Interactions and Fields

- Quantum Picture of Interactions
- Yukawa Theory
- Boson Propagator
- Feynman Diagrams
- Electromagnetic Interactions
- Renormalization and Gauge Invariance
- Strong Interactions
- Weak and Electroweak Interactions
- Gravitational Interactions
- Cross-sections
- Decays and resonances
- The Δ^{++} pion-proton resonance
- The Z^0 resonance
- Resonances in astrophysics

Quantum Picture of Interactions

- Quantum Theory views action at a distance through the exchange of quanta associated with the interaction
- These exchanged quanta are virtual and can "violate" the conservation laws for a time defined by the Uncertainty Principle:
  - $\Delta E \, \Delta t \cong \hbar$

Yukawa Theory

- During the 1930's, Yukawa was working on understanding the short range nature of the nuclear force ($R \approx 10^{-15}$m)
- He postulated that this was due to the exchange of massive quanta which obey the Klein-Gordon eqtn:

$$\partial^2\psi / \partial t^2 = c^2(\nabla^2 - m^2c^2/\hbar^2)\,\psi$$

Yukawa Theory

$$\partial^2\psi / \partial t^2 = c^2(\nabla^2 - m^2c^2/\hbar^2)\,\psi$$

for a static potential, this becomes:

$$\nabla^2\psi = (m^2c^2/\hbar^2)\,\psi$$

We can interpret ψ as the potential U(r) and solve for U:

$$U(r) = g_0\, e^{-r/R} / 4\pi\, r,$$

where $R = \hbar/mc$,

and g_0 is a constant (the strength)

Yukawa Theory

The range of the nuclear force was known,

$$R \approx 10^{-15}\text{m}$$

Therefore, the mass of this new exchange particle could be predicted:

$$R = \hbar/mc,$$

$$mc^2 = \hbar c/R \approx 200 \text{ MeV-fm}/1 \text{ fm} \approx 200 \text{ MeV}$$

- The pion with mass 140 MeV/c^2 was discovered in 1947! (the muon was discovered in 1937 and mis-identified as Yukawa's particle, the "mesotron")
- We now realize that this interaction is actually a residual interaction, so Yukawa was a bit fortunate to find a particle with the predicted mass

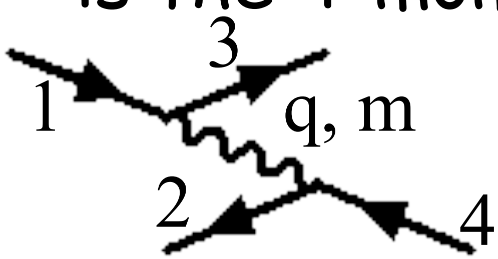
Boson Propagator

The rate for a particular interaction mediated by boson exchange is proportional to the "propagator" squared, where the "propagator" is written as:

$$f(q) = g_0 g \,/\, (q^2 + m^2),$$

where $q^2 = (\Delta p)^2 - (\Delta E)^2$,

is the 4-momentum transfer



$$\Delta p = p_3 - p_1 = p_2 - p_4$$

$$\Delta E = E_3 - E_1 = E_2 - E_4$$

Boson Propagator

- This "propagator" can be derived by taking the Fourier transform of the potential:

$$f(q) = g \int U(r)\, e^{iq \bullet r}\, dV$$

- Therefore, the "propagator" describes the potential in momentum space
- Then, the boson "propagator" is:

$$f(q) = g_0 g \,/\, (|q|^2 + m^2)$$

where q is the momentum of the boson, and m is its mass.

Boson Propagator

The "propagator" can be generalized to four-momentum transfer:

$$f(q) = g_0 g \;/\; (q^2 + m^2),$$

where now $q^2 = (\Delta p)^2 - (\Delta E)^2$,

is the 4-momentum transfer

Rates are proportional to the propagator:

$W = |f|^2 \times$ Phase Space . . .

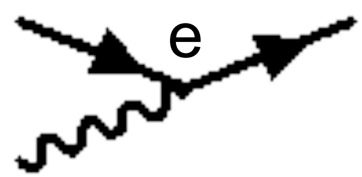
Feynman Diagrams

- Interactions can be depicted with Feynman diagrams
  - electrons 
  - photons 
  - positrons 
    - (equivalent to electron moving backward in time)

  - electron emits a photon

$$A \sim e$$

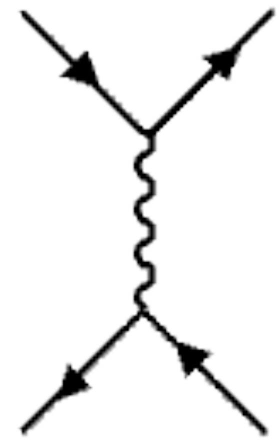
  - electron absorbs a photon



$$A \sim e$$

Feynman Diagrams

- Virtual particles
  - lines joining vertices represent virtual particles (undefined mass)



- Vertices are represented by coupling constants, and virtual particles by propagators

