Homework 3

Due: January 30, 2013

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14. 5.27

15. 5.37

16. 5.42

17. 5.45

18. 5.56

19. 5.58

20. 5.61

Prob 5.27

Equation (5.63) $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}} d\tau'$

a) Show that 5.63 is consistent w/ $\vec{\nabla}\cdot\vec{A}$

$$\vec{\nabla}_r\cdot\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{\nabla}_r\cdot\left[\frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\right]d\tau'$$

$\vec{\nabla}_r$ moves under $\int$ sign because it is with respect to the field point, $\vec{r}$

$$= \frac{\mu_0}{4\pi}\int\left(\vec{J}(\vec{r}\,')\cdot\vec{\nabla}_r\frac{1}{\mathscr{r}}\right)d\tau'$$

$\vec{J}(\vec{r}\,')$ does not depend on $\vec{r}$

note: $\vec{\nabla}_r\frac{1}{\mathscr{r}} = -\vec{\nabla}_{r'}\frac{1}{\mathscr{r}}$ because $\mathscr{r} = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$

$$= -\frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\cdot\vec{\nabla}_{r'}\frac{1}{\mathscr{r}}d\tau'$$

look at $\vec{\nabla}_{r'}\cdot\left[\frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\right] = \frac{1}{\mathscr{r}}\vec{\nabla}_{r'}\cdot\vec{J}(\vec{r}\,') + \vec{J}(\vec{r}\,')\cdot\vec{\nabla}_{r'}\frac{1}{\mathscr{r}}$

$$\Rightarrow \vec{\nabla}_r\cdot\vec{A}(\vec{r}) = -\frac{\mu_0}{4\pi}\int\left\{\underbrace{\vec{\nabla}_{r'}\cdot\left[\frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\right]}_{\text{change to surface integral}} - \underbrace{\frac{\vec{\nabla}_{r'}\cdot\vec{J}(\vec{r}\,')}{\mathscr{r}}}_{\to 0 \text{ because of continuity equation}}\right\}d^3\tau'$$

$\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0\right)$

$$= -\frac{\mu_0}{4\pi}\oint\frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\cdot d\vec{S}$$

make volume large enough to enclose all of $\vec{J}(\vec{r}\,')$

$$\Rightarrow \oint\frac{\vec{J}(\vec{r}\,')\cdot d\vec{S}}{\mathscr{r}} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}_r\cdot\vec{A} = 0}$$

b) Show that 5.63 is consistent w/ $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$

take $\vec{\nabla}_r \times \vec{A} = \frac{\mu_0}{4\pi}\vec{\nabla}_r \times \int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}d\tau'$

$= \frac{\mu_0}{4\pi}\int \vec{\nabla}_r \times \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}d\tau'$, $\vec{\nabla}_r$ independent of $\vec{r}\,'$

$= \frac{-\mu_0}{4\pi}\int \vec{J}(\vec{r}\,') \times \vec{\nabla}_r \frac{1}{\mathscr{r}} d\tau'$

$= -\frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,') \times \frac{-\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$

$$\boxed{\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'}$$

c) show 5.63 is consistent w/ $\nabla^2\vec{A} = -\mu_0\vec{J}$

$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}d\tau'$

$(\vec{\nabla}\cdot\vec{\nabla})\vec{A} = \vec{\nabla}_r^2\vec{A} = \frac{\mu_0}{4\pi}\int \nabla_r^2\left[\frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\right]d\tau'$

$= \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left[\vec{\nabla}_r\cdot\vec{\nabla}_r\left(\frac{1}{\mathscr{r}}\right)\right]d\tau'$

$= \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left[\vec{\nabla}_r\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\right)\right]d\tau'$

$= \frac{-\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left[4\pi\delta^3(\vec{\mathscr{r}})\right]d\tau'$

