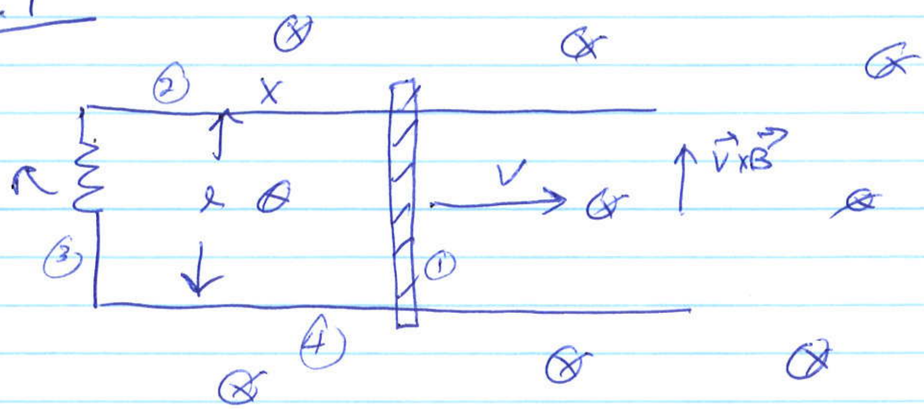
HW #6

7.7



$\vec{B}$ into page

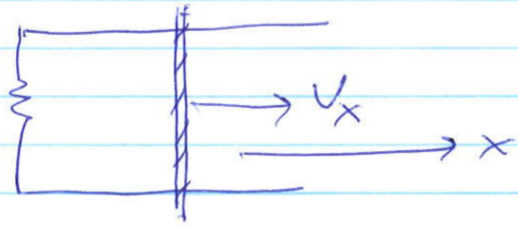
a) $\mathcal{E} = \oint \vec{v}\times\vec{B}\cdot d\vec{l} = vBl = IR \rightarrow \boxed{I = \frac{vBl}{R} \text{ and flows CCW}}$

b) what is force of bar

$\vec{F} = \int I\,d\vec{l}\times\vec{B} =$ ① IlB to left + ② IBx down + ③ IlB to right + IBx up

"U" is rigid ⇒ only force is on ① and is $\boxed{|\vec{F}| = IBl \text{ to left}}$

c)



$m_{bar}\dot{V}_x = -IBl = -\frac{V_x B^2 l^2}{R}$

$\ln\frac{V_x}{V_0} = \frac{-B^2 l^2}{m_{bar}R}(t - \cancel{t_0}^{\,0})$

$\rightarrow \boxed{V_x = V_0 e^{-(B^2 l^2 t/m_{bar}R)}}$

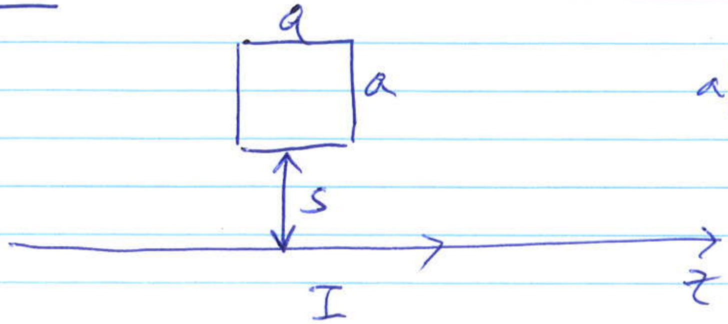
d) $P = IV = I^2 R$

$E = \int_0^\infty P\,dt = \int R\left(\frac{Bl}{R}\right)^2 V_0^2 e^{-\left(\frac{2B^2l^2}{m_{bar}R}\right)t}\,dt$

$= -\frac{B^2 l^2}{R} V_0^2 \frac{m_{bar}R}{2B^2 l^2} e^{-\left(\frac{2B^2l^2}{m_{bar}R}\right)}\Big|_0^\infty$

$\boxed{E = \frac{1}{2} m_{bar} V_0^2}$

7.8



a) find Φ_B through loop

$$\vec{B}_w = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

from Ampere's law

a)

$$\Rightarrow \Phi_B = \int \frac{\mu_0 I}{2\pi s}\hat{\phi}\cdot(ds\,dz)\hat{\phi}$$

$$\boxed{\Phi_B = \frac{\mu_0 I}{2\pi} a \ln\left(\frac{s+a}{s}\right)}$$

b) if loop is pulled in $\hat{s}$ direction with speed v find $\Phi_B(t)$ and then $\mathcal{E}$ and its direction

$$\Phi_B(t) = \frac{\mu_0 I a}{2\pi}\ln\left[\frac{vt+a+s_0}{vt+s_0}\right]$$, where s_0 is the original pos^n of loop

$$\frac{d\Phi_B}{dt} = \frac{\mu_0 I a}{2\pi}\left[\frac{v}{s_0+a+vt} - \frac{v}{vt+s_0}\right]$$

