Demonstrating worker quality through strategic absenteeism.

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This draft: 10/1/1998

Abstract

Determining the productivity of individual workers engaged in team production is difficult. Monitoring expenses may be high, or the observable output of the entire team may be some single product. One way to collect information about individual productivity is to observe how total output changes when the composition of the team changes. While some employers may explicitly shift workers from team to team for exactly this reason, the most common reasons for changes in team composition are at least partly voluntary: vacation time and sick days. In this paper, we develop a model of optimal absenteeism by employees which accounts for strategic interactions between employees. We assume the employer uses both observed changes in output and the strategies of the employees to form beliefs about a given worker's type. We argue that the model we develop is applicable to a variety of workplace situations where signaling models are not, because it allows a worker's decisions to provide information about other workers, as well as about himself.

JEL codes C72, J22. Keywords: Absenteeism, Non-cooperative Games, Signaling, Teamwork.

The authors wish to thank Rick Harbaugh, John Hillas, Van Kolpin, Henke Norde and Dries Vermeulen for their helpful comments and suggestions.

1 Introduction

In 1996 the Board of Governors of the Federal Reserve, responding to a string of fraud cases, told banks that

"One of the many basic tenets of internal control is that a banking organization ensure that employees in sensitive positions be absent from their duties for a minimum of two consecutive weeks. Such a requirement enhances the viability of a sound internal control environment because most frauds or embezzlements require the continual presence of the wrongdoer."

Member banks were then instructed that

"...a minimum of two consecutive weeks absence be required of employees in sensitive positions." (Board of Governors, 1996).

This is an extreme example - one where productivity is actually negative - of a situation where an employer can learn something about an individual worker's job performance by observing what happens during his absence from work. Such situations often involve team production. With team production, determining the productivity of individual workers can be difficult either because monitoring expenses are high, or because the observable output of the entire team is some single product.

Employers will want to know individual productivity so that they can reward more productive workers, or punish less productive ones. At the same time, high-quality workers will want to convince the employer of their type, while low types will want to hide theirs. One way that information about individual productivity can be obtained is to change the composition of the team and observe how total output changes. While some employers may

require workers to take time off from the team, as the Fed recommends above, changes in team composition are also made voluntarily by workers when they absent themselves from work using vacation time and sick days. In this paper we analyze the strategic behavior of employees in such situations and use this analysis to provide new explanations for some aspects of vacation behavior and vacation policies.

The basic model in the paper has one employer with two workers, one high type and one low type. The employer can measure team output periodically, but cannot observe individual output. Each worker can produce either high output or low output, but the high type has a higher probability of producing high output than does the low type. These probabilities are common knowledge. Workers know their own type and their co-workers type, while the employer only knows that he has one worker of each type. We only consider the case where the employer has one of each type of worker, rather than a random draw of workers, for the following reason. If the employer knows the output probabilities associated with each type, then after enough observations of team output he would know with high probability whether his team consisted of two low-type workers, one high-type worker and one low-type worker, or two high-type workers. Only when the employer has a mix will the employees be concerned with conveying further information to the employer.

Each employee can choose to work continuously or take a fixed period of time off. For exposition, we use the case of vacations. We set the vacation length at two weeks, and for simplicity we also assume that output is observed at the end of every two week period. Employees must make their vacation decision without knowing the other's decision, and they cannot take vacations simultaneously. If both workers work continuously, then the employer gets no new information on their types. If one worker takes time off, then the employer has one observation of the individual output of the other worker and forms an expectation about the type of that worker accordingly. Note that this also enables the employer to form

an expectation about the type of the worker who takes time off, since the employer already knows one of them is the low type and the other one is the high type. If both workers take vacations, the employer has an observation of the individual output of each worker and forms an expectation of the type of each worker accordingly. After observing the employees' actions and measuring output, the employer sets wages for the workers based on his estimates of the types of the workers, and then pays those wages for a one year period, including the vacations, if any.

While the model in this paper allows for signaling behavior by the employees, it is different from what are usually known as signaling models. In the usual signaling model there is one employer and one worker. In the extensive form, Nature first determines the worker's type, and knowing his type the worker then decides how much, say, education to obtain. When deciding this, he knows that his wage will depend on the employer's beliefs about what a worker of a different type would do, in his place. However, this other type worker is not actually in the game. The worker's payoff is determined solely by the employer's beliefs and by his own strategy, which in turn depends on those beliefs. This model is a reasonable description of situations where the employer's beliefs about a given worker are independent of his beliefs about another worker. It is appropriate in situations where the employer deals with one worker, or perhaps with a series of workers one at a time, or where the employer has a very large pool of employees or potential employees, or more generally where what an employer believes about one worker does not affect what he believes about another.

However, it is *not* appropriate in situations where what an employer believes about one particular worker's type depends on his beliefs about another's type. In these situations a given worker can, by altering his strategy, affect not only the employer's beliefs about his type, but also the employer's beliefs about the type of another worker. In turn, a worker must worry not only about what *his* actions tell the employer about his type, but also about

how his actions affect the incentives of the *other* worker to alter his behavior, and thus indirectly affect the employer's beliefs about the first worker.

Our model accounts for this possibility by having each worker play a strategic game with the employer and with the other worker. The employer's beliefs depend on the strategies of each worker, and so do the workers' payoffs. We believe that, in addition to the particular application we develop in this paper, these sorts of models are well suited to a variety of situations where the usual signaling models are not.

2 Literature Review

In the economic literature, vacations and sick days are generally treated in combination with absenteeism, defined as employees taking time off from regularly scheduled work. We will begin this review with relevant literature on absenteeism, then related work on signaling in employment situations, then work on team production. Parts of the discussion on absenteeism is based on the review of the literature in Brown and Sessions (1996).

In the textbook explanation of labor supply, workers choose the amount of labor they wish to supply, given a wage determined by their marginal product, which is assumed to be constant with respect to the number of hours worked. Vacations, sickdays and unapproved absences are merely different names for the time that the employee does not work. In such a model there is no absenteeism, since there is no such thing as regularly scheduled work.