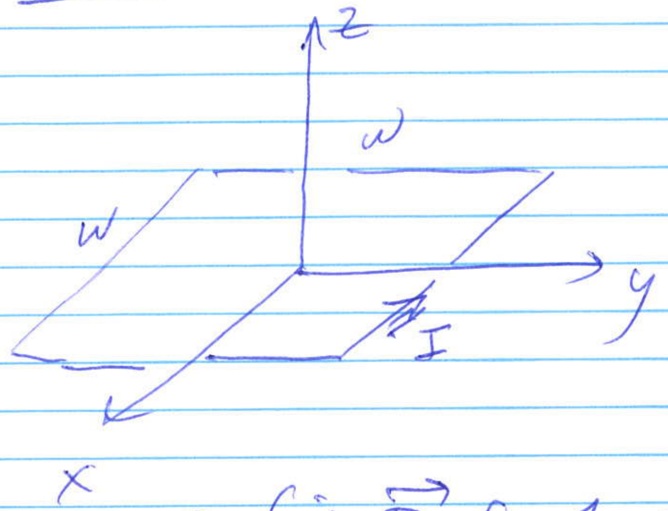
because of $\delta^3(\vec{r})$

$$\nabla_r^2 \vec{A} = -\frac{\mu_0}{4\pi}\left[4\pi \vec{J}(\vec{r})\right]$$

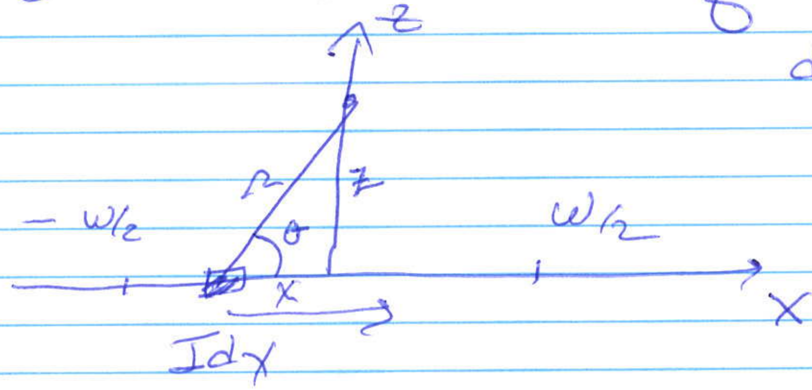
$$\boxed{\nabla_r^2 \vec{A} = -\mu_0 \vec{J}(\vec{r})}$$

Prob 5.37

Find $\vec{B}$ on the z-axis.



① find $\vec{B}$ for 1 side of square (x-side)



$$dB_y = \frac{\mu_0}{4\pi}\frac{I\,dx\sin\theta}{x^2+z^2}$$

now, $\begin{cases} \sin\theta = \dfrac{z}{r} = \dfrac{z}{(x^2+z^2)^{1/2}} \\ \text{and} \\ \cos\theta = \dfrac{x}{r} \end{cases}$

change integration variable to θ,

$$\cos\theta\,d\theta = -\frac{\frac{1}{2}z\,2x\,dx}{(x^2+z^2)^{3/2}} = -\frac{\sin\theta\cos\theta\,dx}{\sqrt{x^2+z^2}}$$

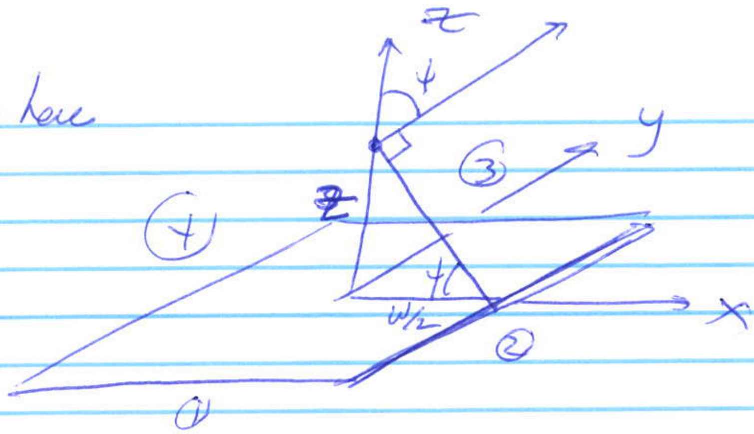
$$\Rightarrow dB_y = +\frac{\mu_0}{4\pi}I\Big(\underbrace{-\frac{d\theta}{\sin\theta}\sqrt{x^2+z^2}}_{dx}\Big)\frac{\sin\theta}{(x^2+z^2)}$$

