1. For any ring \( R \), there is an action of the symmetric group \( S_n \) on \( R[x_1, \ldots, x_n] \) permuting the variables. A polynomial in \( R[x_1, \ldots, x_n] \) that is fixed by every permutation in \( S_n \) is called \textit{symmetric}. Why does the set of symmetric functions in \( R[x_1, \ldots, x_n] \) form a subring?

2. The \textit{elementary symmetric functions} in \( R[x_1, \ldots, x_n] \) are defined to be

\[
\begin{align*}
s_1 &:= x_1 + x_2 + \cdots + x_n \\
s_2 &:= \sum_{i < j} x_i x_j \\
&\vdots \\
s_n &:= x_1 \cdots x_n
\end{align*}
\]

The Symmetric Functions Theorem (Theorem 16.1.6) tells us that any symmetric polynomial in \( R[x_1, \ldots, x_n] \) can be written as a polynomial in the elementary symmetric functions, that is, as \( p(s_1, \ldots, s_n) \in R[s_1, \ldots, s_n] \subseteq R[x_1, \ldots, x_n] \).

Write the symmetric polynomial \( x_1^2 + x_2^2 + x_3^2 \in \mathbb{Q}[x_1, x_2, x_3] \) as a polynomial in the elementary symmetric functions \( s_1, s_2, s_3 \).

3. Using the proof of the Primitive Element theorem, what are some possible primitive elements for the extension \( \mathbb{Q}(\sqrt{2}, \sqrt{3}) \) of \( \mathbb{Q} \)?