1. Show that cancellation works in integral domains: if \( R \) is an integral domain and \( a, b, c \in R \) such that \( ac = bc \), show that \( b = c \).

2. Show that if \( R \) is an integral domain, the degree of the product of two polynomials \( f(x), g(x) \) in \( R[x] \) is the sum of the degrees of \( f \) and \( g \). Find an example of a ring \( R \) that isn’t an integral domain and some polynomials with coefficients in \( R \) that don’t obey this rule!

3. Show that if \( R \) is an integral domain then so is \( R[x] \).

4. Show that when you divide \( f(x) = x^2 + \frac{1}{2} \) by \( x - 3 \) in \( \mathbb{Q}[x] \), the remainder is the constant polynomial \( f(3) \).

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