$$\int dB_y = -\int_{\theta_1}^{\theta_2}\frac{\mu_0}{4\pi}I\frac{\sin\theta}{z}\,d\theta \quad \text{, where } \theta_1 = \cos^{-1}\left(\frac{w}{2\sqrt{z^2+\frac{w^2}{4}}}\right)$$

$$B_y = \frac{\mu_0 I}{4\pi z}\cos\theta\Big|_{\theta_1}^{\theta_2}$$

$$\boxed{B_y = \frac{\mu_0 I}{2\pi z}\left[\frac{w}{2\sqrt{z^2+\frac{w^2}{4}}}\right]}$$

② we have



we want the z-component of field value + the previous z goes to

$$z' = \sqrt{z^2 + \frac{w^2}{4}}$$

and we then project onto the new z-axis with

$$\cos\psi = \frac{w/2}{\sqrt{z^2 + \frac{w^2}{4}}}$$

$$\Rightarrow \text{new } B_z = \frac{\mu_0 I}{4\pi} \frac{1}{\sqrt{z^2 + w^2/4}} \frac{w}{\sqrt{z^2 + \frac{w^2}{2}}} \frac{w/2}{\sqrt{z^2 + w^2/4}}$$

$$B_z = \frac{\mu_0 I w^2}{8\pi} \frac{1}{\left(z^2 + \frac{w^2}{4}\right)\sqrt{z^2 + w^2/2}}$$

③ for all 4 sides

$$\boxed{B_{z,total} = \frac{\mu_0 \overbrace{I w^2}^{m}}{2\pi} \frac{1}{\sqrt{z^2 + \frac{w^2}{2}}\left(z^2 + \frac{w^2}{4}\right)}}$$

④ $z \gg w \Longrightarrow B_{z,total} \simeq \frac{\mu_0 m}{2\pi} \frac{1}{z}\left(1 - \frac{w^2}{4z^2}\right)\frac{1}{z^2}\left(1 - \frac{w^2}{4z^2}\right)$

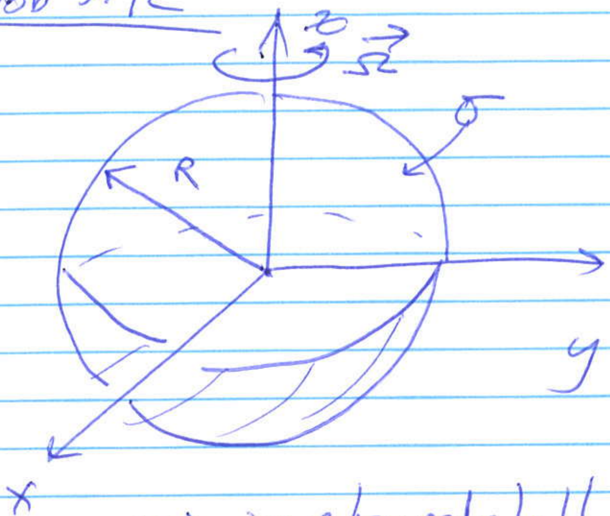
$$\simeq \frac{\mu_0 m}{2\pi z^3}\left(1 - \frac{w^2}{2z^2}\right)$$

$$\simeq \left(\frac{\mu_0}{4\pi}\right)\frac{2m}{z^3}$$

$\theta = 0,\ r = z$

recall: $\vec{B}_d = \frac{\mu_0}{4\pi} m \left(\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right) = \frac{\mu_0 m}{4\pi}\left(\frac{2}{z^3}\hat{z} + 0\right)$