Electromagnetic Interactions

- The fine structure constant specifies the strength of the EM interaction between particle and photons:

$$\alpha = e^2 / 4\pi\hbar c = 1 / 137.0360.....$$

- Emission and absorption of a photon represents the basic EM interaction

vertex amplitude = $\sqrt{\alpha}$ = e

- cannot occur for free particle

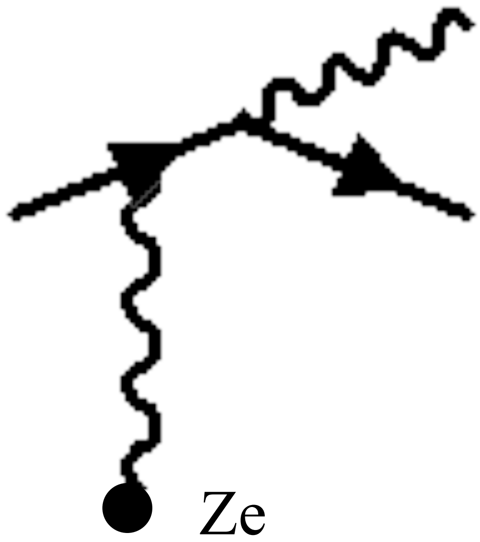
Electromagnetic Interactions

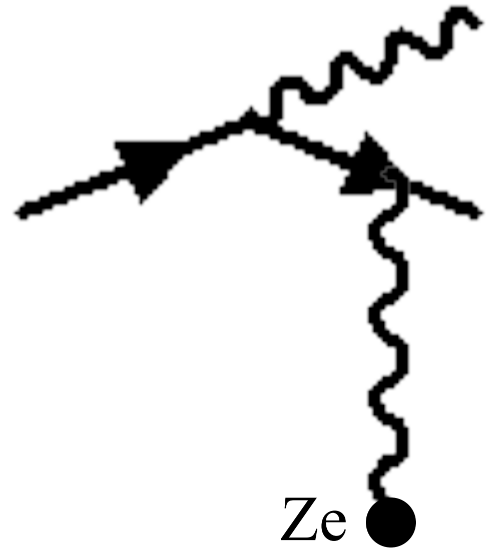
- Coulomb scattering between two electrons:

- Amplitude: α / q^2
- Cross Section = |Amp|^2: α^2 / q^4
  - the Rutherford scattering formula

Electromagnetic Interactions

- Bremstrahlung:
  - electron emits photon in field of the nucleus

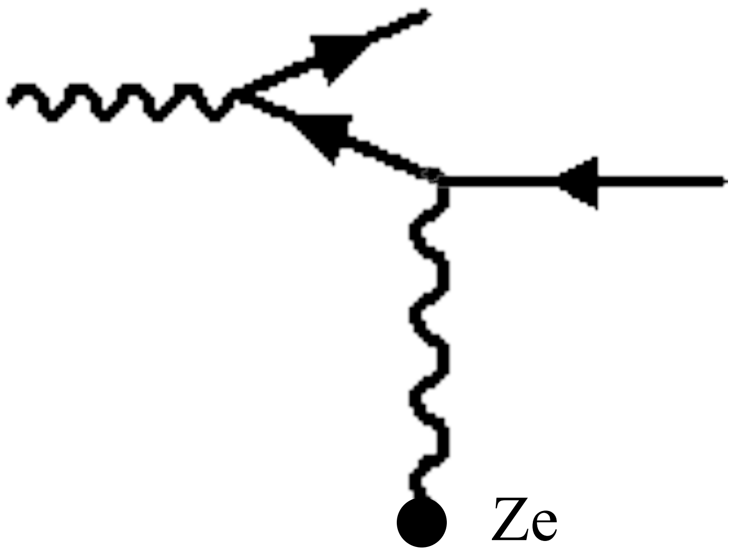




  - cross section: $\sim \alpha^3 Z^2$

Electromagnetic Interactions

- Pair production ($\gamma \rightarrow e^+e^-$)



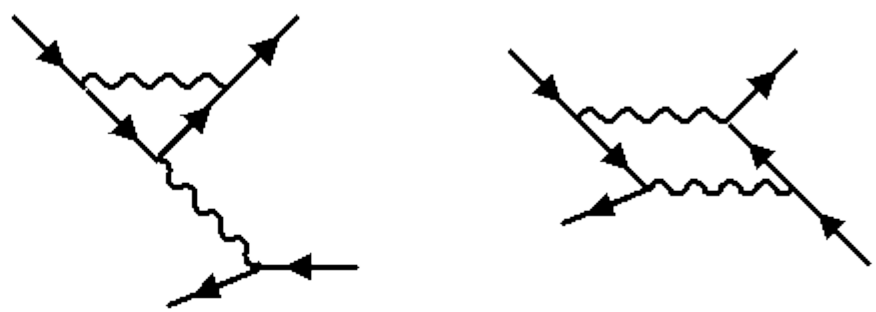
cross section:
$\sim \alpha^3 Z^2$

- This process is closely related to bremsstrahlung ("crossed diagrams")

Electromagnetic Interactions

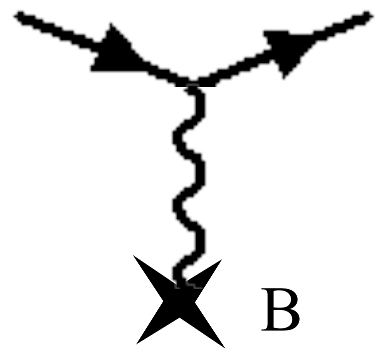
- Higher order processes
  - the diagrams we have seen so far are leading order diagrams, but the rate for a process will be the sum of all orders:
  - For example, Bhabha scattering: $e^+e^- \rightarrow e^+e^-$
    - leading order:

    - higher order:



