Physics 610
Problem Set 2
due: Thursday, November 6, 2014

In working each of these problems, error on the side of verbosity, showing all steps, without assuming the professor knows what is in your mind.

1. In the low energy limit, the exchange of a virtual particle of mass m corresponds to a potential with a range $\sim 1/m$.

   (a) What is the range for the Z^0 gauge boson exchange?

   (b) Compare this to the size of the hydrogen atom.

   (c) Comment on your expectation for the effect of the Z^0 exchange on atomic binding energies?

2. By varying the QED Lagrangian relative to the EM field A_μ, show that Maxwell's equations are obtained.

3. The Z^0 width to lepton pairs $\Gamma_{Z\to e^+e^-}$ has been measured to be 83.91±0.12 MeV. From this measured value and the measured values for M_Z and $sin^2\theta_W$, calculate $g^2/4\pi$. Compare this value to the value obtained from using

$$\frac{g^2}{4\pi} = \frac{1}{sin^2\theta_W}\alpha$$

   where α is evaluated at the weak scale ($\alpha \approx 1/128$).

4. The cross-section for $e^+e^- \to Z^0 \to f\overline{f}$ is proportional to the square of the coupling of the Z^0 to the electron times the square of the coupling of the Z^0 to the fermion, f. Calculate

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

   as a function of $sin^2\theta_W$, where $\sigma_L(\sigma_R)$ is the cross-section for left-handed (right-handed) electrons. Evaluate this for $sin^2\theta_W = 0.23$ and compare it to the experimental value given in the Particle Data Book. Comment on the comparison.

5. Consider an SU(2) invariant theory where the Higgs field is chosen to be an SU(2) triplet of <u>real</u> fields, like W_1, W_2, and W_3. That choice might seem like a good way to try to give mass to the gauge bosons, with three W fields and 3 Higgs fields.

   (a) Write the Lagrangian for the Higgs sector.

   (b) Write the Higgs Lagrangian in terms of the covariant derivative, $\partial_\mu - ig\mathbf{T}\cdot\mathbf{W}_\mu$.

   (c) Study the terms that give mass to the W's. The triplet representation of T_k can be chosen as $-i\epsilon_{ijk}$. Show that only W_1 and W_2 get mass by this procedure.