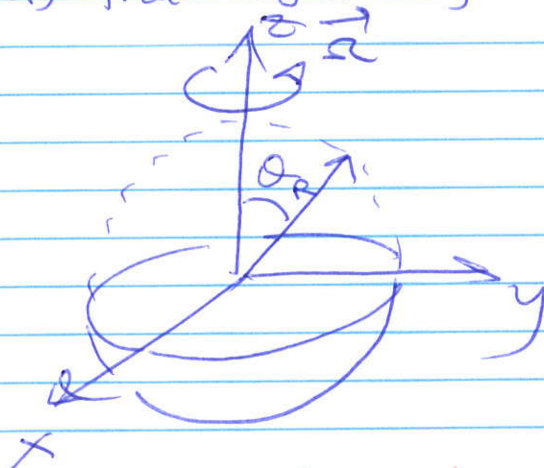
Prob 5.42



find the force on the upper hemisphere produced by the field of lower hemisphere

spinning charged shell

a) find the field of a spinning charged hemispherical shell



Consider the field on a shell of radius R and arbitrary θ as shown.

a) from class note $\vec{B} = \frac{\mu_0 \sigma \Omega_0 R^4}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

b) $\vec{K} = \sigma\vec{\Omega}\times\vec{r} = \sigma\Omega_0 R\sin\theta\,\hat{\phi}$

$\Rightarrow d\vec{F} = \underbrace{\sigma\Omega_0 R\sin\theta\,\hat{\phi}}_{\vec{K}}\,\underbrace{ds}_{\text{area element}} \times \vec{B}$

Use field from Ex. 11

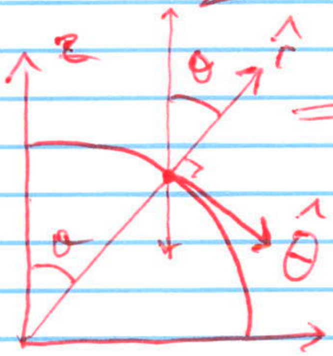
Use $\vec{B} = \vec{B}_{ave} = \frac{\vec{B}_< + \vec{B}_>}{2}$

$\vec{B}_< \propto \vec{\Omega} \Rightarrow$ exerts no downward force

$$d\vec{F} = \sigma\Omega_0 R\sin\theta\, dS\left(\frac{\mu_0\sigma\Omega_0 R^4}{2r^3}\right)\left(2\cos\theta\,\hat{\theta} + \sin\theta\left(-\hat{r}\right)\right)$$

<u>note</u> $r = R$

$$d\vec{F} = \frac{\mu_0\sigma^2\Omega_0^2 R^2}{2}\left[2\sin\theta\cos\theta\,\hat{\theta} + \sin^2\theta\left(-\hat{r}\right)\right]\underbrace{\sin\theta\, d\theta\, R^2\, d\phi}_{dS}$$



$\Rightarrow$ (i) F_r projected onto z axis is $F_r\cos\theta$

(ii) F_θ projected onto z axis is $F_\theta\cos\left(\frac{\pi}{2}-\theta\right)$

$$dF_z = \frac{\mu_0\sigma^2\Omega_0^2R^4}{2}\left[2\sin^2\theta\cos\theta - \cos\theta\sin^2\theta\right]\sin\theta\, d\theta\, d\phi$$

$$F_z = \frac{\cancel{2}\pi\sigma^2\Omega_0^2R^4}{\cancel{2}}\mu_0\left[\int_0^{\pi/2}\cos\theta\sin^2\theta\, d(-\cos\theta)\right]$$

