Astronomy 323
Homework 2
Due: Monday, 2018 January 29

6. Show that the current proper distance to the particle horizon, the most distant place we can see, for a matter dominated, $k = 0$ universe with no cosmological constant, is given by $r_h R_\circ = 3ct_\circ$. Here, r_h is the comoving radial coordinate of the particle horizon, $R_\circ$ is the current scale factor, and $t_\circ$ is the current age of the universe. Why is this answer not simply $ct_\circ$? Hint: light moves along null geodesics where $ds^2 = 0$ and the scale factor for the universe described in this problem is $R(t) = R_\circ(t/t_\circ)^{2/3}$.

7. A universe has curvature $k = 0$ and $\Omega_\Lambda = 1$. Let $R(0) = R_\circ$. Find the comoving coordinate of a galaxy that sits at the particle horizon, r_h, at time t. Comment on your result.

8. You are a two-dimensional creature living on a sphere with radius R. Find the circumference of a circle of radius r drawn on the sphere. Suppose you can only measure distances to $\pm$ 1 cm, how large of a circle would you need to draw to convince yourself the Earth is spherical in shape.

9. Suppose the energy density of the cosmological constant, ε_Λ, was equal to the critical energy density. How much energy is contained in the energy of the cosmological constant in a sphere of radius 1 Astronomical Unit? Compare this energy to the rest mass energy of the Sun. Does the energy in the cosmological constant play a large role in the dynamics of the Solar System? Explain your answer.

10. By making the substitutions

$$x = r\sin\theta\cos\phi \tag{1}$$
$$y = r\sin\theta\sin\phi \tag{2}$$
$$z = r\cos\theta \tag{3}$$

demonstrate that equations (3.12) and (3.13) represent the same metric.