Math 5320, 3/9/18 Name:

Worksheet 21: More about field extensions

1. After the last lecture, we know that $\mathbb{Q}(\sqrt{2}) \cong \mathbb{Q}[x]/(x^2+2)$ is a field. Since it's a field, every nonzero element must have (multiplicative) inverse. What is the multiplicative inverse of x?

2. We talked about the fact that if $F \subset K$ is a field extension, then K is also an F-vector space. Here the axioms of a vector space to help us meditate on why this is true:

   An F-vector space is a commutative group V under an operation we call $+$ with identity 0, along with scalar multiplication by elements of the field satisfying the following axioms:

   - $1v = v$ for all $v \in V$
   - $(ab)v = a(bv)$, $(a+b)v = av + bv$ and $a(v+w) = av + aw$ for all $a, b \in F, v, w \in V$ (associative and distributive laws)

   It's also true if we have an injective ring homomorphism $F \subset R$ and R is an integral domain, then R is also an F-vector space. What's the basis of the polynomial ring $F[x]$ as an F-vector space?

3. What is a basis for $\mathbb{Q}(\sqrt{2})$ as a $\mathbb{Q}$-vector space?

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