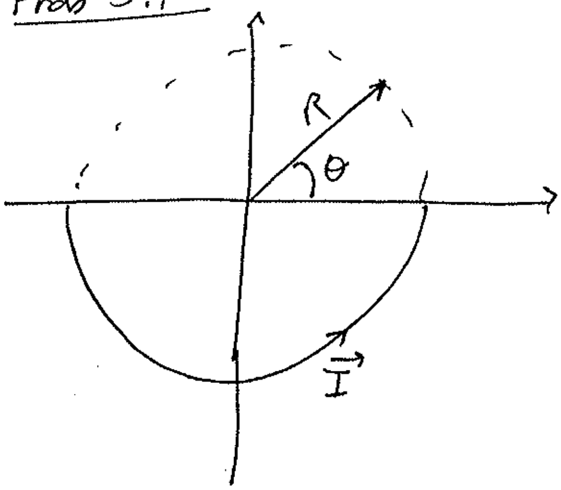
$$= \cancel{2}\pi\sigma^2\Omega_0^2R^4\mu_0\int_0^{\pi/2}\cos\theta\left(1-\cos^2\theta\right)d(-\cos\theta)$$

$$= \cancel{2}\pi\mu_0\sigma^2\Omega_0^2R^4\left[-\frac{\cos^2\theta}{2}+\frac{\cos^4\theta}{4}\right]_0^{\pi/2}$$

$$= \cancel{2}\pi\mu_0\sigma^2\Omega_0^2R^4\left[+\frac{1}{2}+\left(0-\frac{1}{4}\right)\right]$$

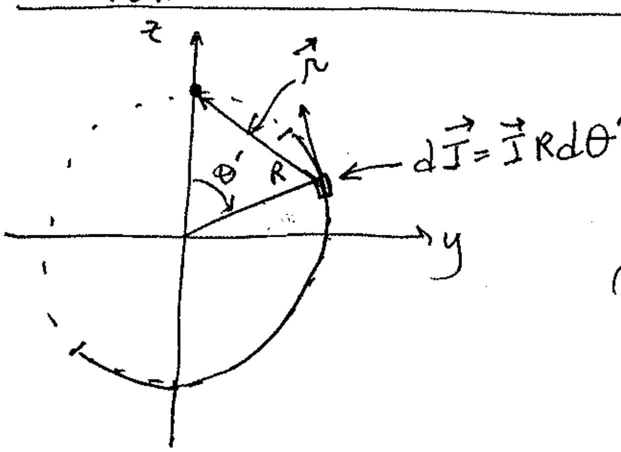
$$= \frac{\pi\mu_0\sigma^2\Omega_0^2R^4}{4}$$

Prob 5.45



find $\vec{B}$ at (R, θ)

Let's rotate the coordinates so that the field point sits on the z-axis



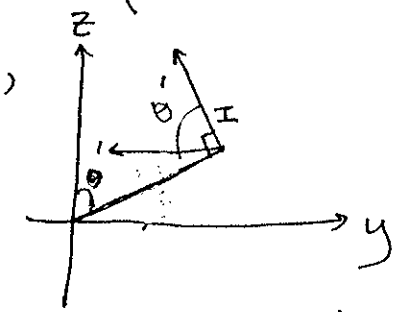
$\leftarrow d\vec{J} = \vec{I} R\, d\theta'$

$$d\vec{B} = \hat{x}\,\frac{\mu_0}{4\pi}\,\frac{I R\, d\vec{\theta}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

write the vectors in Cartesian coordinates

(i) $\vec{I} = I(0, -\cos\theta', \sin\theta')$

from,



(ii) $\vec{\mathscr{r}} = (0, 0 - R\sin\theta', R - R\cos\theta')$

$$\Rightarrow \vec{I} R\, d\theta' \times \vec{\mathscr{r}} = I R\, d\theta'\left(-R[1-\cos\theta']\cos\theta' + R\sin\theta'\sin\theta', 0, 0\right)$$

$$= I R^2 d\theta'\left(-\cos\theta' + 1, 0, 0\right)$$

and so,

$$d\vec{B} = \hat{x}\,\frac{\mu_0}{4\pi}\,\frac{I R^2 (1-\cos\theta')\, d\theta'}{(R^2\sin^2\theta' + R^2 - 2R^2\cos\theta' + R^2\cos^2\theta')^{3/2}}$$

