NAME ____

Physics 413: Introduction to Electrodynamics
Test 1
Wednesday, 6 February 2013

Do 2 out of the 3 following questions. Mark clearly the 2 questions you wish to have graded.

Question 1:

Two circular wire loops, each of radius R, carry oppositely directed currents $\pm$ I. The loops are coaxial separated by distance D. Let the symmetry axis lie along the z-axis.

a. Find the on-axis magnetic field **B**. (12 points)

b. Find an expression for the on-axis **B** for $r \gg$ R,D. What is the dominant multipole field at $r \gg$ R,D? (6 points)

c. Make a rough sketch of the on-axis **B**-fields (B *vs.* z) of the individual loops and the total field. Place the plots on one set of axes. (4 points)

d. An electron is placed and held on the axis of the coils. The spin axis of the electron makes an angle $\psi = 0°$ with the line connecting centers of the loops. On your plot mark the locations where the electron is in equilibrium. If the electron is placed at these equilibria and allowed only to move along the z-axis, describe the motion of the electron after it is released from each equilibrium location. (3 points)

Question 2:

An infinite current sheet has uniform surface current density **K**. The sheet sits in the xy-plane.

a. Find **B** for the sheet. (7 points)

b. Find **A** for the sheet. Does your result surprise you? (7 points)

c. In the Coulomb Gauge, what constraint is placed on **A**? Why is the Coulomb Gauge named the Coulomb Gauge? Show that your answer for **A** in part (b) satisfies the Coulomb Gauge. (4 points)

c. A charged particle at height z $= h_\circ$ has charge q, mass m, and initial downward velocity $\mathbf{v} = v_\circ(-\hat{z})$. Find the range for the initial downward velocity for which the particle does not pass through the current sheet. Describe the motion of the particle in this case. Be as quantitative as you can. If the particle has $v_\circ$ so that it will pass through the current sheet, describe its motion. Assume that it can pass freely through the current sheet. (7 points)

Question 3:

A sphere of charge spins at rate $\Omega = \Omega_\circ \hat{z}$. The sphere has radius R and charge density $\rho = C_\alpha r^\alpha$, where α is an integer,

$$C_\alpha = \frac{3+\alpha}{4\pi}\frac{Q}{R^{3+\alpha}}, \tag{1}$$

and Q is the total charge.

a. Find the dipole moment $\mathbf{m}$ for the sphere, for general α. Compare the magnitudes of $\mathbf{m}$ for $\alpha = 0$ and 2. Do your results make sense? (9 points)

b. Find $\mathbf{B}_c$, the magnetic field at the center of the sphere, for general α. Compare the magnitudes of $\mathbf{B}$ for $\alpha = 0$ and 2? Do your results make sense? (9 points)

c. Find $\mathbf{B}$ on the spin axis for $z \gg \mathrm{R}$. Consider a spherical shell of total charge Q and radius R, surface density, $\sigma = Q/(4\pi\mathrm{R}^2)$. This corresponds to the $\alpha \gg 1$ limit. If $\mathrm{R}/z = 0.1$, roughly how large is the error introduced if only the lowest order nonzero multipole field is kept? (7 points)