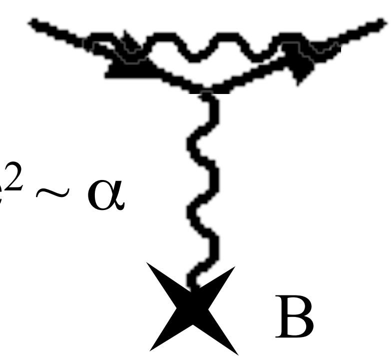
Electromagnetic Interactions

- Example of higher order processes: the electron magnetic moment
  - lowest order:

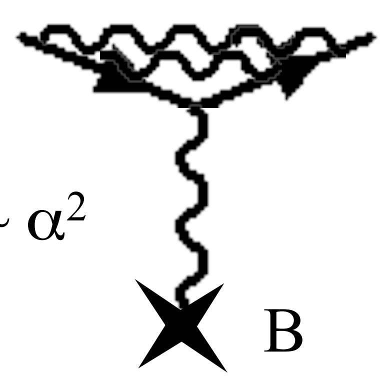


  - higher order:

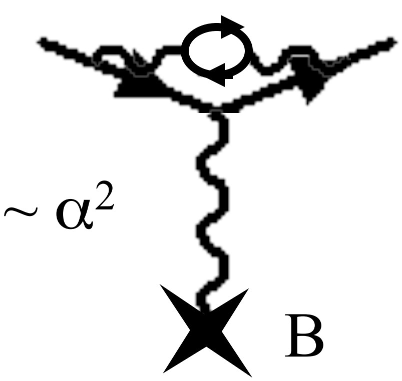
Correction ~ e^2 ~ α



~ α^2



~ α^2



Electromagnetic Interactions

- Electron magnetic moment:
  - a Dirac electron has a magnetic moment of

$\mu = g\, \mu_B\, s,$ $s = 1/2$ $g = 2$ $\mu_B = e\hbar/mc$

$(g-2)/2$ is the anomaly due to higher order terms

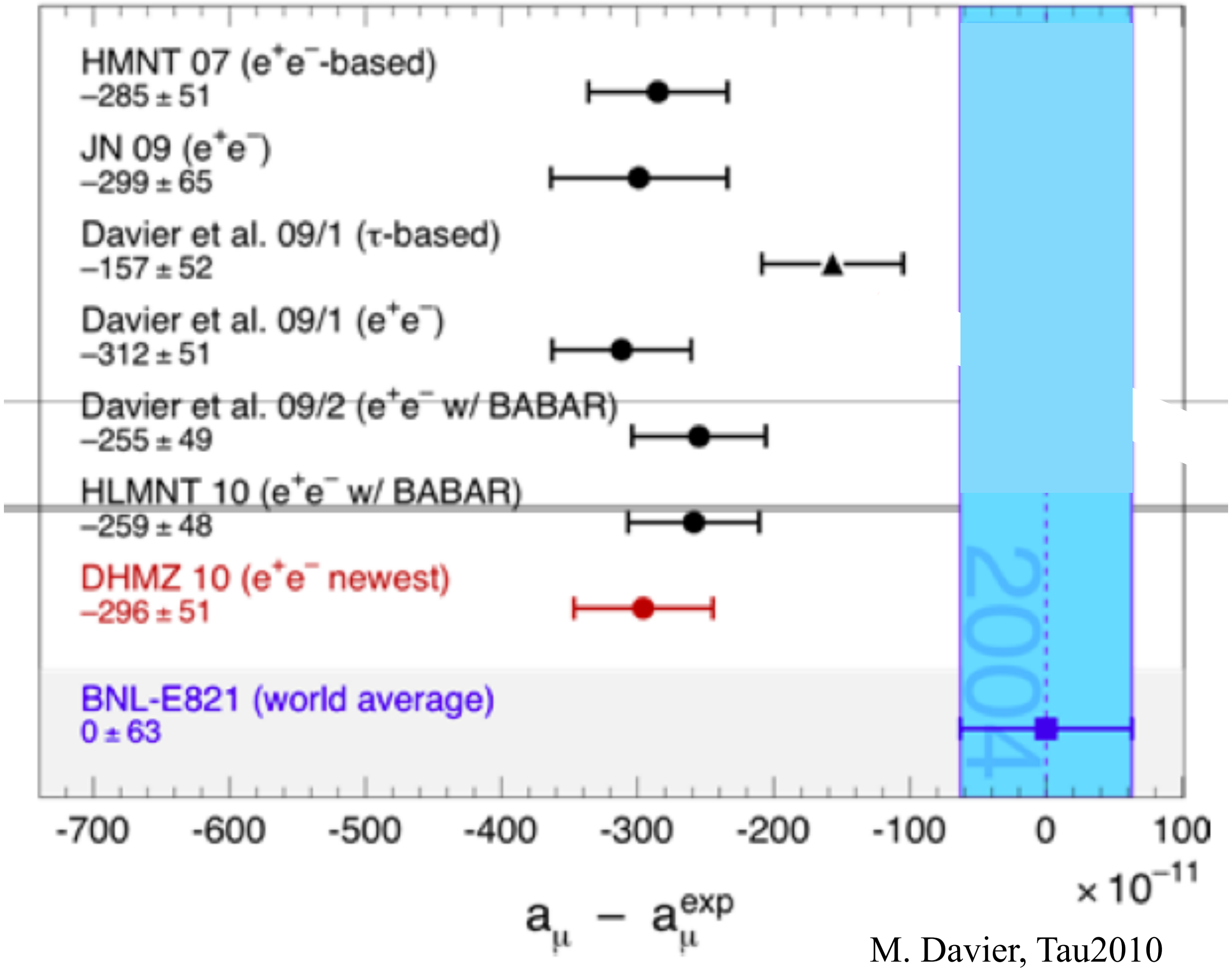
$$(g-2)^{th}/2 = 0.5\,(\alpha/\pi) - 0.32848\,(\alpha/\pi)^2 + 1.19\,(\alpha/\pi)^3 + ..$$
$$= (115965230 \pm 10) \times 10^{-11}$$

