

1. Let R be an integral domain and $p, q \in R$. Show that if p and q divide one another, they are associates.

2. (a) Consider the elements 6 and $2 + 2\sqrt{-5}$ in $\mathbb{Z}[\sqrt{-5}]$. Both of these elements are divisible by 2 and $1 + \sqrt{-5}$. Using the complex norm on $\mathbb{Z}[\sqrt{-5}]$, show that neither 2 nor $1 + \sqrt{-5}$ divides the other.

- (b) Also, show that 2 and $1 + \sqrt{-5}$ are both “maximal” as common divisors of 6 and $2 + 2\sqrt{-5}$ in the sense that $\frac{6}{2}$ and $\frac{2+2\sqrt{-5}}{2}$ have no divisors in common and $\frac{6}{1+\sqrt{-5}}$ and $\frac{2+2\sqrt{-5}}{1+\sqrt{-5}}$ have no divisors in common.