Deardorff and Stafford (1976) and Weiss (1985) look at models where the employer uses both capital and labor as inputs to explain why employers want to determine the length of the working day, rather than leave this choice up to workers. The argument is that the employer prefers that his capital be continuously, and wants a work schedule that ensures this. Given this schedule and the market wage rate, some employees may prefer to work more or less than what is optimal for the employer, depending on their marginal rate of substitution of leisure for money. Absenteeism occurs when the employees with the higher tastes for leisure reduce their hours worked below the employer's target. A similar argument could be used to explain why employers want to set the number of days worked per year and workers may not want to work according to that schedule. This approach forms the basis of a fairly large empirical literature which seeks to determine what sorts of factors make employees likely to absent themselves from work. Dunn and Youngblood (1986), for example, find that workers with higher preferences for leisure, relative to their wage, are absent more often. Leigh (1991) finds that wages, and even paid sick leave, do not have a significant effect on absence.

The model described above is an obvious oversimplification in that it treats the marginal rate of substitution between not working and money as the same at any point in time. In reality, transient and idiosyncratic events such as sickness or the desire to go duck hunting on opening day will temporarily increase the marginal utility of time off. Knowing this, employees should be willing to accept lower average wages in return for generous vacation and sick leave policies. Allen (1981) estimates a hedonic model and finds evidence in support of this. Another possibility is that the productivity of workers declines with time worked. Biddle and Hamermesh (1990) cite evidence that sleep increases productivity, and a similar argument could be made for vacations.

In combination, the work cited above provides a neoclassical explanation for vacations, sick days, and absenteeism. There is also a large literature covering many forms of signaling and screening in employment situations, beginning with Spence (1973). Several papers consider the problem of an employer trying to screen out low quality workers by setting the terms of employment. Bull and Tedeschi (1989) propose that employers may set probationary periods, during which employees are monitored, to scare off less productive workers.

Weiss and Wang (1990) and Loh (1994) develop similar models, and Loh finds evidence that probationary contracts do induce self selection. Rebitzer and Taylor (1995) argue that employers may require workers to work long hours to prove that they have a low taste for leisure, and Landers et al. (1996) show that a signaling model along these lines can explain why associates in law firms work long hours. Obviously, such models do not explain the equally ubiquitous phenomena of paid or mandatory vacations.

There is also a literature on team production following Alchian and Demsetz (1972), which has considered the problem of shirking, not worker ability. While we believe the model we develop in this paper could account for some interesting aspects of behavior if modified to allow shirking, in this paper we assume that workers differ in only in ability.

Finally, there is a small literature on differences in vacation practices across countries. Bell and Freeman (1994) for example, examine why Americans work more than Germans. Failing to find empirical support for other explanations, they attribute the difference to a greater variance in the rewards to success in the U.S. than in Germany.

3 The model

3.1 Preliminaries

One employer has two workers, A and B, one of whom is high type and the other low type. The workers know their type and their co-worker's type. The employer knows he employs one high-type worker and one low-type worker, but not whether A is the high type and B the low one or vice versa, and a priori it assigns a probability of $\frac{1}{2}$ to each of these possibilities. The employer wants to reward the high-type worker and punish the low type, by redistributing the total amount of wages paid. Wages are assumed to be paid (or at least

determined) annually. If the employer knew the workers' types with certainty, he would pay the high type wage w_h and the low type wage w_l with $w_l < w_h$, but he is reluctant to scare the high-type worker away because of a mistake in assessment of the workers type. So the employer distributes the total wage pie according to an $\alpha \in [0,1]$, which is interpreted as "pay A wage $(1-\alpha)w_l + \alpha w_h$ and pay B wage $\alpha w_l + (1-\alpha)w_h$ ". We assume that if the employer believes that A is the high-type worker with probability β (and, consequently, that B is the low-type worker with probability $1-\beta$), then he will pay A a wage $(1-\beta)w_l + \beta w_h$. That is, he will choose the action $\alpha = \beta$. (This behavior will be optimal for the employer if his utility function is, for example, $u(\alpha \mid \beta) = 1 - |\alpha - \beta|$.) This assumption reduces the problem of predicting the employer's action to one of predicting his beliefs about the types of his workers.

The employer gets information about the types of the players by observing output when the team composition changes, that is when one of the two workers is on vacation and the other works alone. A vacation is always one period (two weeks) long, and gives each worker the same utility v > 0. The employer prohibits both workers from taking vacations during the same period, and the workers must decide whether or not to take a vacation at the beginning of the year, without knowledge of the other worker's decision. In each period each worker produces either low output q_l or high output q_h , where $q_l < q_h$. Output is subject to a shock, so that the high-type worker produces q_h with probability $p_h \in (0,1)$ (and q_l with probability $1 - p_h$) and the low-type worker produces q_h with probability $p_l \in (0,1)$, where $p_l < p_h$. We assume output is determined by ability, so that the workers cannot change the probability of high output by changing effort.

3.2 Updating beliefs using output changes

In this section, we will assume the employer uses only the observations of output to update his beliefs. Later we will show how he can also update using knowledge of the actual and optimal strategies of the two types. The results of this section are necessary steps for analyzing the case of strategic behavior, developed in the subsequent section. In addition, they show how the employer should set wages if the employer does not use the decision to take or not take a vacation as a signal, or if the employer is arbitrarily dictating who does and does not take a vacation. First, we find the employer's posterior beliefs in the three possible situations: neither worker takes a vacation, one takes a vacation, both take vacations.

- 1. Neither worker takes a vacation. The employer gets no new information and the posterior beliefs are equal to the prior beliefs, $\beta = \frac{1}{2}$.
- 2. One worker takes a vacation.
 - If A takes a vacation and B does not, then the employer has an observation of B working alone and it uses this information to update beliefs. Suppose B produces q_h while A is gone. If B is the high-type worker, then this happens with probability p_h and if he is the low-type worker this happens with probability p_l . Hence, after observing B producing q_h , the employer believes B is the low-type worker with probability $\frac{p_l}{p_l+p_h}$, so $\beta = \frac{p_l}{p_l+p_h}$. Note that because $p_l < p_h$ it follows that $\beta < \frac{1}{2}$, which implies that the posterior probability that the employer assigns to B being the low-type worker is lower than the prior probability, as we might expect. If B produces q_l while A is gone, in a similar manner we derive $\beta = \frac{1-p_l}{1-p_l+1-p_h} > \frac{1}{2}$.
 - If B takes a vacation and A does not, then we find that $\beta = \frac{p_h}{p_l + p_h} > \frac{1}{2}$ if A produces q_h while B is gone and $\beta = \frac{1 p_h}{1 p_l + 1 p_h} < \frac{1}{2}$ if A produces q_l while B is

gone.