$$\text{experiment} = (115965218.073 \pm 0.028) \times 10^{-11}$$

PRL **100**, 120801 (2008)

this measurement provides very accurate value for the fine structure constant = 1/137.035 999 084(51)

Muon g-2



M. Davier, Tau2010

$\mu = g\,\mu_B\,s,$

$s = 1/2 \qquad g = 2$

$\mu_B = eh/mc$

$a = (g-2)/2$

Observed Difference with Experiment:

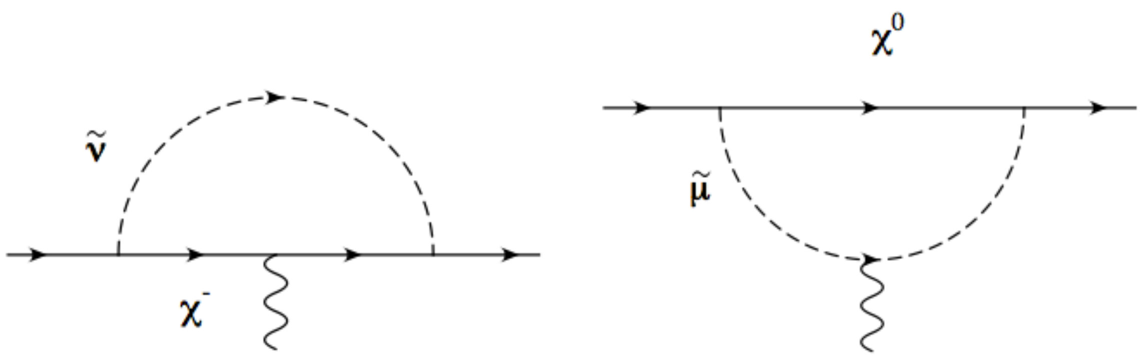
$a_\mu^{exp} - a_\mu^{SM} = (29.6 \pm 8.1) \times 10^{-10}$

➡ 3.6 "standard deviations"

g-2 could mean New Physics ?

Supersymetric particles (Marciano, Munich, 2011)

- Most likely (popular?)
- $a_\mu(\mathrm{SUSY}) = \mathrm{sgn}(\mu)\, 130 \times 10^{-11} \left(\frac{100\mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta$
- $\mathrm{sgn}(\mu) = +$, $\tan\beta = 3 - 40$, $m_{\mathrm{SUSY}} = 100 - 500$ GeV



If SUSY: $\mathrm{sgn}(\mu)+$, dark matter easier, SUSY at LHC likely, EDMS, ...

Renormalization and Gauge Invariance

- Electron line represents “bare” electron
- Observable particles are “dressed” by “infinite” number of virtual photons:
  - logarithmically divergent
- These divergences are swept away through renormalization:
  - “Bare” electron mass and charge is always multiplied by divergent integrals. We know this product must be the physical values of the mass and charge, so we set them to be, and the divergences are removed

Renormalization and Gauge Invariance

- In order for a theory to be "renormalizable" it must satisfy local gauge invariance
  - Examples of gauge invariance are familiar in EM and quantum
    - gauge transformations of scalar and vector potential in E&M do not change physical effects
    - wavefunction can change by an arbitrary phase without altering physics

Renormalization and Gauge Invariance

- The coupling constants that appear in the theory are actually not "constants", but "run" with energy.
  - This is due again to virtual processes
  - For example, $\alpha = 1/137$ at very low energy, but
    $\alpha = 1/128$ at $\sqrt{s} = M_Z$

Strong Interactions

- Decay of the Σ baryons

Baryon	Composition	Q-value, MeV	Decay Mode	Lifetime, s
Σ^0 (1192)	uds	77	$\Lambda\gamma$	10^{-19}
Σ^+ (1189)	uus	116	$p\pi^0$	10^{-10}
Σ^0 (1385)	uds	135	$\Lambda\pi^0$	10^{-23}

Strong Interactions

- Decay of the Σ baryons

Baryon	Composition	Q-value, MeV	Decay Mode	Lifetime, s
Σ^0 (1192)	uds	74	$\Lambda\gamma$	10^{-19}
Σ^+ (1189)	uus	189	$p\pi^0$	10^{-10}
Σ^0 (1385)	uds	208	$\Lambda\pi^0$	10^{-23}

  – These are the Q-values given in Perkins' Table 2.1, which are wrong. Correct values are 77, 116, and 135

