1 Pascal’s triangle

1. Multiply to find the following polynomials:
   (a) \((x + 1)^3\)
   (b) \((x + 1)^4\)
   (c) \((x + 1)^5\)
   (d) \((x + 2)^3\)
   (e) \((x - 1)^3\)

2. How many ways are there to choose 2 marbles from a bag of 5 marbles?

3. How many ways are there to choose 3 marbles from a bag of 5 marbles?

4. How many ways are there to choose 4 marbles from a bag of 5 marbles?

2 Roots of unity and the complex plane

Some more cool examples of where cyclic groups arise are roots of unity, which live inside the complex plane.

1. Find all the solutions to the equation \(x^2 = 1\), that is, all the “square roots of unity”. Check that these form a group under multiplication.

2. Find all the solutions to the equation \(x^3 = 1\) (hint: you will need the complex numbers). Check that these form a group under multiplication.

3. Find all the solutions to the equation \(x^4 = 1\). Check that these form a group under multiplication.

4. Graph the solutions to each of the previous problems. What do they look like?

5. Use Euler’s formula \(e^{i\theta} = \cos(\theta) + i\sin(\theta)\) to write your solutions to the previous questions as complex exponentials. What patterns do you notice?

6. Can you express what all the \(n\)-th roots of unity will look like in terms of complex exponentials?

7. Do the cube roots of unity form a group under multiplication? Do the fourth roots of unity form a group under multiplication? Do the \(n\)-th roots of unity form a group under multiplication?

8. How many generators will these groups have?