3. Both workers take vacations. If both A and B take a vacation, then the employer has an observation of A working alone and an observation of B working alone. Obviously, if both A and B produce the same output while working alone, then the employer gets no new information about their types and his posterior beliefs are equal to his prior beliefs, so $\beta = \frac{1}{2}$. But suppose A produces q_h while B is gone and B produces q_l while A is gone. This happens with probability $p_h(1-p_l)$ if A is the high-type worker and B the low-type worker and it happens with probability $p_l(1-p_h)$ if A is the low-type worker and B the high-type worker. Hence the posterior belief of the employer that A is the high-type worker is $\beta = \frac{p_h(1-p_l)}{p_h(1-p_l)+p_l(1-p_h)} > \frac{1}{2}$. Or, suppose A produces q_l while B is gone and B produces q_h while A is gone. Then the employer's posterior beliefs will be $\beta = \frac{(1-p_h)p_l}{(1-p_h)p_l+(1-p_l)p_h} < \frac{1}{2}$.

Now we are ready to start analyzing the workers' vacation problem, given that the employer will only use his observations of output to update his beliefs and set wages. First, we find (expected) wages for the three basic cases: neither worker takes a vacation, one takes a vacation, both take vacations.

- 1. Neither worker takes a vacation. Then the employer's posterior belief is $\beta = \frac{1}{2}$ and the workers get wages of $\frac{1}{2}w_l + \frac{1}{2}w_h$ each, with certainty.
- 2. One worker takes a vacation.
 - Suppose A is the high-type worker and B the low-type worker.
 - If A takes a vacation and B does not, then with probability p_l the employer observes B producing q_h while A is gone and with probability $(1 p_l)$ the

employer observes B producing q_l while A is gone. If the employer observes q_h , his posterior belief that A is the high-type worker is $\beta = \frac{p_l}{p_l + p_h}$ and if the employer observes q_l his posterior belief is $\beta = \frac{1-p_l}{1-p_l+1-p_h}$. If the posterior belief of the employer is β , then he will pay A a wage $(1-\beta)w_l + \beta w_h$ and B a wage $\beta w_l + (1-\beta)w_h$. Hence, the expected wage of A is $p_l \left[\left(1 - \frac{p_l}{p_l + p_h} \right) w_l + \frac{p_l}{p_l + p_h} w_h \right] + (1-p_l) \left[\left(1 - \frac{1-p_l}{1-p_l+1-p_h} \right) w_l + \frac{1-p_l}{1-p_l+1-p_h} w_h \right]$. We can re-arrange this as $(1-a)w_l + aw_h$, where $a := \frac{p_l + p_h - 2p_l p_h}{(2-p_l-p_h)(p_l+p_h)} \in \left(\frac{1}{2},1\right)$.

- If B takes a vacation and A does not, then with probability p_h the employer observes A producing q_h while B is gone and with probability $(1 - p_h)$ the employer observes A producing q_l while B is gone. In the same manner as before, we compute that in this case the expected wage of A is

$$p_h \left[\left(1 - \frac{p_h}{p_l + p_h} \right) w_l + \frac{p_h}{p_l + p_h} w_h \right]$$

$$+ (1 - p_h) \left[\left(1 - \frac{1 - p_h}{1 - p_l + 1 - p_h} \right) w_l + \frac{1 - p_h}{1 - p_l + 1 - p_h} w_h \right]$$

$$= (1 - a) w_l + a w_h.$$

• If A is the low-type worker and B the high-type worker and exactly one of the workers takes a vacation, then we use symmetry of the types and the results above to conclude that the expected wage of A is $aw_l + (1-a)w_h$.

So, when the employer only uses output to determine type, not vacation behavior itself, we find that it does not make any difference insofar as the expected wages of the workers are concerned whether the high type takes a vacation and the low type does not, or the other way around. While the probability of high or low output is affected by which worker type works alone, in both cases the employer gets exactly one

observation of a worker working alone. Given the assumption that one worker is of each type, this one observation reveals just as much information about the vacationing worker as about the one who remains at work and therefore the expected wages are the same in both cases.

- 3. Both workers take vacations. By assumption these vacations are at different times, so the employer now has two observations, one of each worker working alone. Since either worker can produce high or low output, there are four possible contingencies. We denote by q_h^A the event that A produces high output while B is gone and by q_l^A the event that A produces low output while B is gone. q_h^B and q_l^B are defined analogously.
 - Suppose again that A is the high-type worker and B the low-type worker. Then, the employer observes q_h^A and q_h^B with probability $p_h p_l$, q_l^A and q_l^B with probability $(1 p_h)(1 p_l)$, q_h^A and q_l^B with probability $p_h(1 p_l)$, and q_l^A and q_h^B with probability $(1 p_h)p_l$. Combining this with the posterior beliefs of the employer after different observations that we computed before, we find that the expected wage of A is

$$p_{l}p_{h}\left[\frac{1}{2}w_{l} + \frac{1}{2}w_{h}\right] + (1 - p_{l})(1 - p_{h})\left[\frac{1}{2}w_{l} + \frac{1}{2}w_{h}\right]$$

$$+p_{h}(1 - p_{l})\left[\left(1 - \frac{p_{h}(1 - p_{l})}{p_{h}(1 - p_{l}) + p_{l}(1 - p_{h})}\right)w_{l} + \frac{p_{h}(1 - p_{l})}{p_{h}(1 - p_{l}) + p_{l}(1 - p_{h})}w_{h}\right]$$

$$+(1 - p_{h})p_{l}\left[\left(1 - \frac{(1 - p_{h})p_{l}}{(1 - p_{h})p_{l} + (1 - p_{l})p_{h}}\right)w_{l} + \frac{(1 - p_{h})p_{l}}{(1 - p_{h})p_{l} + (1 - p_{l})p_{h}}w_{h}\right]$$

$$= (1 - b)w_{l} + bw_{h},$$

where
$$b := \frac{p_l + p_h - 4p_l p_h + p_l^2 + p_h^2}{2(p_l - 2p_l p_h + p_h)} \in (a, 1).$$

 \bullet If A is the low-type worker and B the high-type worker, then we use symmetry

of the types and the results above to conclude that the expected wage of A is $bw_l + (1-b)w_h$.