Strong Interactions

- Decay of the Σ baryons

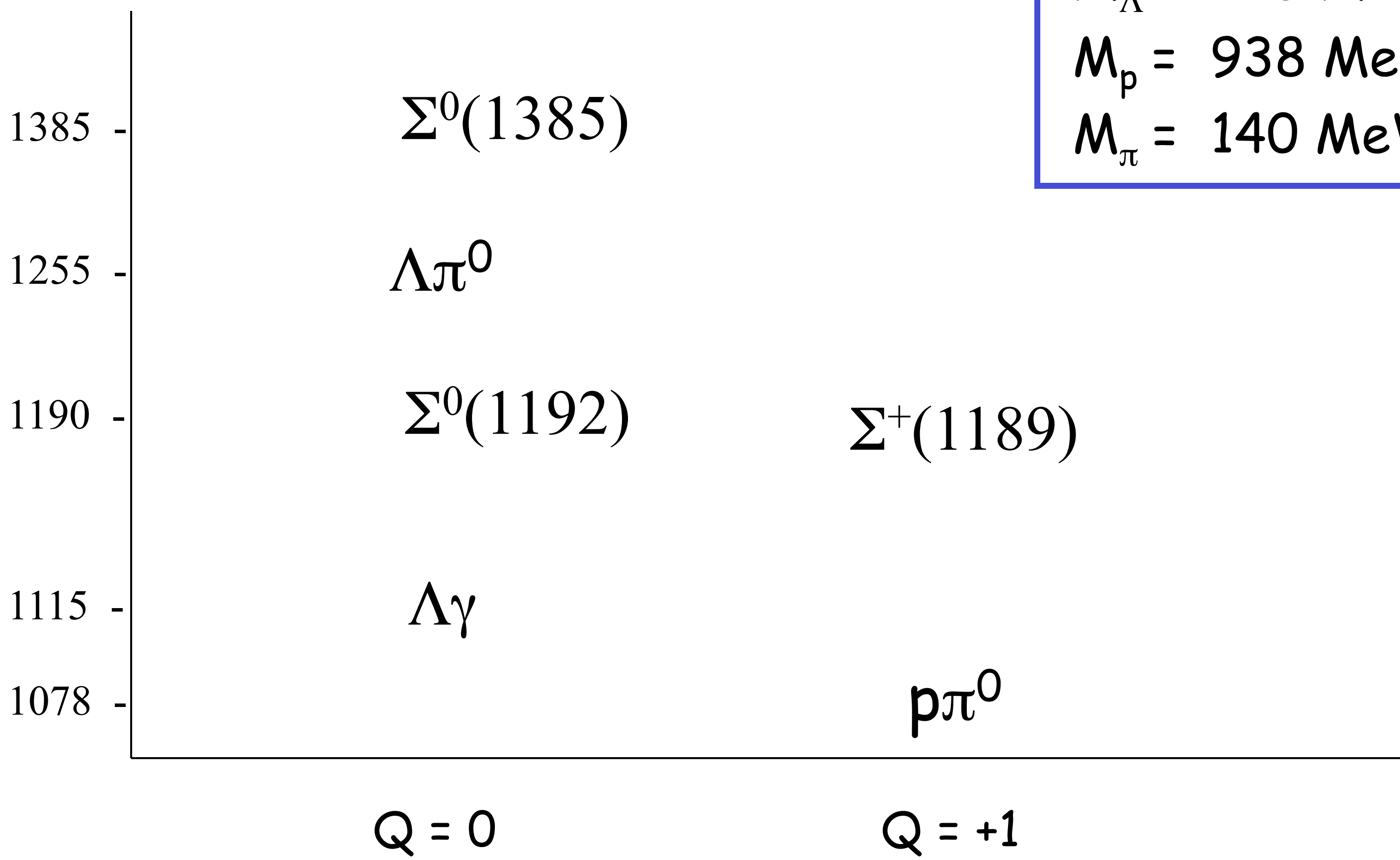
Baryon	Composition	Q-value, MeV	Decay Mode	Lifetime, s
Σ^0 (1192)	uds	77	$\Lambda\gamma$	10^{-19}
Σ^+ (1189)	uus	116	$p\pi^0$	10^{-10}
Σ^0 (1385)	uds	135	$\Lambda\pi^0$	10^{-23}

  - why is the Σ^0 (1192) 3 MeV heavier than the Σ^+ (1189) ?
  - why does the Σ^+ (1189) live a billion times longer than the Σ^0 (1192) ?

