Math 5320, 2/9/18 Name:

Worksheet 12: An application of Zorn's Lemma to maximal ideals

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1. Let $\varphi : R \to S$ be a ring homomorphism. Show that the image of φ, denoted by $\varphi(R)$, is a subring of S.

(See also section A.3 of the textbook for a discussion of Zorn's lemma)

A *partial ordering* of a set S is a relation $s \leq s'$ which may hold between some elements of S and satisfies the following axioms for all $s, s', s'' \in S$:

(i) $s \leq s$

(ii) if $s \leq s'$ and $s' \leq s''$, then $s \leq s''$

(iii) if $s \leq s'$ and $s' \leq s$, then $s = s'$.

A partial ordering is called a *total ordering* if, in addition

(iv) for all $s, s' \in S$, either $s \leq s'$ or $s' \leq s$.

An element $m \in S$ is *maximal* if there is no s such that $m \leq s$ except for m itself.

If A is a subset of a partially ordered set S, then an *upper bound* for A is an element $b \in S$ such that for all $a \in A$, $a \leq b$.

A partially ordered set S is *inductive* if every totaly ordered subset T of S has an upper bound.

Lemma (Zorn's Lemma)**.** *An inductive partially ordered set S has at least one maximal element.*

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2. Let R be a ring and I and ideal in R. Use Zorn's Lemma to prove that I is contained in a maximal ideal.

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