The fact that b > a shows that the expected wage of the high-type worker increases as the employer gets more observations.

Since the workers know their own and each other's types, and their expected payoffs only depend on their types, we can represent the independent decisions of the workers on whether or not to take a vacation as a game in strategic form in which we abstract from the workers' names (A or B) and identify the workers by their types. We assume that the workers' utility is separable in the expected wage and the utility v derived from a vacation. So, if a worker has an expected wage w, then his utility is w if he does not take a vacation and it is w + v if he does take a vacation. The strategic game between the workers is represented in the following diagram, where the utility of the high-type worker is given first in each cell.

low-type worker

		no vacation	vacation
	no vacation	$\frac{1}{2}w_l + \frac{1}{2}w_h,$	$(1-a)w_l + aw_h,$
high-type worker		$\frac{1}{2}w_l + \frac{1}{2}w_h$	$aw_l + (1-a)w_h + v$
	vacation	$(1-a)w_l + aw_h + v,$	$(1-b)w_l + bw_h + v,$
		$aw_l + (1-a)w_h$	$bw_l + (1-b)w_h + v$

This strategic game is easily analyzed. Since $b > a > \frac{1}{2}$, the high-type worker is always better off taking a vacation, no matter what the low-type worker does (note that $w_l < w_h$). This results from the fact that as the high-type worker takes a vacation, he makes sure the

employer gets an observation of the low-type worker working alone and this, as we have seen above, increases the expected wage of the high-type worker. He also gets the utility from taking a vacation.

The optimal strategy of the low-type worker depends on whether the utility derived from a vacation is enough to compensate for the lower expected wage that results from the extra output observation his vacation provides. If the high-type worker doesn't take a vacation, then the low-type worker loses $\left(a-\frac{1}{2}\right)\left(w_h-w_l\right)$ in expected wage if he takes a vacation and if the high-type worker does take a vacation, then the low-type worker loses $(b-a)\left(w_h-w_l\right)$ in expected wage if he takes a vacation. Since $a-\frac{1}{2}>b-a^1$, it follows that the low-type worker is always better off taking a vacation if $v>\left(a-\frac{1}{2}\right)\left(w_h-w_l\right)$. If $v<(b-a)\left(w_h-w_l\right)$, the low-type worker is always better off not taking a vacation. In the intermediate cases where $(b-a)\left(w_h-w_l\right)< v<\left(a-\frac{1}{2}\right)\left(w_h-w_l\right)$, the low-type worker's best reply is to mimic the vacation behavior of the high-type worker does not take one.

This concludes our analysis of the situation if the employer updates his beliefs using only his observations of output. However, we found that in this case the high-type worker will always take a vacation. So if the employer observes one worker taking a vacation and the other worker not taking a vacation, then he should conclude that the worker taking a vacation must be the high-type worker. But, if he does so, the employer's posterior beliefs will not be as we described above, because these beliefs are now based not only on output observations, but also on the decisions of the workers whether or not to take a vacation. Furthermore, if the employer's beliefs take this additional information into account, then the payoffs to the employees will also be different than above. Since it is reasonable to believe that the employer will use vacation decisions as signals of the types of the workers, we must consider

¹This can be easily verified from the formulas for a and b.

this more complex situation. We do this using the following extensive-form game.

3.3 The game

Figure 1 is a abbreviated representation of the extensive form of the game. We exclude the last stage, where the employer chooses the parameter α which determines the redistribution of wages, since by assumption the employer simply sets this equal to his beliefs β . So the last stage shown is the move by nature which determines the realized output levels. We analyze this game using the *sequential equilibrium* of Kreps and Wilson (1982) to impose restrictions on beliefs about information states off the equilibrium path. A sequential equilibrium consists of beliefs π and a strategy profile σ such that the following two conditions hold.

- 1. π is fully consistent with σ : there exists a sequence of completely mixed strategies that converge to σ such that the beliefs corresponding to these completely mixed strategies converge to π . Note that the beliefs corresponding to a completely mixed strategy are completely determined by Bayes' rule.
- 2. Each player's strategy in σ is sequentially rational at every information state with beliefs π : σ maximizes the players' payoffs given π at every information state.

A strategy profile σ is called *sequential equilibrium scenario* if there exist beliefs π such that π and σ together form a sequential equilibrium.

The sequential equilibrium restriction is sensible because it requires that the beliefs about states that are reached with probability zero must still be reasonable, in the sense that they can be derived from mixed strategies that are arbitrarily close to the strategies actually played. We now ask if the four possible pure strategy combinations are sequential equilibrium scenarios.

Note that we have set the game up with two workers, labeled A and B, one assigned by nature to be low type and the other high type. Therefore the strategies we consider are properly given in terms of decisions by the workers A and B. However, it seems more intuitive to think of the workers as the low type and the high type, and so we also give the strategies using those labels, in parentheses.

Lemma 1 The strategy in which A and B both decide to take a vacation, irrespective of their types, is a sequential equilibrium scenario. (The strategy in which both types of workers take a vacation is a sequential equilibrium scenario.)

Proof. Denote the strategy of worker A to always take a vacation by Y^A and the strategy of B to always take a vacation by Y^B .² To prove that these strategies form a sequential equilibrium scenario we have to find beliefs π such that (Y^A, Y^B) and π satisfy conditions 1 and 2 for a sequential equilibrium.

The beliefs that 'do the job' are as follows: if both workers take a vacation, and the employer has two observations, then the posterior beliefs are the beliefs β based on output that we have computed before. If one worker takes a vacation and the other doesn't, then the posterior belief of the employer is that the worker taking vacation is the high-type worker, no matter what output the other worker produced when working alone. If neither worker takes a vacation, then the employer's posterior beliefs are equal to his prior beliefs. We refer to this set of beliefs as π .

