Physics 610
Problem Set 3
Due: Thursday, December 4, 2014

In working each of these problems, error on the side of verbosity, showing all steps, without assuming the professor knows what is in your mind.

1. (a.) Show that the Standard Model Higgs potential can be written as

$$V = \frac{1}{2} m_h^2 h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4.$$

   (b.) Describe the meaning of each of the 3 terms.

   (c.) How might you look for the presence of the 2nd term? Be specific about the experiment.

2. In gauge-mediated SUSY-breaking (GMSB) models SUSY breaking is communicated to the visible sector by the standard model gauge interactions. Since these interactions are stronger than gravity, the messenger scale in these models is lower than the Planck scale, and the SUSY-breaking scale is also lower than in SUGRA (typically $\sqrt{F} \leq 10^6$ GeV).

   Distinctive features of GMSB theories which have important phenomenological consequences are the following. Due to the low SUSY-breaking scale, the gravitino, which is the supersymmetric partner of the graviton, has a mass in the range 10^{-4} - 10^{-2} eV (to be compared to 1 TeV in SUGRA), and is therefore the LSP. Since it interacts very weakly with matter, it gives rise to missing energy in the final state. The next-to-lightest supersymmetric particle (NLSP) is in general either the lightest neutralino χ_1^0, which decays according to $\chi_1^0 \to \gamma\widetilde{G}$, or a slepton (usually the stau), which decays according to $\widetilde{l} \to l\widetilde{G}$; since the coupling between the gravitino and the NLSP decreases with increasing values of the SUSY-breaking scale, in GMSB models the NLSP lifetime depends on

$$c\tau \sim 100\mu m \left(\frac{100GeV}{m}\right)^5 \left(\frac{\sqrt{F}}{100TeV}\right)^4$$

   where $c\tau$ and m are the NLSP lifetime and mass, respectively.

   (a.) Choose a value of $\sqrt{F}$ (see above) and m, based on your understanding of the current expectations.

   (b.) Estimate the value of $c\tau$ for the values in (a.).

   (c.) Discuss how you might mount an experiment to search for this decay.

   (ref - F. Gianotti, "Searches for supersymmetry at high-energy colliders: the past, the present and the future," New Journal of Physics 4 (2002) 63.163.25 (http://www.njp.org/) http://iopscience.iop.org/1367-2630/4/1/363.)

3. Imagine an experiment searching for pair production of the selectron and the anti-selectron in electron-positron annihilation at $\sqrt{s} = 500$ GeV (the ILC).

(a) Draw the Feynman diagrams for two ways to produce this final state.

(b) Calculate the energy of the produced selectrons at this collider in terms of the mass of the selectron, $M_{\tilde{e}}$, and $\sqrt{s}$, assuming the initial electron and positron have the same collision energy.

(c) Suppose the selectron decays to an electron and the LSP, which is the lightest neutralino $\tilde{\chi}_1^0$. Draw the Feynman diagram for this decay.

(d) Calculate the minimum and maximum energies (E_{min} and E_{max}) of the resulting electrons in terms of the $M_{\tilde{e}}$, $\sqrt{s}$, and the mass of the lightest neutralino, $M_{\tilde{\chi}_1^0}$.

(e) Calculate the minimum and maximum possible values for the total missing energy in such events.

(f) Suppose the experiment measures $E_{min} = 40$ GeV and $E_{max} = 200$ GeV. Determine the selectron and lightest neutralino masses for this case.

(g) Consider the main backgrounds to this measurement and comment on them.

4. The existence of large extra dimensions has been proposed as a possible solution to the heirarchy problem (that is, the question of why the Planck scale of $M_{Pl} \sim 10^{19}$ GeV is so much larger than the weak scale of 1 TeV). If we live on a 4D hypersurface, but gravity exists in a (4+n)D bulk, the fundamental scale of gravity (M_*) may also be 1 TeV, but the effect of gravity reduced by the extra dimensions, making the force appear very weak. This leads to the result that at large distances ($r > R$, the scale of the extra dimensions) gravity acts as

$$F \sim 1/r^2,$$

and at short distances ($r < R$)

$$F \sim 1/r^{2+n},$$

and the fundamental scale, M_*, is related to M_{Pl} by the size of the large extra dimensions.

(a) Assuming the n large extra dimensions all have the size R, show

$$M_{Pl}^2 \sim R^n M_*^{n+2}.$$

(b) Given that 4D gravity explains motion on the scale of the Earth-Sun distance, what can you say about the limits on the number of extra dimensions n? (Assume $M_* \sim 1$ TeV.)

(c) To what length scale would you have to make measurements to rule out n=2? (Again, assume $M_* \sim 1$ TeV.)

(d) Suppose $M_* = 1$ TeV. What values for n might be detectable in collider experiments. (Consider the typical distance scales for such experiments.)

5. The Synchrotron radiation energy loss for a particle in one turn of a storage ring is:

$$\Delta E = \frac{4\pi}{3}(e^2\beta^3\gamma^4/R)$$

or

$$\Delta E[GeV] = 88.5 \times 10^{-6}(E_{beam}[GeV])^4/R[m]$$

where R is the radius of the ring.

(a) Calculate the energy loss rate (Power) of all particles in the ring, assuming an average beam current, I_{av}, expressing the result as a function of I_{av}, E (the beam energy) and R.

(b) Calculate from the formula above the beam energy lost by a 100 GeV electron beam on each turn of the LEPII project. Also calculate the total power of lost energy in the ring. (Note the average beam current at LEPII was about 5 mA, and other parameters can be found in the Particle Data Group report.)

(c) There is a proposal to reuse the LEPII ring for a Higgs Factory. This is called LEP3. The proposed beam energy is 120 GeV and the beam current is 7.2 mA. Calculate the power that would be lost in the LEP3 ring. Compare this to the total power available to the collider facility of 200 MW.

(d) Consider the parameters for a future electron-positron collider:

$$I_{beam} = 20 \text{ mA}$$
$$E_{beam} = 250 \text{ GeV}$$

Calculate the power (in Watts) contained in the beams.

(e) Calulate the required radius of the 250 GeV storage ring that would bring the synchrotron radiation losses down to 5% of the beam power, assuming the same beam power as calculated in (d). Does this motivate a linear collider? Why?