Strong Interactions

- Decay of the Σ baryon

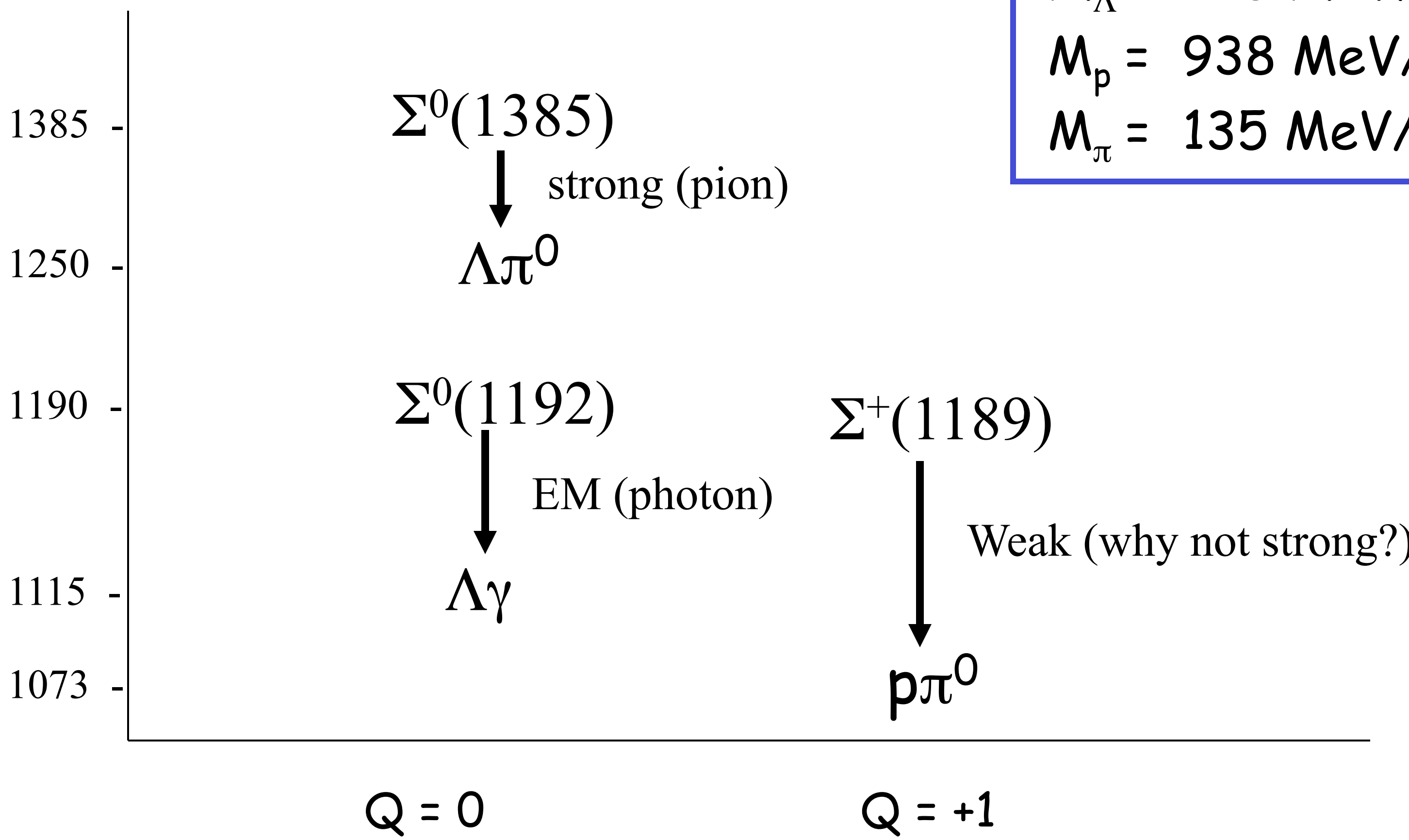
M_Λ = 1115 MeV/c^2
M_p = 938 MeV/c^2
M_π = 140 MeV/c^2



Strong Interactions

- Decay of the Σ baryon

M_Λ = 1115 MeV/c^2
M_p = 938 MeV/c^2
M_π = 135 MeV/c^2



Strong Interactions

- Decay of the Σ baryons

Baryon	Composition	Q-value, MeV	Decay Mode	Lifetime, s
Σ^0 (1192)	uds	77	$\Lambda\gamma$ (EM)	10^{-19}
Σ^+ (1189)	uus	116	$p\pi^0$(weak)	10^{-10}
Σ^0 (1385)	uds	135	$\Lambda\pi^0$(strong)	10^{-23}

- strong
  - Σ^0 (1192) cannot decay strongly because $M_\Lambda + M_\pi = 1250$ MeV/c^2
  - Σ^+ (1189) cannot decay strongly (same reason)
- EM
  - Σ^+ (1189) cannot decay Emly because there is no lighter charged, strange baryon (Σ^+ (1189) is lightest charged, strange baryon)

Strong Interaction

- The ratios of the decay rates give an approximate indication of the relative strengths of the interactions:

$$\{\tau[\Sigma^0(1192)]/\tau[\Sigma^0(1385)]\}^{1/2} \approx 100 \approx \alpha_s/\alpha$$

