Homework 1

Due: January 14, 2013 (revised due date, January 16, 2013):w

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1. 5.3

2. 5.4

3. 5.11

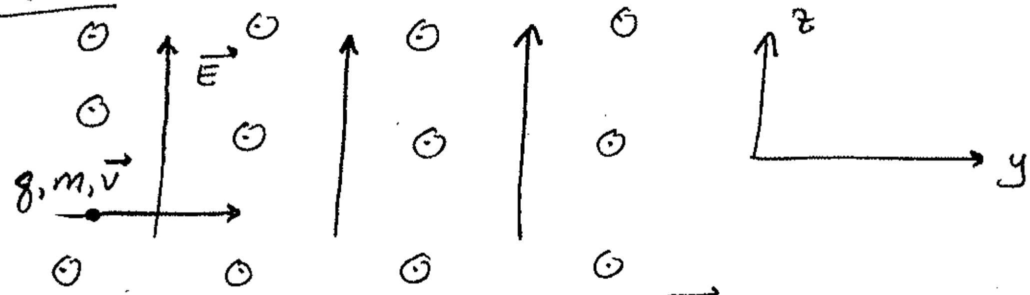
4. 5.12

5. 5.39

6. 5.40

Prob 5.3

(a)



Because $\vec{F}_B = q(\vec{v} \times \vec{B})$ and $q = -e < 0$

$\Rightarrow$ we need $\vec{B}$ out of page for electrons to balance $-e\vec{E}$

$$\vec{F}_{es} + \vec{F}_B = -eE\hat{z} - evB(-\hat{z}) = 0 \Rightarrow \boxed{E = vB}$$

(b) Turn off $E \Rightarrow$ cyclotron motion. For a radius of gyration R, find (q/m) of the particles.

$$R_{gyro} = \frac{mv}{qB} \qquad (5.3)$$

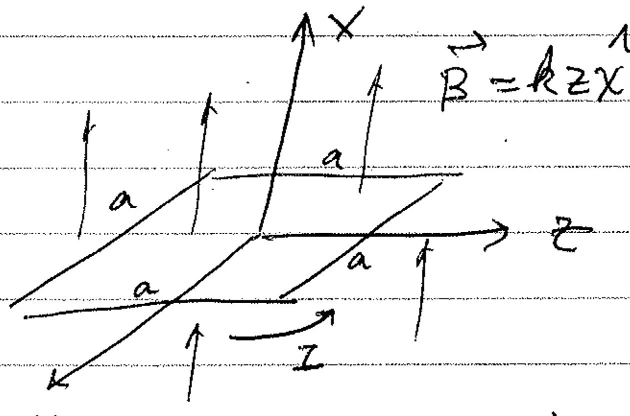
$\leftarrow$ from (a)

$$= \frac{m}{qB}\left(\frac{E}{B}\right)$$

$$\Rightarrow \boxed{\frac{q}{m} = \frac{E}{B^2 R_{gyro}}}$$

Prob 5.4

Suppose that $\vec{B} = kz\hat{x}$, where k is a constant. Find the force on a square loop (side a), lying in the yz plane and centered at the origin, if it carries a current I flowing CCW when you look down from the x-axis.



$$\vec{F} = \oint I\, d\vec{l} \times \vec{B}$$

$z = \frac{a}{2}$

$$= \hat{y}\int_{-a/2}^{a/2} Ikz\,dz + \hat{z}\int_{-a/2}^{a/2} Ikz\,dy - \hat{y}\int_{-a/2}^{a/2} Ikz\,dz$$

$z = -\frac{a}{2}$

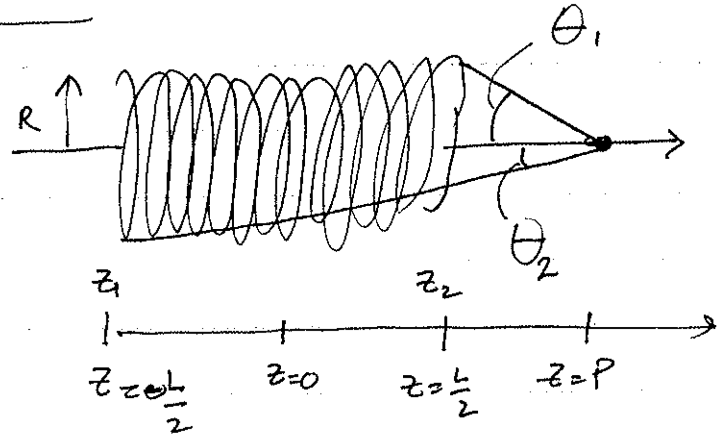
$$-\hat{z}\int_{-a/2}^{a/2} Ikz\,dy$$

$$= \hat{y} Ik \overbrace{\left(\frac{a^2}{8} - \left[+\frac{a^2}{8}\right] - \frac{a^2}{8} + \left[+\frac{a^2}{8}\right]\right)}^{0}$$