With these beliefs, and given that the other worker takes a vacation, both the high-type worker and the low-type worker want to take a vacation, because not taking a vacation will lead the employer to believe that he is the low-type worker and result in wage w_l . But taking a vacation too will lead to a higher expected wage $(bw_l + (1-b)w_h)$ for the low-type worker

 $^{^{2}}Y$ stands for yes, take a vacation, N will stand for no, do not.

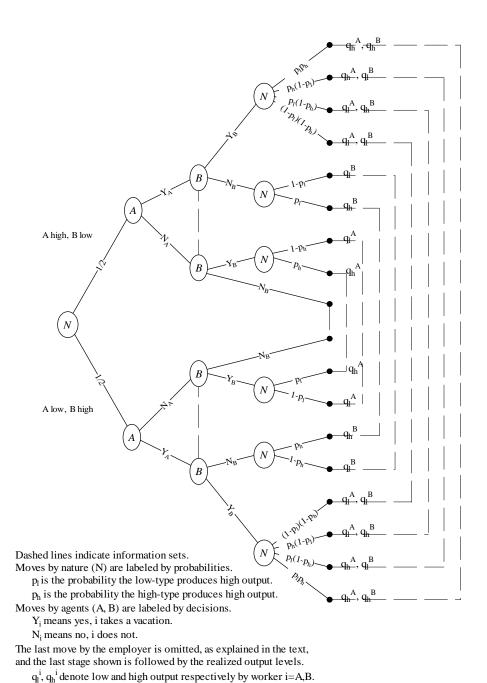


Figure 1: Abbreviated extensive form game.

and $(1 - b)w_l + bw_h$ for the high-type worker) and the vacation itself brings a utility of v. So, condition 2 is satisfied: (Y^A, Y^B) is sequentially rational with beliefs π .

To prove that π is fully consistent with (Y^A, Y^B) , consider the sequence of completely mixed strategies $(\sigma_t)_{t=1}^{\infty}$: according to σ_t both A and B choose to take a vacation with probability $1-\frac{1}{t}$ if they are the low-type worker and with probability $1-\frac{1}{t^2}$ if they are the high-type worker. Clearly, the strategies σ_t converge to (Y^A, Y^B) . The posterior beliefs that are obtained from the σ_t 's using Bayes's rule converge to the beliefs π . Checking this is straightforward, and we will only show one case. Suppose worker A takes a vacation and worker B doesn't and worker B produces q_h while working alone. According to strategy σ_t this happens with probability $\left(1-\frac{1}{t^2}\right)\frac{1}{t}p_l$ if B is the low-type worker and with probability $\left(1-\frac{1}{t}\right)\frac{1}{t^2}p_h$ if B is the high-type worker. Therefore, the posterior belief of the employer is that A is the high-type worker is $\pi_t = \frac{\left(1-\frac{1}{t^2}\right)\frac{1}{t}p_l}{\left(1-\frac{1}{t^2}\right)\frac{1}{t}p_l}$. Taking the limit gives the desired result: $\lim_{t\to\infty}\pi_t = \lim_{t\to\infty}\frac{\left(1-\frac{1}{t^2}\right)p_l}{\left(1-\frac{1}{t^2}\right)p_l} = \frac{p_l}{p_l} = 1$. Note that this is the posterior belief according to π .

Lemma 2 The strategy in which both A and B decide to take a vacation if they are the high-type worker and not to take a vacation if they are the low-type worker is not a sequential equilibrium scenario. (The strategy where the high-type worker takes a vacation and the low-type does not is not a sequential equilibrium scenario.)

Proof. Denote the strategy of worker A to take a vacation if he is the high-type worker and not to take a vacation if he is the low-type worker by (Y_h^A, N_l^A) and the strategy of B to take a vacation if he is the high-type worker and not to take a vacation if he is the low-type worker by (Y_h^B, N_l^B) . To prove that these strategies do not form a sequential equilibrium scenario we show that for any beliefs π that are fully consistent with $((Y_h^A, N_l^A), (Y_h^B, N_l^B))$ condition 2 for a sequential equilibrium is violated. Choose any beliefs π that are fully

consistent with $((Y_h^A, N_l^A), (Y_h^B, N_l^B))$. It follows that if one worker takes a vacation and the other doesn't, the posterior belief of the employer is that the worker taking vacation is the high-type worker, no matter what output the other worker produced when working alone. This result is obtained in a way similar to the case developed in the proof of lemma 1. Now suppose A is the high-type worker and B the low-type worker. Then A will take a vacation and B should not take a vacation according to his strategy (Y_h^B, N_l^B) . However, this will result in a wage w_l for B. If, however, B diverts from his strategy and takes a vacation, then he will not have a lower expected wage, no matter what the posterior beliefs of the employer will be, and on top of that B will derive utility from having a vacation. This shows that (Y_h^B, N_l^B) is not sequentially rational for B with beliefs π .

The intuition behind lemma 2 is that if the employer uses vacation behavior as a signal, then the low-type worker reveals himself to be low type if he does not take a vacation and the high-type worker does. Hence, not taking a vacation brings about a low wage and also deprives the worker of the utility that he would derive from taking a vacation. A worker's output does not matter to the employer's posterior beliefs and therefore to wages, because it provides only probabilistic information about his type which will never be enough to offset the firm's deterministic belief based on the vacation behavior.

In the following lemma we only consider symmetric strategies, i.e. strategies in which A and B make the same choices if they are of the same type. This is not a severe restriction: it just amounts to saying that it is the types of the workers that are important, not the names. If the strategies are not symmetric, the beliefs of the employer after both workers take a vacation can be practically anything, and this makes it impossible to check for sequential equilibria. If the strategies are not symmetric, the beliefs of the employer after both workers take a vacation can be practically anything and this makes it impossible to check for sequential equilibria. It may very well be possible that the strategy described in lemma 3 is a

sequential equilibrium scenario if we allow for non-symmetric strategies. Note that we have only encountered symmetric strategies in lemmas 1 and 2. However, in those lemmas this was only a matter of convenience, we did not have to restrict the analysis to such strategies. In lemma 3, however, we use this restriction in our proof of the lemma.

