Astronomy 323
Homework 3
Due: Wednesday, 2018 February 7

11. For a $k = 0$ universe with $\Omega_\Lambda = 1$, (a) find the comoving radius r_E of galaxies that will be on our event horison at time t in the future. Show that r_E shrinks exponentially; (b) Assume that $H_\circ = 70$ km s^{-1} Mpc^{-1} and find within how many years galaxies in the Virgo cluster (distance ~ 15 Mpc) reach the event horizon.

12. (a) Show that the rate of change of the redshift, z, is given by

$$\frac{\mathrm{d}z}{\mathrm{d}t_\circ} = H_\circ(1+z) - H(z) \tag{1}$$

where $H(z) = \dot{a_{em}}/a_{em}$ is the Hubble parameter at the time of emission, t_{em}. (b) Show that for $k = 0$ and $\Lambda = 0$,

$$H(z) = H_\circ(1+z)^{1.5}. \tag{2}$$

Given this and assuming $H_\circ = 70$ km s^{-1} Mpc^{-1}, find the change in redshift after 10 years for a source at $z = 1$.

13. In a positively curved universe with $\Omega_{\circ,m} > 1$, $\Omega_\Lambda = 0$, and $k = +1$, show that the present age of the universe is given by

$$H_\circ t_\circ = \frac{\Omega_\circ}{2(\Omega\circ - 1)^{1.5}} \cos^{-1}\left(\frac{2-\Omega_\circ}{\Omega_\circ}\right) - (\Omega_\circ - 1)^{-1} \tag{3}$$

For $H_\circ = 70$ km s^{-1} Mpc^{-1}, plot $t_\circ$ for various $\Omega_\circ$.

14. As an alternative to a cosmological constant, the universe may contain another quantum field, *quintessence* which also has a positive energy density and negative w. (a) For a flat universe containing non-relativistic matter and *quintessence* with $w =$ -0.5 and $\Omega_{\circ,matter} \leq 1$, find a(t) for which the energy densities in matter and *quintessence* are equal. (b) Find a(t) for the universe. (c) What are the forms for a(t) in the limits in the regions dominated by matter and dominated by *quintessence*? (d) What is the current age of the universe in terms of $H_\circ$ and $\Omega_{\circ,matter}$?

15. Einstein introduced Λ to stabilize a mass filled universe against expansion or contraction.

a. Find an expression for Λ in terms of the density.

b. Find an expression for the curvature k. Is the universe open, closed, or flat?

c. Explain why Einstein's model is an unstable equilibrium so that any departure from equilibrium (expansion or contraction) would tend to continue.