac{C_5^6}{C_5^{10}}$$

**Example 5.2** We want to choose 5 people from 20 people to organize a traveling group. Are there how many different ways to choose a group?

**Solution:** In this example order is not important, therefore, there are  $C_5^{20}$ .

**Example 5.3** Suppose that 10 defective computers are included in a shipment of 1000 computers. If you test 20 computers in this 1000 computers, what is the probability of event  $A$  that you found two defective computers?

**Solution:**

$$\mathbb{P}(A) = \frac{C_2^{10} C_{18}^{990}}{C_{20}^{1000}}.$$

**Definition 5.2** The intersection of events  $A$  and  $B$ , denoted by  $A \cap B$ , is all the simple events belonging to both  $A$  and  $B$ . The union of events  $A$  and  $B$ , denoted by  $A \cup B$ , is all the simple events belonging to either  $A$  or  $B$ . The complement of event  $A$ , denoted by  $A^c$ , is all the simple events belonging to sample space  $\Omega$  but  $A$ .

**Example 5.4** Consider the random experiment of rolling a fair die. Define  $A = \{E_1, E_2, E_3, E_4\}$ ,  $B = \{E_1, E_3, E_5\}$ . What are  $A \cap B$ ,  $A \cup B$ , and  $A^c$ ?

A more general definition of probability is defined as follows:

**Definition 5.3** Given a sample space  $\Omega$ , if we define a function  $\mathbb{P}$  on  $\Omega$  by:

- (1) The empty set is denoted by  $\phi$ .  $\mathbb{P}(\phi) = 0$ .
- (2) For each simple event  $E_i$ , its probability  $\mathbb{P}(E_i)$  is defined and  $0 \leq \mathbb{P}(E_i) \leq 1$ . For a sequence of mutually exclusive events  $A_i$ , let  $A = \cup_{i=1}^{\infty} A_i$ , then

$$\mathbb{P}(A) = \sum_{i=1}^{\infty} \mathbb{P}(A_i).$$

- (3) For each event  $B$ ,  $0 \leq \mathbb{P}(B) \leq 1$  and  $\mathbb{P}(\Omega) = 1$ . Then,  $\mathbb{P}$  is called a probability on  $\Omega$ .

**Example 4.10**

In a pocket there are 3 black and 2 white balls. Balls are identical except their colors. Randomly drawing a ball and observing its color. How many simple events in the sample space? Can you define a reasonable probability on the sample space? Is this equally likely?

**Solution:**

$$\Omega = \{W, B\}$$
$$\mathbb{P}(W) = \frac{2}{5}, \quad \mathbb{P}(B) = \frac{3}{5}.$$