Lemma 3 The strategy in which both A and B decide to take a vacation if they are the low-type worker and not to take a vacation if they are the high-type worker is not a sequential equilibrium scenario if all non-symmetric strategies are discarded. (The strategy where the low-type worker takes a vacation and the high-type worker does not is not a sequential equilibrium scenario, if all non-symmetric strategies are discarded.)

Proof. Denote the strategy of worker A to take a vacation if he is the low-type worker and not to take a vacation if he is the high-type worker by (Y_l^A, N_h^A) and the strategy of B to take a vacation if he is the low-type worker and not to take a vacation if he is the high-type worker by (Y_l^B, N_h^B) . To prove that these strategies do not form a sequential equilibrium scenario we show that for any beliefs π that are fully consistent with $((Y_l^A, N_h^A), (Y_l^B, N_h^B))$ condition 2 for a sequential equilibrium is violated.

Choose any beliefs π that are fully consistent with $((Y_l^A, N_h^A), (Y_l^B, N_h^B))$. Let $(\sigma_t)_{t=1}^{\infty}$ be a sequence of completely mixed symmetric strategies that converges to $((Y_l^A, N_h^A), (Y_l^B, N_h^B))$: according to σ_t both A and B choose to take a vacation with probability $1 - \varepsilon_t$ if they are the low-type worker and with probability δ_t if they are the high-type worker, where $\lim_{t\to\infty} \varepsilon_t = \lim_{t\to\infty} \delta_t = 0$.

Suppose both workers take a vacation, such that the employer has two observations of output. We will show that the posterior belief according to π necessarily is the belief β based on output. Suppose A produces high output while working alone and B produces low output while working alone. According to strategy σ_t this happens with

probability $\delta_t (1 - \varepsilon_t) p_h (1 - p_l)$ if A is the high-type worker and B the low-type worker and with probability $(1 - \varepsilon_t) \delta_t p_l (1 - p_h)$ if A is the low-type worker and B the high-type worker. Therefore, the posterior belief of the employer that A is the high-type worker is $\pi_t = \frac{\delta_t (1 - \varepsilon_t) p_h (1 - p_l)}{\delta_t (1 - \varepsilon_t) p_h (1 - p_l) + (1 - \varepsilon_t) \delta_t p_l (1 - p_h)}$. Taking the limit gives the desired result: $\lim_{t \to \infty} \pi_t = \lim_{\varepsilon_t \to 0, \delta_t \to 0} \frac{(1 - \varepsilon_t) p_h (1 - p_l)}{(1 - \varepsilon_t) p_h (1 - p_l) + (1 - \varepsilon_t) p_l (1 - p_h)} = \frac{p_h (1 - p_l)}{p_h (1 - p_l) + p_l (1 - p_h)}$. Note that this is the posterior belief according to β . Similar computations show that π necessarily is also the belief β based on output for other production levels observed if both workers take a vacation.

It follows fairly easily that if one worker takes a vacation and the other doesn't, then the posterior belief of the employer according to π is that the worker taking vacation is the low-type worker, no matter what output the other worker produced when working alone. Further, if both workers do not take a vacation, then the employer's posterior belief is equal to his prior belief and he believes that A is the high-type worker with probability $\frac{1}{2}$.

Now, we will prove that the strategies (Y_l^A, N_h^A) and (Y_l^B, N_h^B) are not sequentially rational with beliefs π .

Suppose A is the high-type worker and B the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and does not take a vacation. Then B should take a vacation according to his strategy (Y_l^B, N_h^B) . This will result in a wage w_l for B and the utility v from having a vacation. If, however, B diverts from his strategy and does not take a vacation, then he will have a higher expected wage, namely $\frac{1}{2}w_l + \frac{1}{2}w_h$, but he will lose the utility from having a vacation. Hence, it is optimal for B to take a vacation if $w_l + v \ge \frac{1}{2}w_l + \frac{1}{2}w_h$, or $v \ge \frac{1}{2}(w_h - w_l)$.

Now, suppose B is the high-type worker and A the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and takes a vacation. If B sticks with his strategy (Y_l^B, N_h^B) and does not take a vacation he gets wage w_h . If he does take a vacation, he gets a lower expected wage, $(1 - b)w_l + bw_h$, but he will gain the utility v from having a

vacation. Hence, it is optimal for B not to take a vacation if $w_h \ge (1-b)w_l + bw_h + v$, or $v \le (1-b)(w_h - w_l)$.

We have shown that for (Y_h^B, N_l^B) to be sequentially rational for B with beliefs π it must hold that $v \geq \frac{1}{2}(w_h - w_l)$ and $v \leq (1 - b)(w_h - w_l)$. It is clearly impossible for these two conditions to be satisfied at the same time because $b > \frac{1}{2}$.

Intuitively, if it pays for the low-type worker to take a vacation despite the fact that this identifies him as low type, then the utility of a vacation must be high enough that the high-type worker will also want to take a vacation, despite the wage penalty he will get from pooling. Longer vacations will strengthen this result, since they will increase b.

The last symmetric strategies we consider are the strategies in which neither A nor B decide to take a vacation.

Lemma 4 The strategy in which A and B both decide not to take a vacation, irrespective of their types, is a sequential equilibrium scenario iff $v \leq \frac{1}{2}(w_h - w_l)$. (The strategy where both the high-type and the low-type workers do not take vacations is a sequential equilibrium scenario iff $v \leq \frac{1}{2}(w_h - w_l)$.)

Proof. Denote the strategy of worker A to never take a vacation by N^A and the strategy of B to never take a vacation by N^B . Assume that $v \leq \frac{1}{2}(w_h - w_l)$. To prove that the strategies (N^A, N^B) form a sequential equilibrium scenario we have to find beliefs π such that (N^A, N^B) and π satisfy conditions 1 and 2 for a sequential equilibrium.

Beliefs that 'do the job' are as follows: if both workers take a vacation, and the employer has two observations, then the posterior beliefs are the beliefs β based on output that we have computed before. If one worker takes a vacation and the other does not, then the posterior belief of the employer is that the worker taking vacation is the low-type worker, no matter what output the other worker produced when working alone. If neither worker takes

a vacation, then the employer's posterior beliefs are equal to his prior beliefs. We refer to this set of beliefs as π .

