$$\vec{B} = \frac{\mu_0 \Omega_0 \rho_0}{3r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]\frac{r^5}{5}$$

$$+ \frac{2\mu_0\rho_0}{3}\Omega_0\left(\frac{R^2}{2} - \frac{r^2}{2}\right)\hat{z}$$

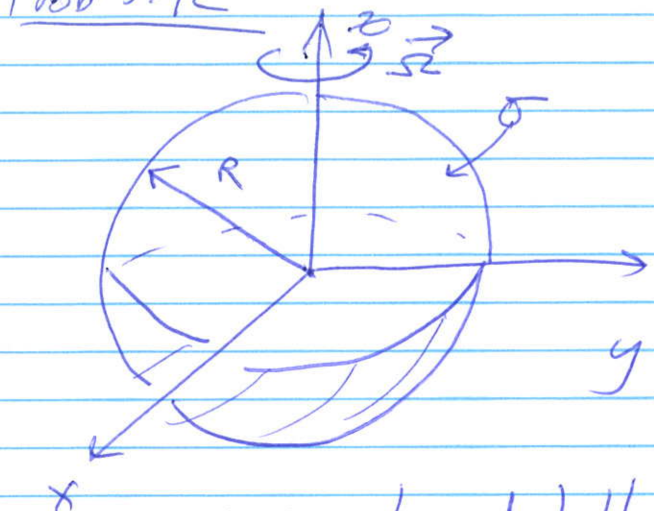
note: $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$$= \frac{\mu_0 \Omega_0 \rho_0}{3}\left[\frac{2r^2}{5}\cos\theta\,\hat{r} + \frac{r^2}{5}\sin\theta\,\hat{\theta} + \left(R^2 - r^2\right)\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right)\right]$$

$$= \frac{\mu_0 \Omega_0 \rho_0}{3}\left[\hat{r}\left(\frac{2r^2}{5}\cos\theta + \left[R^2 - r^2\right]\cos\theta\right) + \hat{\theta}\left(\frac{r^2}{5}\sin\theta - \left[R^2 - r^2\right]\sin\theta\right)\right]$$

$$\boxed{\vec{B} = \frac{\mu_0 \Omega_0 \rho_0}{3}\left[\hat{r}\cos\theta\left(R^2 - \frac{3}{5}r^2\right) + \hat{\theta}\sin\theta\left(\frac{6r^2}{5} - R^2\right)\right]}$$

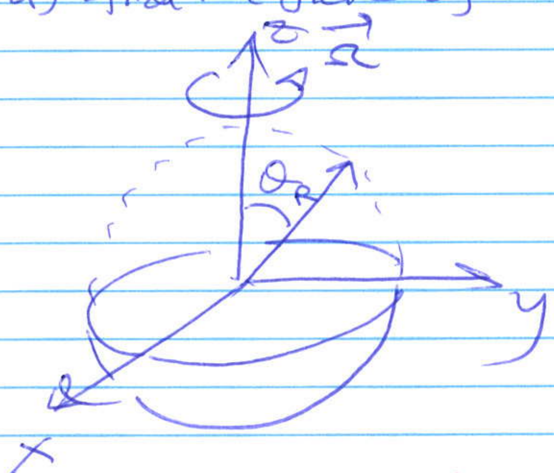
Prob 5.42



find the force on the upper hemisphere produced by the field of lower hemisphere

spinning charged shell

a) find the field of a spinning charged hemispherical shell



Consider the field on a shell of radius R and arbitrary θ as shown.

a) from class note $\vec{B} = \frac{\mu_0 \sigma \Omega_0 R^4}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$

b) $\vec{K} = \sigma \vec{\Omega} \times \vec{r} = \sigma \Omega_0 R \sin\theta\,\hat{\phi}$

$$\Rightarrow d\vec{F} = \underbrace{\sigma \Omega_0 R \sin\theta\,\hat{\phi}}_{\vec{K}}\ \underbrace{ds}_{\text{area element}} \times \vec{B}$$

Use field from Ex. 11

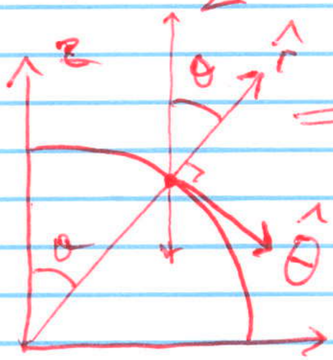
Use $\vec{B} = \vec{B}_{ave} = \frac{\vec{B}_< + \vec{B}_>}{2}$

$\vec{B}_< \propto \vec{\Omega} \Rightarrow$ exerts no downward force

$$d\vec{F} = \sigma\Omega_0 R \sin\theta \, dS \left(\frac{\mu_0 \sigma \Omega_0 R^4}{2r^3}\right)\left(2\cos\theta\,\hat{\theta} + \sin\theta\left(-\hat{r}\right)\right)$$

<u>note</u> $r = R$

$$d\vec{F} = \frac{\mu_0\sigma^2\Omega_0^2 R^2}{2}\left[2\sin\theta\cos\theta\,\hat{\theta} + \sin^2\theta\left(-\hat{r}\right)\right]\underbrace{\sin\theta\, d\theta\, R^2\, d\phi}_{dS}$$



$\Rightarrow$ i) F_r projected onto z axis is $F_r \cos\theta$

ii) F_θ projected onto z axis is $F_\theta \cos\left(\frac{\pi}{2} - \theta\right)$

$$dF_z = \frac{\mu_0\sigma^2\Omega_0^2R^4}{2}\left[2\sin^2\theta\cos\theta - \cos\theta\sin^2\theta\right]\sin\theta\, d\theta\, d\phi$$

$$F_z = \frac{\cancel{2}\pi\sigma^2\Omega_0^2R^4}{\cancel{2}}\mu_0\left[\int_0^{\pi/2}\cos\theta\sin^2\theta\, d(-\cos\theta)\right]$$

$$= \cancel{2}\pi\sigma^2\Omega_0^2R^4\mu_0\int_0^{\pi/2}\cos\theta\left(1-\cos^2\theta\right)d(-\cos\theta)$$

$$= \cancel{2}\pi\mu_0\sigma^2\Omega_0^2R^4\left[-\frac{\cos^2\theta}{2} + \frac{\cos^4\theta}{4}\right]_0^{\pi/2}$$

$$= \cancel{2}\pi\mu_0\sigma^2\Omega_0^2R^4\left[+\frac{1}{2} + \left(0 - \frac{1}{4}\right)\right]$$

$$= \frac{\pi\mu_0\sigma^2\Omega_0^2R^4}{4}$$