$$+ \hat{z} Ik\left[\frac{a}{2}\overbrace{\left(\frac{a}{2} - \left[-\frac{a}{2}\right]\right)}^{a} - \left(-\frac{a}{2}\right)\overbrace{\left(\frac{a}{2} - \left[-\frac{a}{2}\right]\right)}^{a}\right]$$

$$\vec{F} = \hat{z} I k a^2$$

Prob 5.11



Find the $\vec{B}$-field on the axis of a tightly wound solenoid consisting of N turns per unit length wrapped around a cylindrical tube of radius R and carrying current I.

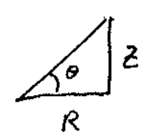
Example 6 $\Longrightarrow$ $B_z = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$ (5.34)

for the axis of a circular loop.

J = current per unit length along z = NI = $\left[\frac{A}{m}\right]$

$$d\vec{B}_s = \frac{\mu_0 J dz}{2} \frac{R^2}{(R^2+z^2)^{3/2}} \hat{z}$$

$$\rightarrow \vec{B}_s = \frac{\mu_0 N I R^2}{2} \int_{z_1}^{z_2} \frac{dz}{(R^2+z^2)^{3/2}};$$



let: ① $\frac{z}{R} = \tan\theta$

$dz = R\,d\theta / \cos^2\theta$

② $\frac{R}{\sqrt{R^2+z^2}} = \cos\theta$

$$= \frac{\mu_0 N I R^2}{2} \int_{R\tan\theta_1}^{R\tan\theta_2} \frac{R\,d\theta}{\cos^2\theta} \frac{\cos^3\theta}{R^3}$$

$$= \frac{\mu_0 N I}{2} \left(\sin\theta\right)_{R\tan\theta_1}^{R\tan\theta_2}$$

$$= \frac{\mu_0 N I}{2} \left[\frac{-z_1}{\sqrt{R^2+z_1^2}} + \frac{z_2}{\sqrt{R^2+z_2^2}}\right]$$

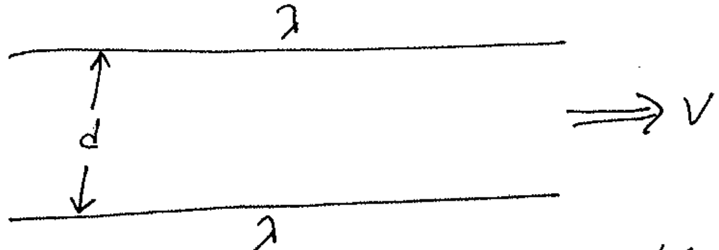
$$= \frac{\mu_0 N I}{2} \left[\frac{L/2}{\sqrt{R^2+L^2/4}} - \frac{-L/2}{\sqrt{R^2+L^2/4}}\right]$$

$$\boxed{\vec{B}_s = \frac{\mu_0 N I L}{2\sqrt{R^2 + L^2/4}}\,\hat{z}}$$

if $L \to \infty$,

$$\boxed{\vec{B}_s \simeq \mu_0 N I \,\hat{z}}$$

Prob 5.12



Q: How large must v be in order that the magnetic force balance the electrical repulsion?

$$(i)\quad \vec{E}_\lambda = \frac{\lambda \hat{s}}{2\pi s \epsilon_0} \longrightarrow \vec{F}_E = \lambda l \vec{E}_\lambda \Rightarrow \frac{\vec{F}_E}{l} = \frac{\lambda^2}{2\pi d \epsilon_0}\hat{s} \equiv \text{force per unit length}$$