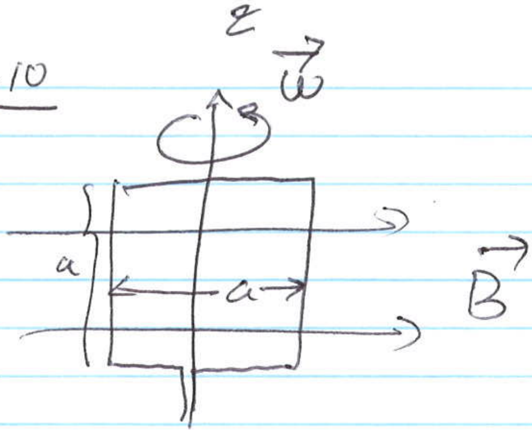
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 I a}{2\pi}\left[\frac{v(vt+s_0)-v(s_0+a+vt)}{(s_0+vt)(a+s_0+vt)}\right]$$

$$\boxed{\mathcal{E} = +\frac{\mu_0 I a}{2\pi}\left[\frac{+av}{(s_0+vt)(s_0+vt+a)}\right]}$$

$\Rightarrow I = \dfrac{\mathcal{E}}{R}$ and flows in CCW direction

c) if $v \to \hat{z}$ direction $\Rightarrow \dot{\Phi}_B = 0 \to \mathcal{E} = 0$

7.10



a) find $\mathcal{E}(t)$:

$$\Phi_B = \oint \vec{B} \cdot d\vec{s}$$

maximum flux is Ba^2. Varies as

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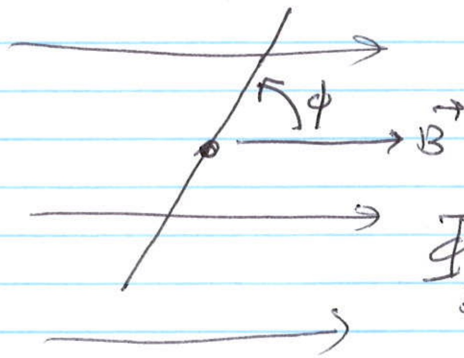