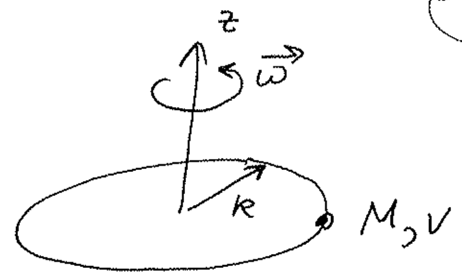
$$d\vec{B} = \hat{x}\,\frac{\mu_0}{4\pi}\,\frac{IR^2}{(2R^2)^{3/2}}\,\frac{d\theta'}{(1-\cos\theta')^{1/2}}$$

$$\vec{B} = \hat{x}\,\frac{\mu_0}{4\pi}\,\frac{I}{2\sqrt{2}\,R}\int_{\theta}^{\theta+\pi}\frac{d\theta'}{(1-\cos\theta')^{1/2}}$$

$$= \hat{x}\,\frac{\mu_0 I}{8\pi\sqrt{2}\,R}\,\sqrt{2}\,\ln\left[\tan\frac{\theta'}{4}\right]_{\theta}^{\theta+\pi} \leftarrow \text{from Tables}$$

$$\boxed{\vec{B} = \hat{x}\,\frac{\mu_0 I}{8\pi R}\,\ln\left[\frac{\tan\left(\frac{\theta+\pi}{4}\right)}{\tan\left(\theta/4\right)}\right]}$$

Prob 5.~~5~~6



a) find the ratio of $\frac{m}{L}$ for $\Longrightarrow$

, i.e., g = gyromagnetic ratio

$$= \frac{m}{L}$$

$$\Longrightarrow g = \frac{\overbrace{\left(Q \frac{V}{2\pi R}\right)}^{I} \underbrace{\pi R^2}_{\text{Area}}}{MVR} = \boxed{\frac{Q}{2M}}$$

b) what is g for a spinning sphere? Consider the sphere to be made of many rings; each ring has charge q_i, and a mass m_i. q_i and m_i are not constant, but we have that (q_i/m_i) is constant, from the fact that $Q = \lambda_Q \times 2\pi R$ and $M = \lambda_m \times 2\pi R \Rightarrow \frac{Q}{M} = $ constant.

Thus, for all rings $\frac{m_i}{L_i} = \frac{Q_i}{2M_i} = \text{constant} \Rightarrow m_i = \frac{Q_i}{2M_i} L_i$.

The total moment divided by the total L is then

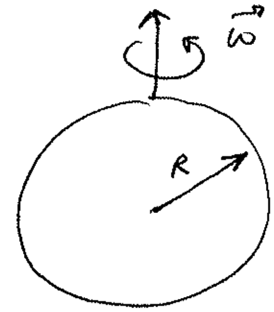
$$g = \frac{\Sigma m_i}{\Sigma L_i} = \frac{\frac{Q}{2M}\Sigma L_i}{\Sigma L_i} = \frac{Q}{2M} \checkmark$$

c) $g = \frac{m}{L} = \frac{m}{\frac{1}{2}\hbar} = \frac{Q}{2M} \Rightarrow \boxed{m = \frac{Q\hbar}{4M}}$

for an electron,

$$m_e = 4.62 \times 10^{-24}\ \text{Am}^2$$

Prob 5.58



A uniformly charged solid sphere of radius R carries total charge Q and is set spinning w/ angular velocity $\vec{\omega}$ about the z-axis.

a) what is the dipole moment of the sphere?