length along wire and

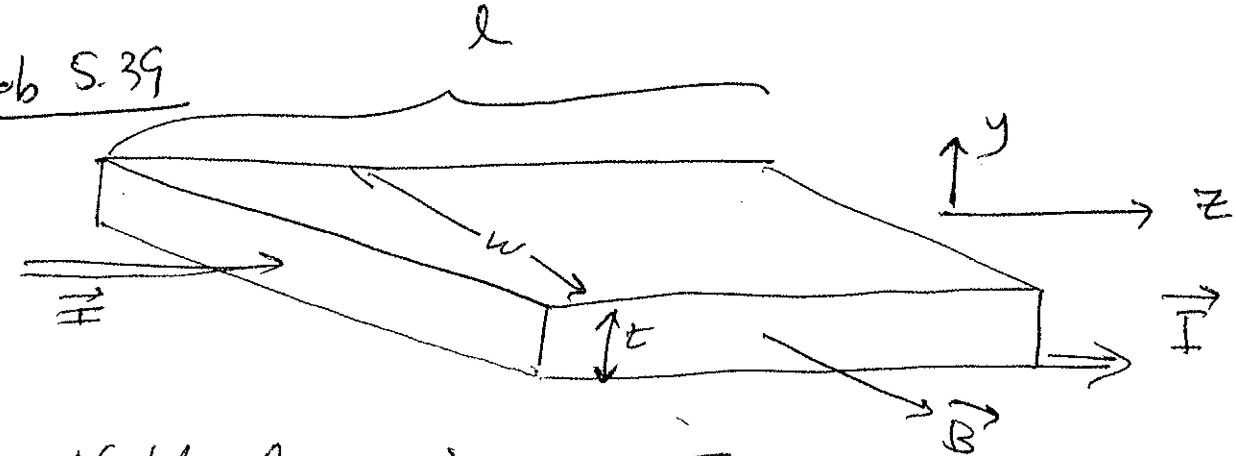
λl = charge

$$(ii)\quad \vec{B}_\lambda = \frac{\mu_0 \overbrace{\lambda v l}^{I_{\text{enclosed}}}}{2\pi s}\hat{\phi} \longrightarrow \vec{F}_B = \underbrace{\lambda v}_{dI} \frac{\mu_0 \lambda v l}{2\pi s}(-\hat{s}) \Rightarrow \frac{\vec{F}_B}{l} = -\frac{(\lambda v)^2 \mu_0}{2\pi s}\hat{s}$$

$$\frac{\vec{F}_B}{l} + \frac{\vec{F}_E}{l} = 0 \Rightarrow \frac{\lambda^2}{2\pi \epsilon_0 d} = \frac{\lambda^2 v^2 \mu_0}{2\pi d} \Rightarrow v^2 = \frac{1}{\mu_0 \epsilon_0} = c^2$$

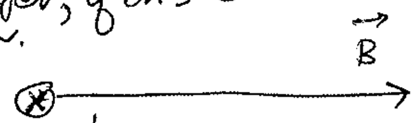
the forces balance when $v \equiv c$; this is not reasonable

Prob 5.39



a) If the charge carrier as positive, in which direction do they deflect?

I into paper, q in same direction.



$\downarrow$ $\vec{V} \times \vec{B}$ is downward

b) Find the resulting ΔV at equilibrium

$$d\vec{F} = I d\vec{l} \times \vec{B} = \underbrace{(\rho \vec{v}\, \overbrace{t l}^{area}\, dz)}_{\vec{I} dz} \times \vec{B}$$

$$\rightarrow \frac{d\vec{F}}{dz} = \rho v t l B(+\hat{y}) = \frac{dq}{dz} E_t \hat{y}$$

now, $dq = \rho(\underbrace{t l dz}_{d^3x})$

$$\rightarrow +\rho v t l B = \frac{\rho t l dz}{dz} E_z$$

$$\rightarrow E_z = +Bv$$

$$\rightarrow \boxed{\Delta V = -Bvt}$$

c) How would your answer change if the charge carriers were negative?

I into paper, q out-of-paper

$\odot \longrightarrow B$

$\uparrow$ $\vec{v}\times\vec{B}$ is upward, however $q\vec{v}\times\vec{B}$ is $\downarrow$ downward

$\Rightarrow$ $q<0$ builds up at the bottom of the slab

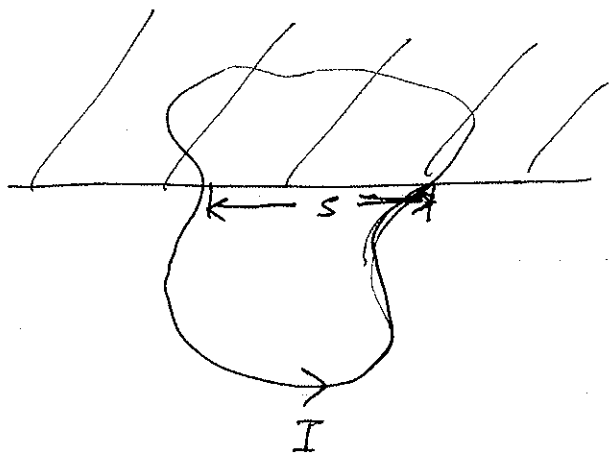
$\rightarrow$ $E_z \downarrow$ ($-\hat{y}$ direction)

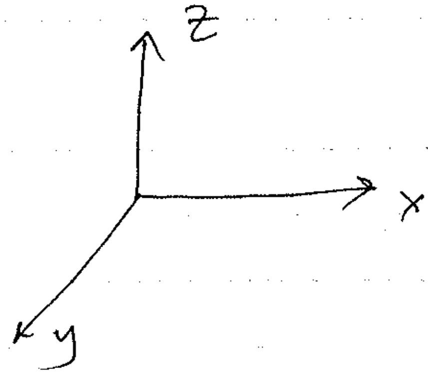
$\Longrightarrow$ ΔV has the same magnitude, but a different sign.