- Since $\alpha = 1/137$, $\alpha_s \approx 1$

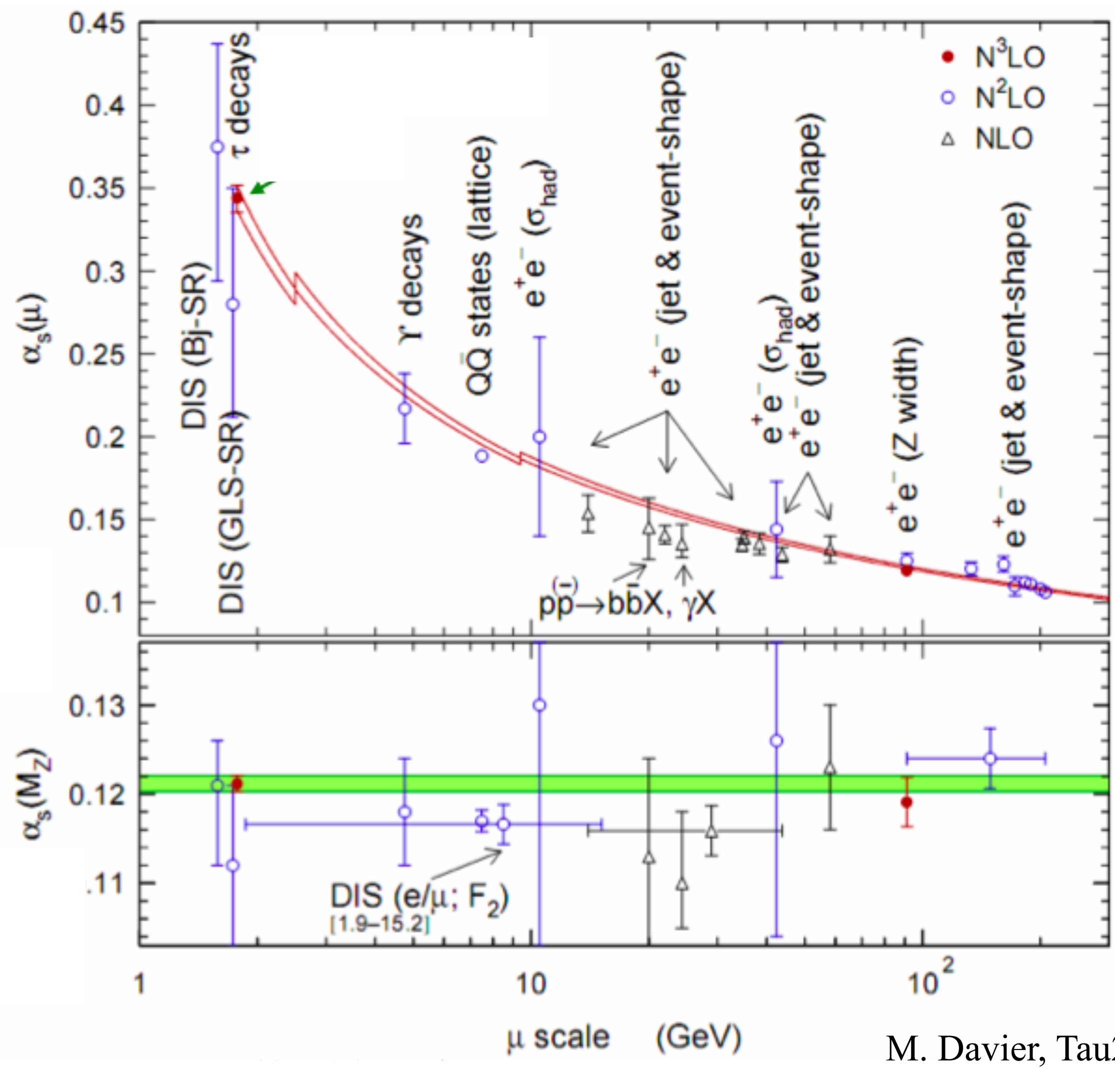
Strong Interaction

- gluon mediator
  - neutral, massless, vector particle (as with EM)
  - carries color charge (unlike EM)
- QCD (Quantum Chromodynamics)
  - Nine possible color combinations for the gluons
    - blue anti-green
    - green anti- red
    - etc.
    - however, one is a color singlet, which means it does not interact, so there are actually eight active gluon states

Strong Interaction

- The QCD (strong) coupling constant “runs”, such that at large distances (within a hadron) the coupling constant is very large and perturbation theory cannot be applied, while at small distances (which occur during violent collisions) the coupling constant is small and perturbation theory is valid
- This large distances behavior is the origin of “confinement” of quarks within hadrons

Running of Strong Coupling



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Strong Interactions

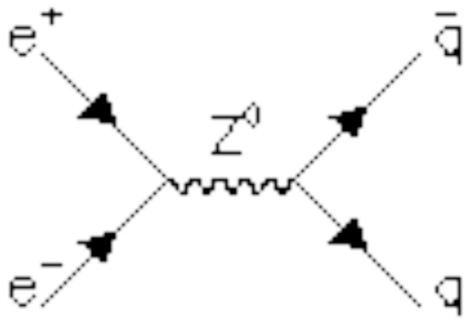
- The quark-antiquark potential is often approximated by

  $V_s = -(4/3)\, \alpha_s / r + k\, r$

- The 1/r portion of this potential is similar to the Coulomb potential
- However, the long range linear term is different
  - confinement
  - sizes and masses of hadrons $\Rightarrow$ k ~ 1 GeV/fm
    - (14 tonnes)

Strong Interactions

- Two jet events



- confining potential

Weak and Electroweak Interactions

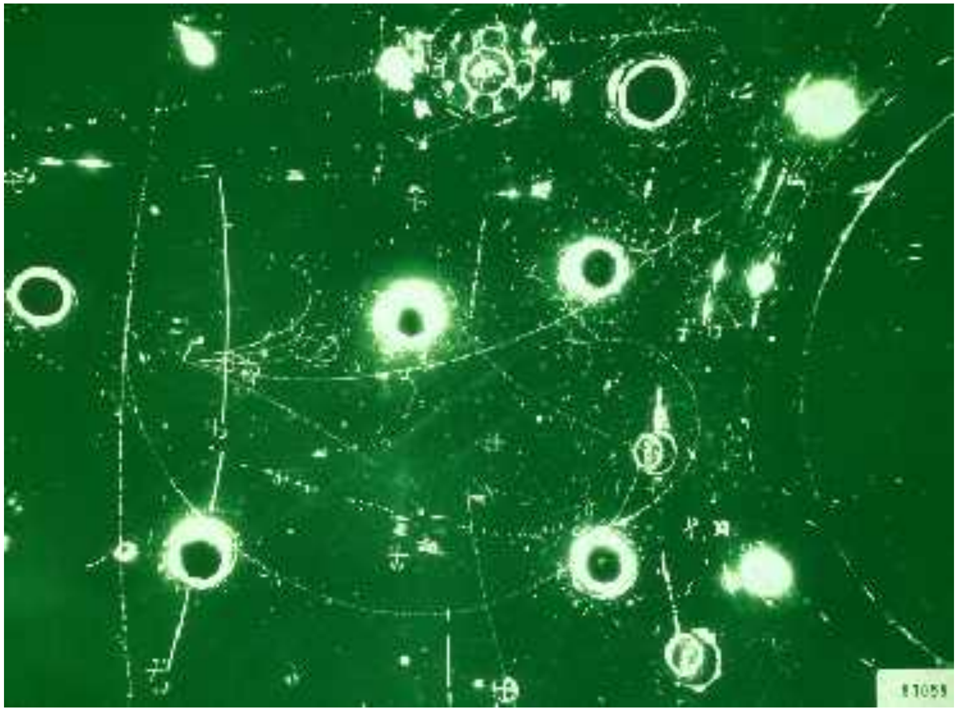
- The ratios of the decay rates give an approximate indication of the relative strengths of the interactions:

