

16. The surface brightness  $\Sigma$  of an object is defined as its observed flux divided by its observed angular area. For a class of objects that are both standard candles and standard yardsticks, what is  $\Sigma$  as a function of redshift  $z$ ? Would observing the surface brightness of this object be a useful way of determining the deceleration parameter?

17. You observe a quasar at  $z = 5$ , and determine that the observed flux of light from the quasar varies on a timescale of 3 days. If the observed variation is due to a variation in the intrinsic luminosity of the quasar, what was the variation in timescale at the time the light was emitted? For the bulk of the light from the quasar to vary on a timescale  $\delta t_e$ , the bulk of the light must come from a region of physical size  $R \leq R_{max} = c \delta t_e$ . What is  $R_{max}$  for the quasar? What is the angular size of  $R_{max}$  in the Benchmark model?

18. Verify that

$$m - M \approx 43.17 - 5 \log_{10} \left( \frac{H_0}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right) + 5 \log_{10} z + 1.086(1 - q_0)z \quad (1)$$

is correct in the limit of small  $z$ .

19. The Draco galaxy has luminosity  $L + (1.8 \pm 0.8) \times 10^5 L_\odot$  with half of its  $L$  contained within a sphere of radius  $120 \pm 12$  pc. The dispersion velocity of the stars in Draco is  $\sigma = 10.5 \pm 2.2 \text{ km s}^{-1}$ . What is the mass of the Draco galaxy? What is its mass-to-light ratio? Describe possible sources of error.

20. A ray of light grazes the surface of the Earth. Through what angle is the light ray bent by gravitational lensing? Ignore effects of the Earth's atmosphere. Repeat your calculation for a white dwarf and for a neutron star.