With these beliefs, and given that the other worker does not take a vacation, neither the high-type worker nor the low-type worker wants to take a vacation. This can be seen as follows: taking a vacation will lead the employer to believe that he is the low-type worker and result in wage w_l and the vacation itself brings a utility of v. But taking no vacation will lead to the higher expected wage $\frac{1}{2}w_l + \frac{1}{2}w_h$. So, taking no vacation is sequentially rational if $\frac{1}{2}w_l + \frac{1}{2}w_h \ge w_l + v$, or $v \le \frac{1}{2}(w_h - w_l)$.

To prove that π is fully consistent with (N^A, N^B) , consider the sequence of completely mixed strategies $(\sigma_t)_{t=1}^{\infty}$: according to σ_t both A and B choose to take a vacation with probability $\frac{1}{t}$ if they are the low-type worker and with probability $\frac{1}{t^2}$ if they are the high-type worker. Clearly, the strategies σ_t converge to (N^A, N^B) . The posterior beliefs that are obtained from the σ_t 's using Bayes' rule converge to the beliefs π . Checking this is a straightforward exercise so we will only do so for one case, as an example.

Suppose worker A takes a vacation and worker B doesn't and worker B produces q_h while working alone. According to strategy σ_t this happens with probability $\frac{1}{t^2}\left(1-\frac{1}{t}\right)p_l$ if B is the low-type worker and with probability $\frac{1}{t}\left(1-\frac{1}{t^2}\right)p_h$ if B is the high-type worker. Therefore, the posterior belief of the employer is that A is the high-type worker is $\pi_t = \frac{\frac{1}{t^2}\left(1-\frac{1}{t}\right)p_l}{\frac{1}{t^2}\left(1-\frac{1}{t}\right)p_l+\frac{1}{t}\left(1-\frac{1}{t^2}\right)p_h}$. Taking the limit gives the desired result:

$$\lim_{t \to \infty} \pi_t = \lim_{t \to \infty} \frac{\frac{1}{t} \left(1 - \frac{1}{t} \right) p_l}{\frac{1}{t} \left(1 - \frac{1}{t} \right) p_l + \left(1 - \frac{1}{t^2} \right) p_h} = \frac{0}{0 + p_h} = 0.$$

Note that this is the posterior belief according to π .

We have chosen a specific set of beliefs π that are fully consistent with (N^A, N^B) and for this set of beliefs we obtained the condition $v \leq \frac{1}{2} (w_h - w_l)$ necessary for (N^A, N^B) to be sequentially rational. We could have chosen other sets of beliefs that are fully consistent with (N^A, N^B) and then we could have found another upper bound on v. However, the upper bound so obtained must be lower than $\frac{1}{2} (w_h - w_l)$. This can be seen as follows. It is not hard to show that the posterior beliefs of the employer if none of the workers takes a vacation must be equal to his prior beliefs if the beliefs are to be fully consistent with (N^A, N^B) . The beliefs after observing one worker taking a vacation can be practically anything. However, they will always lead to a worker taking a vacation receiving an expected wage $w \geq w_l$ and a utility v from the vacation. Hence, taking no vacation is sequentially rational if $\frac{1}{2}w_l + \frac{1}{2}w_h \geq w + v$, or $v \leq \frac{1}{2}w_h - w + \frac{1}{2}w_l \leq \frac{1}{2} (w_h - w_l)$.

In lemmas 1 through 4 we addressed the question of whether the 4 symmetric pure strategy combinations for the workers are sequential equilibrium scenarios. We did not address mixed strategies in those lemmas. We were unable to solve for mixed strategy sequential equilibrium scenarios analytically, so we check for the existence of such scenarios for specific parameter values. We found unique sequential equilibrium scenarios in mixed strategies for all the parameter values we tried.

4 Discussion

We have shown that we can support two different strategies as sequential equilibrium scenarios. One equilibrium scenario is where both workers take vacations, another is where neither worker takes a vacation. The cases where only one type of worker takes a vacation are *not* supportable as sequential equilibrium scenarios under *any* consistent beliefs (considering only symmetric strategies). We begin this section of the paper by considering which of the two

sequential equilibrium scenarios is most plausible. We then ask whether the employer might prefer other outcomes, and consider what sorts of changes in vacation policy might produce results that are more preferable to the employer. The conclusion to the paper will connect these results to actual vacation behavior and policies.

Which equilibrium scenario is achieved depends first on the employer's beliefs, and in the case of the (N^A, N^B) equilibrium scenario there is also the necessary requirement that $v \leq \frac{1}{2} (w_h - w_l)$. Since if this requirement is not satisfied, the (N^A, N^B) equilibrium scenario is not supportable by any consistent beliefs, it is worth asking if this condition is likely to hold. Suppose that w_l and w_h denote annual wages, that a vacation is two weeks long, and that $w_h = 2w_l$. Then the condition $v \leq \frac{1}{2} (w_h - w_l)$ translates to $v \leq \frac{w_l}{2}$. Dividing w_l by 26 to get the bi-weekly wage, this amounts to saying that the utility from a two week vacation cannot be more than 13 times what a worker positively identified as the low type would have received in wages for that two week period. Looking at it the other way, if we assume that the utility from a two week vacation is equal to the average wage for two weeks work, the above condition implies $\frac{1}{2}(\frac{w_l}{26} + \frac{w_h}{26}) \leq \frac{1}{2}(w_h - w_l)$, or that w_h must be at least 8% greater than w_l . Either way, it seems quite possible that this condition may be satisfied in practice, and therefore that, given the appropriate beliefs, the equilibrium scenario may be one where neither worker wants to take a vacation.