$$\vec{m} = \frac{1}{2}\int \vec{r}\,' \times \vec{J}\, dV'$$

$$= \frac{1}{2}\int \vec{r}\,' \times (\rho\,\vec{\omega} \times \vec{r}\,')\, dV'$$

$$= \frac{3Q}{8\pi R^3}\int \vec{r}\,' \times (\vec{\omega} \times \vec{r}\,')\, r'^2 dr' \sin\theta' d\theta' d\phi'$$

$$= \frac{3Q}{4R^3}\int (\hat{z}\,\omega r'^2 \sin^2\theta')\, r'^2 dr' \sin\theta' d\theta'$$

$$= \frac{3QR^2\omega}{20}\int \sin^2\theta' \sin\theta' d\theta'\, \hat{z}$$

$$= \frac{3QR^2\omega}{20}\int_{-1}^{1} (1-\cos^2\theta')\, d(\cos\theta')\, \hat{z}$$

$$= \frac{3QR^2\omega}{20}\left[2 - \frac{2}{3}\right]\hat{z}$$

$$\boxed{\vec{m} = \frac{QR^2\omega}{5}\hat{z}}$$

b) Find the average $\vec{B}$ inside the sphere,

$$\vec{B}_{av} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{R^3} \qquad (5.87)$$

$$= \frac{\mu_0}{4\pi}\left[\frac{2}{5}\frac{Q\omega}{R}\hat{z}\right]$$

c) Find the $\vec{A}$ at large r ($r>>R$)

$$\vec{A}_{dip} \simeq \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{QR^2\omega}{5r^3}(0,0,1)\times(r\sin\theta,\ 0,\ r\cos\theta)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{QR^2\omega}{5r^3}\right)(0, r\sin\theta, 0)$$

d) Find the exact $\vec{A}$ outside the sphere: From Example 11

$$\vec{A}(P) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}}{\mathscr{r}}dV' \qquad \vec{J} = \rho\vec{v} = \rho(\vec{\omega}\times\vec{r}\,') \qquad dV' = r'^2 dr' \sin\theta' d\theta' d\phi' \qquad \mathscr{r}^2 = r'^2 + r^2 - 2rr'\cos\theta'$$

$$= -\hat{y}\left(\frac{\mu_0\rho\omega_0\sin\psi}{2}\right)\int\frac{\cos\theta'\sin\theta' d\theta'}{\sqrt{r^2+r'^2-2rr'\cos\theta'}}r'^3 dr'$$

$$= -\hat{y}\left(\frac{\mu_0\rho\omega_0\sin\psi}{2}\right)\int_{-1}^{1}\frac{\cos\theta' d(\cos\theta')}{\sqrt{r^2+r'^2-2rr'\cos\theta'}}r'^3 dr'$$

<u>note</u>: $P_1(\cos\theta') = \cos\theta'$

5-58.5

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\hat{r}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi\right)\right] - \frac{\hat{\theta}}{r}\left[\frac{\partial}{\partial r}\left(r A_\phi\right)\right]$$

$$= \frac{\hat{r}}{r}\left[2A_\phi \frac{\cos\theta}{\sin\theta}\right] - \frac{\hat{\theta}}{r}\left[\frac{\mu_0\rho\omega_0}{2}\sin\theta\left\{\frac{2R^2 r}{3} - \frac{4r^3}{5}\right\}\right]$$

$$\vec{B} = \frac{\hat{r}}{\cancel{r}}\left[\mu_0\rho\omega_0 \cancel{r}\cos\theta\left(\frac{R^2}{3} - \frac{r^2}{5}\right)\right] - \frac{\hat{\theta}}{\cancel{r}}\left[\mu_0\rho\omega_0 \cancel{r}\sin\theta\left\{\frac{R^2}{3} - \frac{2r^2}{5}\right\}\right]$$

note:
$$\begin{cases} \hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z} \\ \hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z} \\ \hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \end{cases}$$

