

Upper Division I, Winter 2023, Meeting 1

1. The sum of three different positive integers is 7. What is the product of these integers?
Ans: $1 \cdot 2 \cdot 4 = 8$. The students confirmed that there are no other answers by checking what number can be the largest of the three.
2. The only available nonstop routes in the Solar System are: Earth-Mercury, Pluto-Venus, Earth-Pluto, Pluto-Mercury, Mercury-Venus, Earth-Venus, Uranus-Neptune, Neptune-Saturn, Saturn-Jupiter, Jupiter-Mars, Mars-Uranus, Uranus-Jupiter. Can one travel from Earth to Mars?
Ans: It is impossible to travel from Earth to Mars. The students sketched all routes and discovered that there are two separate (disconnected) pieces. The first contains Earth, Mercury, Pluto, and Venus. The second contains Uranus, Neptune, Saturn, Jupiter, and Mars.
3. There are 50 cities in the Nowhere Land. If there are two roads leaving each city, how many roads are there in the Nowhere Land?
Ans: 50. Since there are two roads leaving each city, there are $2 \cdot 50 = 100$ "ends of the roads". Each road has two ends, hence, there are $100 \div 2 = 50$ roads.
4. A certain company has 100 computers. Some of them should be connected by cables. Each cable connects 2 computers. A technician knows that eventually each computer should have 8 cables connected to it. How many cables should he bring to complete this job?
Ans: 400. Since each computer has 8 cables connected to it, there are $8 \cdot 100 = 800$ cable ends. Now, each cable has two ends, so there are $800 \div 2 = 400$ cables.

Homework:

1. Little Timbers soccer team played three games in a tournament. It won once, lost once and tied (scored the same number of goals as the other team) once. In total Little Timbers scored three goals and conceded one goal. What was the score in the game that Little Timbers won?
2. The numbers of degrees of the angles in a triangle are three different integers. What is the minimum possible sum of its smallest and largest angles? What is the maximum possible sum of its smallest and largest angles?
3. A cube is sliced three times as shown on the picture below, to produce eight rectangular cuboids (or rectangular parallelepipeds). In other words, each of three cuts is parallel to one of the faces of the cube) What is the ratio of the total surface area of the eight rectangular cuboids to the surface area of the original cube? Does the answer change if the positions of the cuts change but they are still parallel to the faces of the cube?