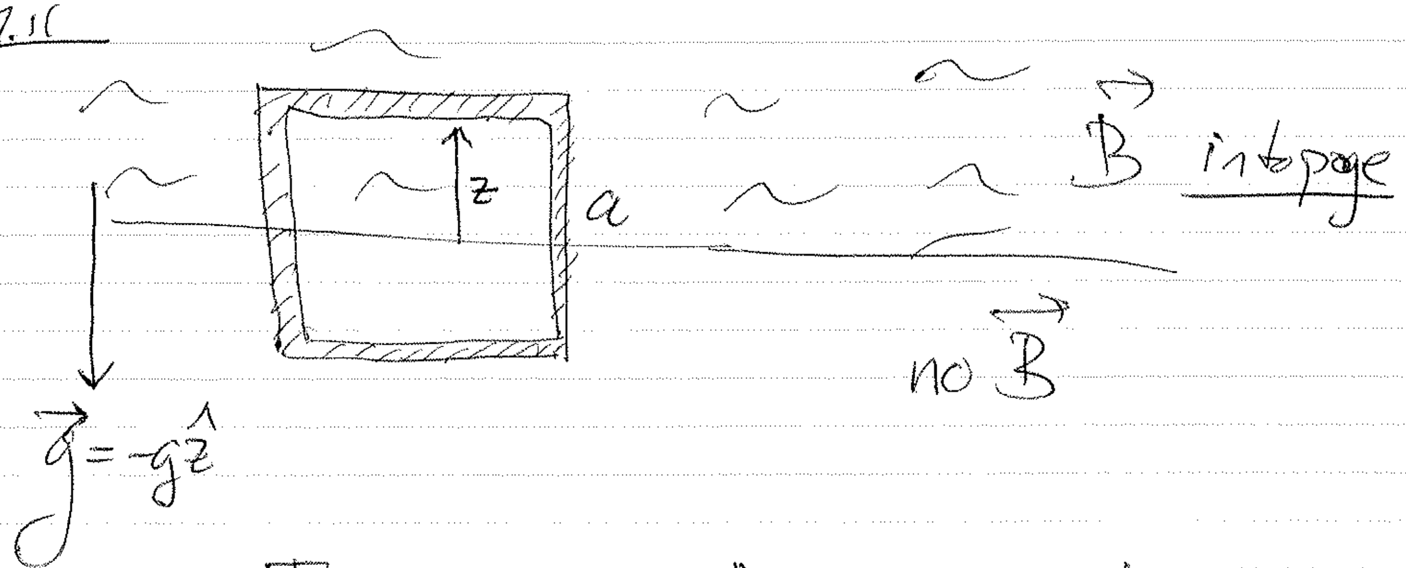
$\phi = \omega t$

$\Phi_B = \Phi_0 \underbrace{\cos\phi}$

projection onto direction of B

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt} = +\Phi_0 \omega \sin \omega t}$$

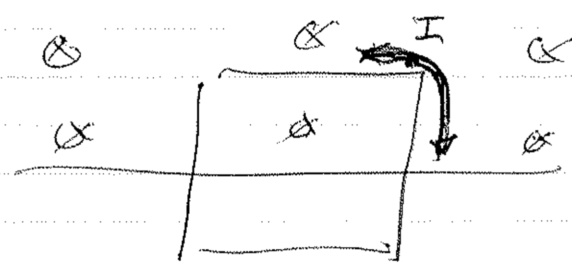
7.11



a) $\Phi_B = Baz$ and "pushes" in CW sense

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -Ba\dot z \Rightarrow I = \frac{Ba\dot z}{R} \text{ in CW sense} \quad \leftarrow \dot z < 0$$

(current goes in ccw sense)



$\Rightarrow \vec F = \int I\, d\vec l \times \vec B$ will point upward

$$\vec F_B = \frac{-Ba\dot z}{R} aB\hat z$$

$$\Rightarrow \cancel{M_{Al}}\ddot z = \cancel{M_{Al}}\dot v_z = -\cancel{M_{Al}}g - \frac{Bav_z}{RM_{Al}}aB$$

Let $\lambda = \frac{B^2a^2}{M_{Al}R}v_z + g \Rightarrow \frac{M_{Al}R}{B^2a^2}\dot\lambda = -\lambda$

$$\lambda = \lambda_0 e^{-\left(\frac{M_{Al}R}{B^2a^2}t\right)}$$

$$\frac{B^2a^2}{M_{Al}R}v_z + g = g e^{-\left(\frac{M_{Al}R}{B^2a^2}t\right)}$$

$$\Rightarrow \boxed{v_z = -g\left(1 - e^{-\frac{M_{al}R}{B^2a^2}t}\right)\frac{M_{al}R}{B^2a^2}}$$

b) Recall:
$$\dot{v}_z = -g - \frac{B^2a^2}{M_{al}R}v_z$$

terminal velocity occurs when $\dot{v}_z = 0$

$$\Rightarrow \underbrace{v_z = -\frac{M_{al}Rg}{B^2a^2} \text{ is terminal velocity}}_{\text{as above}}$$

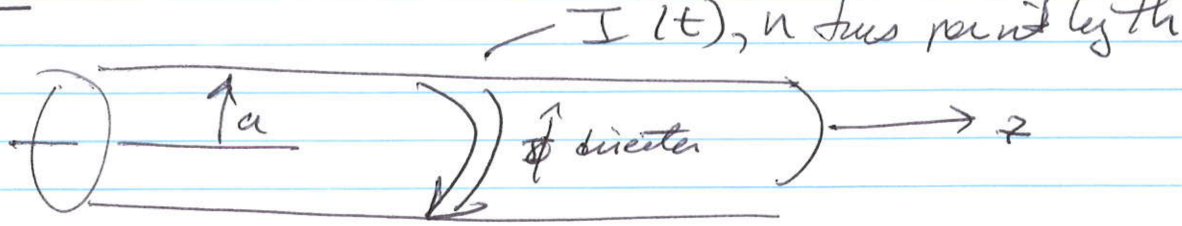
c) to reach $0.9\, V_{\text{Terminal}}$, use

$$\frac{v_z}{v_T} = 0.9 \Rightarrow \frac{v_z = -\frac{M_{al}Rg}{B^2a^2}\left(1 - e^{-\frac{M_{al}R}{B^2a^2}t_{0.9}}\right)}{-\frac{M_{al}Rg}{B^2a^2}} = 0.9$$