$$\left(\frac{\vec{B}}{\mu_0\rho\omega_0}\right) = \hat{x}\left[\sin\theta\cos\theta\cos\phi\left(\frac{R^2}{3} - \frac{r^2}{5}\right) - \cos\theta\sin\theta\cos\phi\left(\frac{R^2}{3} - \frac{2r^2}{5}\right)\right]$$

$$+\hat{y}\left[\sin\theta\cos\theta\sin\phi\left(\frac{R^2}{3} - \frac{r^2}{5}\right) - \cos\theta\sin\theta\sin\phi\left(\frac{R^2}{3} - \frac{2r^2}{5}\right)\right]$$

$$+\hat{z}\left[\cos^2\theta\left(\frac{R^2}{3} - \frac{r^2}{5}\right) + \sin^2\theta\left(\frac{R^2}{3} - \frac{2r^2}{5}\right)\right]$$

note: the $\hat{x}, \hat{y}$ components will integrate to 0 when the ϕ integration is performed. All we will be left with is the $\hat{z}$ integral.

$$\left(\frac{\vec{B}}{\mu_0\rho\omega_0}\right) = \hat{z}\left[\frac{R^2}{3} - \frac{r^2}{5} - \frac{r^2}{5}\sin^2\theta\right]$$

$$= \hat{z}\left[\frac{R^2}{3} - \frac{r^2}{5}\left(1 + \sin^2\theta\right)\right]$$

$$\vec{B} = \hat{z}(\mu_0 \rho \omega_0)\left[\frac{R^2}{3} - \frac{r^2}{5}(1+\sin^2\theta)\right]$$

$$\int \vec{B}\,dV = \hat{z}(\mu_0 \rho \omega_0)\left[\frac{R^2}{3}\int dV - \int \frac{r^2}{5}(1+\sin^2\theta)\,2\pi r^2 dr \sin\theta\, d\theta\right]$$

$$= \hat{z}(\mu_0 \rho \omega_0)\left[\frac{4\pi R^5}{9} - \frac{2\pi}{5}\int r^4 dr\,(1+\sin^2\theta)\sin\theta\, d\theta\right]$$

$$= \hat{z}(\mu_0 \rho \omega_0)\left[\frac{4\pi}{9}R^5 - \frac{2\pi}{25}R^5\int(1+\sin^2\theta)\sin\theta\, d\theta\right]$$

$$= \hat{z}(2\pi\mu_0 \rho \omega_0)\left[\frac{2}{9} - \frac{1}{25}\int_{-1}^{1}(2 \quad \cos^2\theta)\,d(\cos\theta)\right] R^5$$

$$= \hat{z}(2\pi\mu_0 \rho \omega_0 R^5)\left[\frac{2}{9} - \frac{1}{25}\left(4 - \frac{2}{3}\right)\right]$$

$$= \hat{z}(2\pi\mu_0 \rho \omega_0 R^5)\left[\frac{2}{9} - \frac{2}{15}\right]$$

$$= \hat{z}(2\pi\mu_0 \rho \omega_0 R^5)\left(\frac{12}{135}\right) = \hat{z}\left(\frac{3}{2}\mu_0 Q \omega_0 R^2\right)\left(\frac{12}{135}\right)$$

<u>note:</u> $\vec{m} = \frac{Q\omega_0 R^2}{5}\hat{z}$

$$\Rightarrow \int \vec{B}\,dV = \hat{m}\left(\frac{3}{2}\mu_0 \frac{12}{27}\right) = \hat{m}\left(\frac{2\mu_0}{3}\right)$$

and

$$\left[\frac{\int \vec{B}\,dV}{\int dV} = \hat{m}\left(\frac{2\mu_0}{3}\right)\left(\frac{3}{4\pi R^3}\right) = \frac{\mu_0}{4\pi R^3}(2\vec{m})\right]$$

in agreement w/ part (b)