Math 5320, 2/23/18 Name:

Worksheet 17: Proving that factorization terminates in PID's, polynomial long division with monic divisors

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Let R be an integral domain. If factorization doesn't terminate in R, then there is some $a \in R$ that can be properly factored, and at least one of its factors can be properly factored, and at least one of those factors can be properly factored and so on, forever. We can write this as

$$a = a_1b_1 = a_2b_2b_1 = a_3b_3b_2b_1 = \cdots \quad \text{where } a_n = a_{n+1}b_{n+1} \text{is a proper factorization for all } n$$

This means we have an infinite chain of ideals properly contained in one another(set $a = a_0$):

$$(a_0) \subsetneq (a_1) \subsetneq (a_2) \subsetneq \cdots$$

1. Prove that the union of ideals $\bigcup_{n\in\mathbb{N}}(a_n)$ is an ideal in R.

2. Let $f(x, y)$ be an element of $\mathbb{C}[x, y]$. Prove that if $f(x, 0) = 0$, then there is some $g(x, y) \in \mathbb{C}[x, y]$ such that $f(x, y) = y \cdot g(x, y)$.

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