$$\Rightarrow 0.1 = e^{-(M_{al}R\,t_{0.9}/B^2a^2)}$$

$$\Rightarrow \boxed{t_{0.9} = -\frac{B^2a^2}{M_{al}R}\ln(0.1)}$$

7.15



Assume quasi-static approximation

a) $\vec{B}(t) = \mu_0 n I(t)\hat{z}$, $s<a$

$\Rightarrow \Phi_B(t) = \int \vec{B}\cdot d\vec{S}$

$= \int \mu_0 n I(t)\hat{z}\cdot(s\,d\phi\,ds)\hat{z}$

$= \mu_0 n I(t)\pi s^2$

$= \mu_0 n I(t)\begin{cases}\pi a^2 & s>a\\ \pi s^2 & s<a\end{cases}$

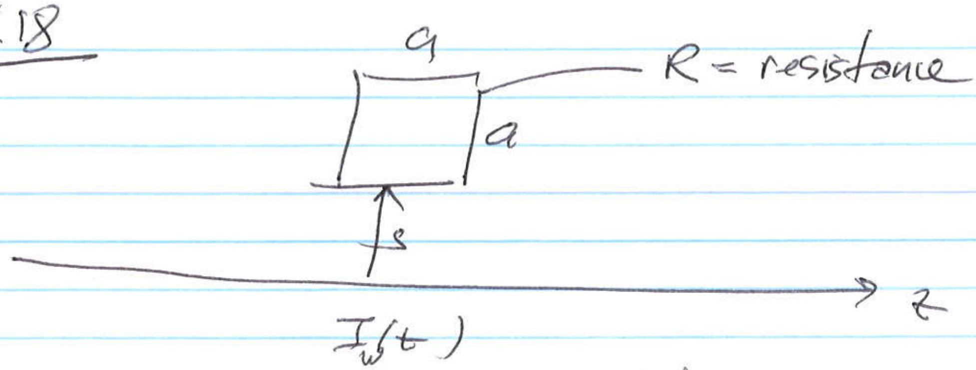
$\Rightarrow +\dfrac{d\Phi_B}{dt} = +\mu_0 n \dot{I}\begin{cases}\pi a^2 & s>a\\ \pi s^2 & s<a\end{cases}$

b) $\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$

$E_\phi 2\pi s = -\mu_0 n \pi \dot{I}\begin{cases}a^2 & s>a\\ s^2 & s<a\end{cases}$

$\vec{E}_\phi = -\begin{cases}\dfrac{\mu_0 n \dot{I} a^2}{2s}\hat{\phi} & s>a\\ \dfrac{\mu_0 n \dot{I} s}{2}\hat{\phi} & s<a\end{cases}$

7.18



a) If $I_w \to 0$ instantaneously $\Rightarrow$ I_w flow around loop in CCW direction

b) what total charge passes through a point on the loop after the wire is cut?

From 7.8
$$\Phi_B = \frac{\mu_0 I_w a}{2\pi} \ln\left(\frac{s+a}{s}\right)$$

for fixed s, $I_w = \Phi_{0w} \to 0$ instantaneously, but let's say $\dot{I}_w$ is fast, but finite and then let it go to ∞

So, $\dot{\Phi}_B = \frac{\mu_0 a}{2\pi} \ln\left(\frac{s+a}{s}\right) \dot{I}_w = -\mathcal{E} = -I_\ell R$

$$\Rightarrow I_\ell(t) = \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \dot{I}_w$$

$$\frac{dQ_\ell}{dt} = \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \frac{dI_w}{dt}$$

Integrate from $t = 0 \to \infty$

$$Q_\ell(\infty) - \cancelto{0}{Q_\ell(0)} = \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right)\left(\cancelto{0}{I_w(\infty)} - I_w(0)\right)$$

$$\boxed{Q_\ell(\infty) = -\frac{\mu_0 a I_{w,0}}{2\pi R} \ln\left(\frac{s+a}{s}\right)}$$

c) Now let $I_u(t) = \begin{cases} 0, & t > 1/\alpha \\ (1-\alpha t) I_0, & 0 \leq t \leq 1/\alpha \end{cases}$

$$\Rightarrow I_i(t) = \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \begin{cases} 0, & t > 1/\alpha \\ -\alpha I_0, & 0 \leq t \leq 1/\alpha \end{cases}$$