d) (i) find $\vec{Q}\cdot\hat{\phi}$ if $\vec{Q}$ is aligned w/ the z-axis.

<u>recall</u>: $\vec{Q} = m(\vec{r}\times\vec{v}) - \frac{\mu_e}{4\pi} q_e q_m \hat{r}$

$$= m(r v_\theta \hat{\phi} - r v_\phi \hat{\theta}) - \frac{\mu_0}{4\pi} q_e q_m \hat{r}$$

$$\Rightarrow \vec{Q}\cdot\hat{\phi} = 0 = m r v_\theta \hat{\phi}\cdot\hat{\phi}$$

$\hat{\phi}$ is always $\perp$ to z-axis

$$\Rightarrow \boxed{v_\theta = 0 \Rightarrow \theta = \text{constant}}$$

(ii) $\vec{Q}\cdot\hat{r} = |\vec{Q}|\cos\theta = -\frac{\mu_e}{4\pi} q_e q_m$

$$\Rightarrow \boxed{|\vec{Q}| = \frac{\mu_0}{4\pi}\frac{q_e q_m}{\cos\theta}}$$

(iii) $\vec{Q}\cdot\hat{\theta} = -|\vec{Q}|\sin\theta = -m r v_\phi = -m r^2 \sin\theta\,\dot{\phi}$

$$\Rightarrow \dot{\phi} = \frac{|\vec{Q}|}{m r^2} = \frac{k}{r^2}$$

where $\frac{|\vec{Q}|}{m} = \frac{\mu_0}{4\pi m}\frac{q_e q_m}{\cos\theta} = k$

e) $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2$ ($\dot{\theta} = 0$)

$$\frac{v^2}{\dot{\phi}^2} = \left(\frac{\dot{r}}{\dot{\phi}}\right)^2 + r^2\sin^2\theta$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)^2 = \frac{v^2}{k^2}r^4 - r^2\sin^2\theta$$

$$\boxed{\left(\frac{dr}{d\phi}\right) = r\sqrt{\frac{v^2 r^2}{k^2} - \sin^2\theta}}$$

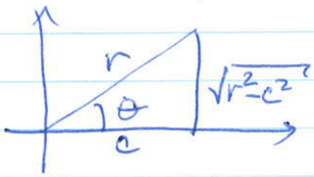
where v, k, θ are constants

f) $$\frac{dr}{d\phi} = r\sqrt{\frac{v^2}{k^2}r^2 - \sin^2\theta}$$

$$= \frac{v}{k}\, r\sqrt{r^2 - \frac{k^2\sin^2\theta}{v^2}}$$

$$= \frac{v}{k}\, r\sqrt{r^2 - c^2}$$

$$\Rightarrow \int \frac{dr}{r\sqrt{r^2-c^2}} = \frac{v}{k}\int d\phi$$



$$\Rightarrow \cos\theta = \frac{c}{r}, \quad \frac{\sqrt{r^2-c^2}}{r} = \sin\theta$$

$$-\sin\theta\, d\theta = -\frac{c}{r^2}dr \Rightarrow dr = \frac{r^2}{c}\sin\theta\, d\theta$$

$$= \frac{c}{\cos^2\theta}\sin\theta\, d\theta$$

$$\Rightarrow \int \underbrace{\frac{c}{\cos^2\theta}\sin\theta\, d\theta}_{dr}\ \underbrace{\frac{\cos\theta}{c}}_{\frac{1}{r}}\ \underbrace{\frac{1}{c\tan\theta}}_{\frac{1}{\sqrt{r^2-c^2}}}$$

$$\Rightarrow \frac{1}{c}\int d\theta = \frac{v}{k}\int d\phi$$

$$\frac{1}{c}\left[\theta - \theta_0\right] = \frac{v}{k}\left[\phi - \phi_0\right]$$

new constant

$$\theta = \frac{cv}{k}\left[\phi - \phi_1\right]$$

$$\cos^{-1}\left[\frac{c}{r}\right] = \frac{cv}{k}\left[\phi - \phi_1\right]$$

new constant

$$\Rightarrow \phi(r) = \frac{k}{cv}\cos^{-1}\left(\frac{c}{r}\right) + \alpha$$