Given this possibility, we ask which equilibrium scenario the employer will prefer, and how he might attempt to ensure that it is achieved. In the model developed above we have assumed that the employer will pay the workers w_l and w_h if he knows their types with certainty, and that if he does not he will pay each worker a weighted average of those wages, with weights equal to his perceived probability that they are of each type. This captures the idea that the employer gets a benefit from rewarding high-type workers and penalizing low-type ones, but bears a cost when he gives these rewards and penalties to the wrong

types. However, this wage function does not say anything about why or by how much the employer might gain from better information about the workers types: it just specifies how he will use the information he has. Now we assume that the employer gets some benefit from better information about types (this will increase with the difference between p_h and p_l), and that he bears some cost if workers take paid vacations. Depending on the relative weights of these two effects, the employer may clearly prefer either equilibrium scenario, or may even prefer that the workers take more vacation time than in the (Y^A, Y^B) outcome, in order to obtain still more information about types.

If he does not like an equilibrium scenario, the employer can try promote a more desired result by altering wages, by altering how he divides wages conditional on his assessments of the workers' types, by not paying wages during vacations, by altering the length of the vacation period, or simply by removing the element of choice and either prohibiting vacations or making them mandatory. These actions can be expected not only to alter how much knowledge the employer derives about his workers' types, but also to alter the kinds of workers who will tend to apply to the firm. While we leave proofs for future work, it seems reasonable that the higher the probability that the employer will learn the true types of his workers, the more likely it is that the low types will be scared away from even applying. (Though this need not always be the case, say if low types have higher tastes for vacation time.) This should provide the employer with another reason to prefer those equilibrium scenarios that tend to provide more information about workers' types.

First we will consider policies that simply attempt to encourage workers to adopt a particular sequential equilibrium scenario, rather than force them to take vacation time. Wages are one such policy tool. (Though there will certainly be other motives for setting particular wages, which may limit the employer's use of them for this purpose.) From lemma 4, decreasing the gap between w_l and w_h will make it less likely that the (N^A, N^B)

equilibrium scenario can be sustained. The intuition is that the smaller wage gap lowers the penalty to taking a vacation and having the employer believe you are low type, which is the belief that sustains that equilibrium scenario. So, employers who want to promote the (Y^A, Y^B) equilibrium scenario, to get more information about types, might want to equalize wages. But this benefit may be mitigated by the fact that equalizing wages will also tend to encourage low types to apply in the first place.

Interestingly, the net effect of a decreased wage gap on the expected wage of workers is uncertain. It reduces the premium to being identified as the high type, but as explained above, it also makes it more likely that the high-type worker will be identified as such. So, paradoxically, the high-type worker may well prefer a lower wage premium, because of the higher probability that he will then be identified as high.

Another variable under the employer's control is the length of vacations. Longer vacations mean that, in the (Y^A, Y^B) pooling equilibrium scenario, the employer will get more observations of output and therefore more information about worker types. In the model we have developed above longer vacations will never be enough to destroy the (Y^A, Y^B) equilibrium scenario, because they only provide probabilistic information about types that will not offset the employers deterministic beliefs from vacation behavior, when only one worker vacations. However, in a model where the employer's beliefs from vacation behavior were uncertain, or in which workers made random errors in their strategies, longer vacations might conceivably destroy the (Y^A, Y^B) equilibrium scenario, by reducing the advantages to the low-type worker of mimicking a high-type worker who took a vacation.

Longer vacations should also make it less likely that the (N^A, N^B) equilibrium can be sustained, because longer vacations will presumably have higher v's, raising the benefit to deviating from the equilibrium scenario by taking a paid vacation. These longer vacations will in general be more expensive to the employer, though, to the extent that workers get

utility from time off, the cost may be mitigated by their willingness to accept lower overall annual wages in return. Still, this cost may well limit the employer's desire to use longer vacation periods as a way of collecting information about types.

This raises the possibility that the firm may prefer not to pay workers during vacations. While we do not derive optimal strategies in this situation, it seems possible that this might produce very different results from those derived from the model above. For example, if the lost wages for the low type are enough larger than v, (Y^A, Y^B) might not be an equilibrium scenario: the low type could prefer to work, even though by doing so the employer identifies him as the low type. On the other hand, the resulting lower wage then reduces the cost of a vacation, so the net effect is uncertain.

One extension to the model we have developed in this paper would be to allow output to depend on effort, rather than ability. Suppose high effort leads to a higher probability of high output, but it is costly, and more costly to some than others. Assume that the employer would like to know these costs, and that he will reward low-cost workers more than high-cost ones. Employees have either low costs or high costs, and again assume that the employer knows he has one of each type. (However, this assumption is more difficult to defend in this case than in the ability case.) While we leave the modeling of vacation choices in this situation for future research, we can say some things about effort, given those choices. The optimal behavior follows from the fact that each worker will want output to drop when they are on vacation, but not when their co-worker is.

Suppose that only one worker takes a vacation. Then the other worker can minimize the expected drop in output during that vacation by slacking off when both are working, and increasing effort when he is the only worker. The worker who vacations can maximize the expected output drop by working hard both before and after the vacation.

However, if both workers take vacations, at different times, things are more complicated.

Each worker can now set two levels of effort, one while both are working and one while the other is on vacation. Clearly each will want to set a high level of effort when they are working alone. However, there are two opposing effects that determine their effort when both are working. If they work hard, output will tend to drop during their own vacation, which makes them look like the high type. But, working hard will also mean that output will tend to drop during the other worker's vacation, which makes them look like the low type. So, the overall level of effort during the period when both work is not obvious.

5 Conclusion

In this paper we have developed a simple model of vacation behavior. The model captures some of the strategic considerations of employees in a situation where the employer uses the output changes that occur when the composition of a team changes to estimate the productivity of individual team members. One of the key distinctions between our model and previous work on signaling in the workplace is that we explicitly address the strategic play among workers, along with that between workers and the employer. We believe this is an important improvement, and that models of the sort we develop in this paper are appropriate for studying a host of games of workplace intrigue. Convincing the boss that you are a productive worker is vital to career advancement, and efforts to do so are always complicated by the fact that your co-workers are simultaneously trying to prove that they are the ones making the business succeed, not you.

The policies discussed above leave open the possibility that the employer will be dissatisfied with the amount of information about types that is revealed in an equilibrium scenario, or that the employer may find that the cost of encouraging the employees to reveal this information voluntarily exceeds his value. As an alternative, the employer could simply require the employees to take some fixed amount of vacation time. This is in fact the policy that the Federal Reserve has implemented for banks, and one that many other firms also follow.

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