Physics 610
Problem Set 1
Due: October 23, 2014

In working each of these problems, error on the side of verbosity, showing all steps, without assuming the professor knows what is in your mind.

1. The Lagrangian $\mathcal{L}$ for the massless electromagnetic field A_μ interacting with a spin-1/2 field ψ of bare mass m is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$\text{and } D_\mu = \partial_\mu + ieA_\mu Q.$$

   Show that $\mathcal{L}$ is invariant under the local gauge transformation below.

$$\psi(x) \rightarrow exp(-ieQ\alpha(x))\psi(x),$$

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\alpha(x).$$

2. As explained by Dawson (http://arxiv.org/abs/0812.2190), a complex scalar $SU(2)_L$ doublet field, Φ, couples to the gauge fields in the Standard Model. Draw a curve illustrating $V(\Phi)$ versus $|\Phi|$ and the presence of a non-zero vacuum expectation value. Label ν on the figure and express its relationship to λ and μ^2.

3. Derive equation (11) of Dawson from equations (5), (8) and (9).

4. From equation (11) of Dawson, obtain an expression for the mass of the photon, and show how the combination of the terms vanish, leaving the photon massless.

5. Re-express the equations (13) of Dawson in terms of sin θ and cos θ, using the measured value of ν. From the masses of the W and Z, calculate tan $\theta = g'/g$. Which of the two couplings is weaker? What happens in the limit $g' \rightarrow g$?

6. Show and explain equation (47) of Dawson, and evaluate to yield equation (48).