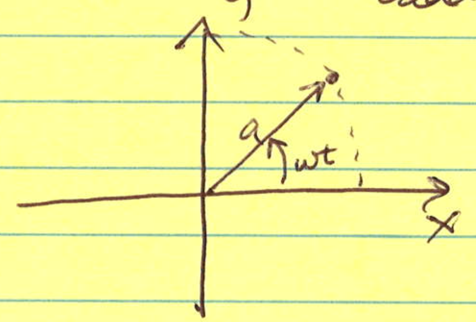
b) Let $z = \underline{a}e^{i\omega t}$. Show that the particle travels in circle of radius $\underline{a}$ at speed $\underline{\omega a}$ and w/acceleration v^2/a directed toward the center of circle

Sol^{n}

(i) $z = x + iy = a\cos\omega t + ia\sin\omega t$

$|z| = \sqrt{a^2\cos^2\omega t + a^2\sin^2\omega t}$

$r = a$, center by the



(ii) $\frac{dz}{dt} = \Big[\underbrace{-\omega a\sin\omega t + i\omega a\cos\omega t}_{\omega i z}\Big]$

$v = \left|\frac{dz}{dt}\right| = \sqrt{\omega^2 a^2\sin^2\omega t + \omega^2 a^2\cos^2\omega t}$

$= \omega a$ ✓

(iii) $\frac{d^2z}{dt^2} = -\omega^2 a\cos\omega t - i\omega^2 a\sin\omega t$

$= -\omega^2\Big[\underbrace{a\cos\omega t + ia\sin\omega t}_{}\Big]$

$-\omega^2 z$

(8) Chapter 2, 16-11

Prove that

(i) $\cos\theta + \cos 3\theta + \cos 5\theta + \cdots + \cos(2n-1)\theta = \dfrac{\sin 2n\theta}{2\sin\theta}$

(ii) $\sin\theta + \sin 3\theta + \sin 5\theta + \cdots + \sin(2n-1)\theta = \dfrac{\sin^2 n\theta}{\sin\theta}$

Soln

note: $e^{i\theta} + e^{i3\theta} + e^{i5\theta} + e^{i7\theta} + \cdots + e^{i(2n-1)\theta} = R$

$(\cos\theta + i\sin\theta) + (\cos 3\theta + i\sin 3\theta) + (\cos 5\theta + i\sin 5\theta) + \cdots$
$+ (\cos[2n-1]\theta + i\sin[2n-1]\theta) = ?$

note: $\dfrac{1}{1-x} = ?$, let's do the division.

$$\begin{array}{r|l} & 1+x+x^2+\cdots \\ 1-x & 1 \\ & \underline{1-x} \\ & x \\ & \underline{x-x^2} \\ & x^2 \\ & \underline{x^2-x^3} \\ & x^3 \end{array}$$

, wow

$1 + x + x^2 + \cdots = \dfrac{1}{1-x}$

or $\sum_{i=0}^{\infty} x^i = \dfrac{1}{1-x}$

note: $\dfrac{e^{i\theta}}{1-e^{2i\theta}} = e^{i\theta} + e^{i3\theta} + e^{i5\theta} + \cdots$

Pretty good, but we have a finite series. We want the series to truncate at $(2n-1)\theta$ of fixed n.

Consider: $\dfrac{e^{2i(n-1)\theta}}{1-e^{-2i\theta}}$

$$
\begin{array}{r l}
 & e^{2i(n-1)\theta} + e^{2i(n-2)\theta} + e^{2i(n-3)\theta} + \cdots + e^{i\theta} \\
1-e^{-2i\theta} \,\big) & e^{2i(n-1)\theta} \\
 & \underline{e^{2i(n-1)\theta} - e^{2i(n-2)\theta}} \\
 & e^{2i(n-2)\theta} \\
 & \underline{e^{2i(n-2)\theta} - e^{2i(n-3)\theta}} \\
 & e^{2i(n-3)\theta}
\end{array}
$$

and our series sums to

$$e^{i\theta} + e^{3i\theta} + e^{i5\theta} + \cdots + e^{i(2n-1)\theta} = \frac{e^{2i(n-1)\theta}}{1-e^{-2i\theta}}$$

$$\vdots \qquad = \frac{e^{2in\theta}}{e^{i\theta}-e^{-i\theta}}$$

$$= \frac{\cos 2n\theta + i\sin 2n\theta}{2i\sin\theta}$$

$$\left.\begin{array}{l} \text{Real, } \cos\theta + \cos 3\theta + \cos 5\theta + \cdots + \cos(2n-1)\theta \\ \text{Img, } \sin\theta + \sin 3\theta + \sin 5\theta + \cdots + \sin(2n-1)\theta \end{array}\right\} = \frac{1}{2}\frac{\sin 2n\theta}{\sin\theta} - \frac{i}{2}\frac{\cos 2n\theta}{\sin\theta}$$