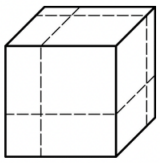
Upper Division I, Winter 2023, Meeting 2

1. The mean of numbers $34, 39, 42, 42, 44, 49, 60, x$ is 46. Find x.
   Ans: 72.

2. Little Timbers soccer team played three games in a tournament. It won once, lost once and tied (scored the same number of goals as the other team) once. In total Little Timbers scored three goals and conceded one goal. What was the score in the game that Little Timbers won?
   Ans: 3:0.

3. The numbers of degrees of the angles in a triangle are three different integers. What is the minimum possible sum of its smallest and largest angles? What is the maximum possible sum of its smallest and largest angles?
   Ans: The first question is equivalent to finding three different positive integers that add up to 180 such that the middle one is as large as possible. The middle number must be smaller than 90. Otherwise, the largest is greater than 90 and the sum of all three numbers is greater than 180. Could 89 work? Yes, the numbers could be 1, 89, and 90. Hence, the minimum possible sum is 91 degrees.
   The second question is equivalent to finding three different positive integers that add up to 180 such that the middle one is as small as possible. The middle number must be greater than 1. Could 2 work? Yes, 1, 2, and 177 are such numbers. Hence, the largest possible sum is 178 degrees.

4. A cube is sliced three times as shown on the picture below, to produce eight rectangular cuboids (or rectangular parallelepipeds). In other words, each of three cuts is parallel to one of the faces of the cube) What is the ratio of the total surface area of the eight rectangular cuboids to the surface area of the original cube? Does the answer change if the positions of the cuts change but they are still parallel to the faces of the cube?

   Ans: 2:1. The students provided two different explanations of this answer.
   (1) Each of the eight rectangular cuboids has three faces that are on the outside of the original cube and three faces that are inside the cube (each was formed by one of three slices). Since the areas of parallel faces are the same, the sum of the areas of the "outside" faces is equal to the sum of the areas of the "inside" faces. Now, for all eight rectangular cuboids together we get that the sum of the areas on the outside (which is equal to the surface area of the original cube) is the same as the sum of the areas on the inside and the answer is 2:1.
   (2) Consider a cube that was sliced once. Two square faces were formed inside the original cube both having the same area as the faces of the cube. With three slices six such faces were formed, which means that the total surface area was doubled.

5. A certain company has 15 computers. Some of them should be connected by cables. Each cable connects 2 computers. A technician sketched a plan according to which each computer should have 3 cables connected to it. How many cables should the technician bring to complete the task according to his plan?
   Ans: The technician made a mistake in his plans. If there are 15 computes with 3 cables connected to each, then there are $15 \cdot 3 = 45$ cable ends. However, the number of cable ends must be divisible by 2 since each cable connects two computers.

6. In the land of numbers there are cities called 1, 2, 3, 4, 5, 6, 7, 8, and 9. Two cities are connected by a road if and only if a two digit number formed by their names is divisible by 3. Can one travel from the city 1 to 9?
   Ans. It's impossible. The students drew a diagram with all possible roads and discovered that 3, 6, and 9 were connected to each other and to no other city. Hence, 9 is reachable only from 3 and 6.

Homework:

1. The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

2. Is it possible to arrange numbers 0 through 9 into a row in such a way that the sum of any two adjacent numbers is divisible either by 5, or by 7, or by 13?

3. There are 30 students in Tom's class. Tom claims that 9 of them have 3 friends each (in that class), 11 have 4 friends each and 10 have 5 friends each. Could Tom be right?