5.58.1

Prob. 5.40

(A)





$\vec{B}$ is in $\hat{y}$-direction

(i) the force is
$$d\vec{F} = (I\, d\vec{\ell} \times \vec{B})$$
$$= I(0 - dz\,B_y,\ 0 - 0,\ dx\,B_y)$$
$$= \underbrace{I B_y}_{\text{constants}}(-dz, 0, dx)$$

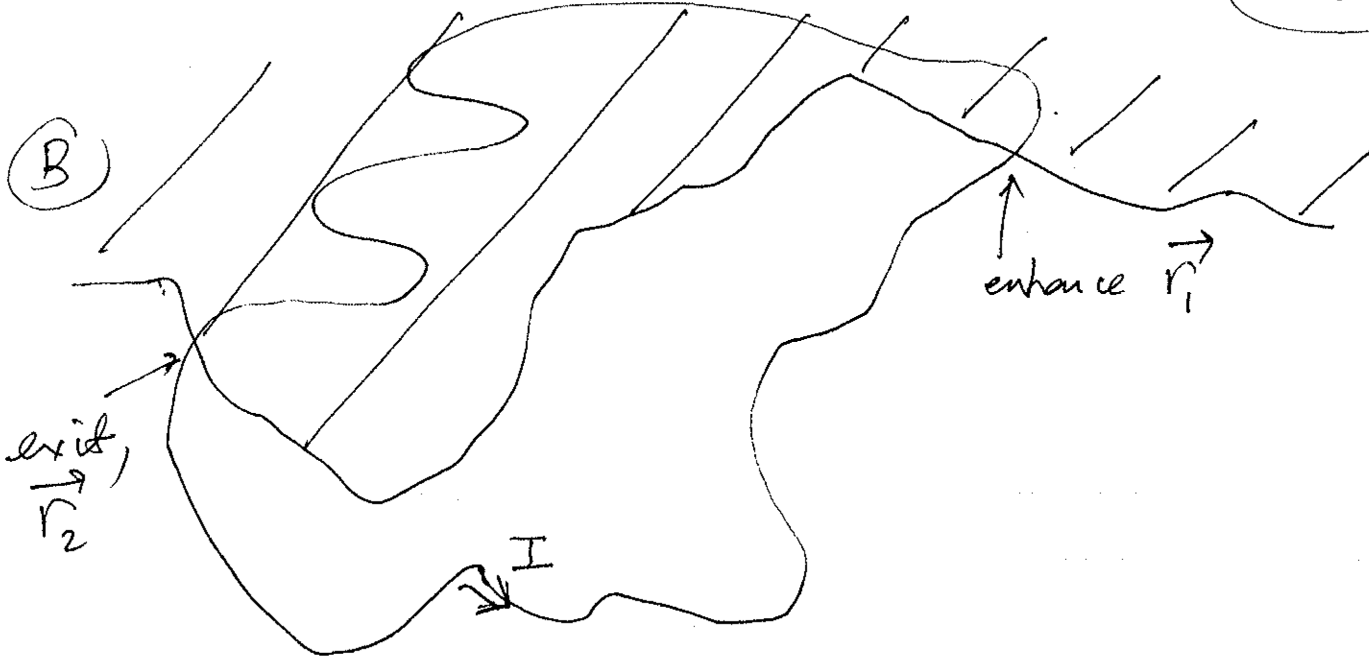
(ii) We see that if integrate $d\vec{F}$ over the path that

a) $-\int_{z_1}^{z_2} dz = 0$ since $z_1 = z_2$

b) $\int_{x_1}^{x_2} dx = (x_2 - x_1) = s$

independent of the path followed. We are only concerned w/ the progress each charge makes

5.58.2



Q: What is the force now?

Well, do the same thing:

$$d\vec{F} = I B_y (-dz, 0, dx)$$

$$\Rightarrow \quad F_x = -I B_y (z_2 - z_1)$$

$$F_z = I B_y (x_2 - x_1)$$

Analogously, the force only depends on the entry and exits locations of the current loop. If $(z_1 \neq z_2) \Rightarrow$ force in the horizontal direction.

If $(x_2 \neq x_1) \Rightarrow$ force in the vertical direction.