$$\{\tau[\Sigma^0(1192)]\,/\,\tau[\Sigma^+(1189)]\}^{1/2} \approx 10^{-5} \approx \alpha_{weak}\,/\,\alpha$$

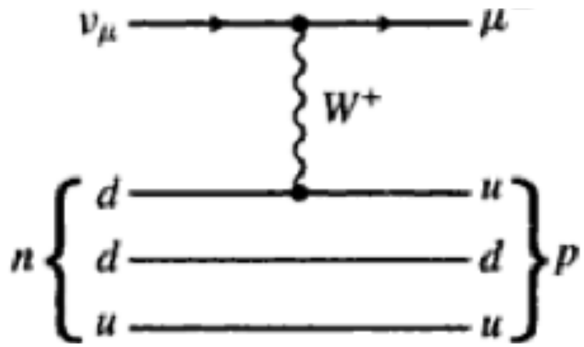
- All leptons and quarks "feel" the weak force, but it is so small that it is swamped by the strong and EM forces, unless they are forbidden by some conservation law

Weak and Electroweak Interactions

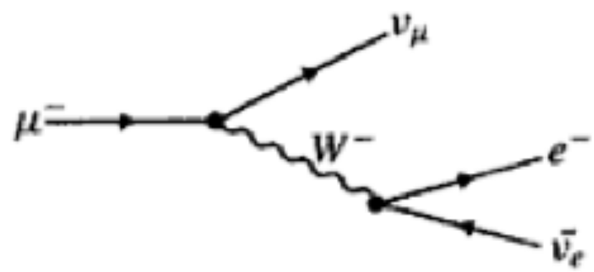
- Nuclear β-decay
- Lepton conservation
- Weak due to very heavy mediating vector bosons:
  - $W^{\pm}$, 80 GeV/c^2
  - Z^0 , 91 GeV/c^2
- charge-current
  - β-decay
- neutral-current



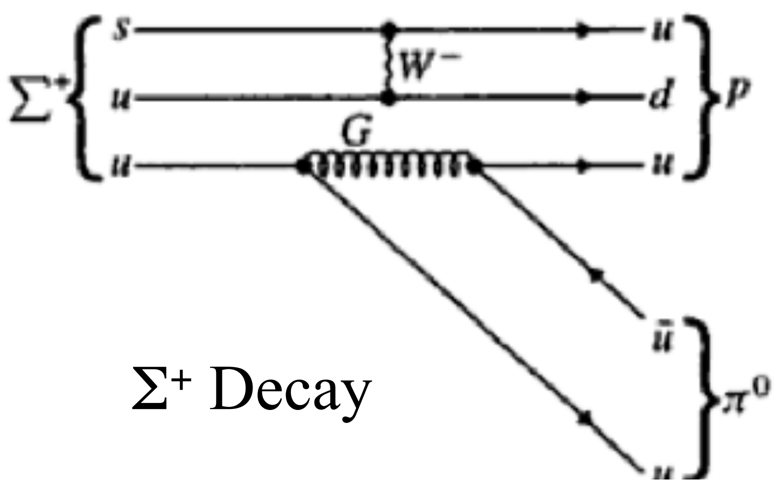
Weak Interactions



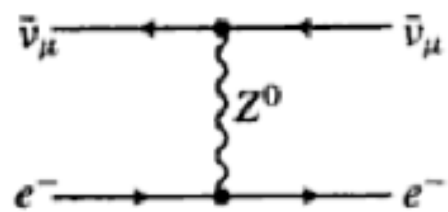
Neutrino Charged Current Interaction



Muon Decay



Σ^+ Decay



Neutral Current

Weak and Electroweak Interactions

- Simplfied picture of the Weak Interaction:
  - Propagator:

    $f(q) = g^2 / (q^2 + M_{W,Z}^2)$
  - for $q^2 << M^2$, $f(q) = g^2 / M_{W,Z}^2$

  - Fermi's early theory of β-decay postulated an interaction with strength $G = 10^{-5}$, which we now recognize as $G = g^2 / M_{W,Z}^2 = 10^{-5}$

  - Once we recognize the origin of the weakness, we can predict the masses of the W and Z

    $M_{W,Z} \sim e/\sqrt{G} \sim \sqrt{4\pi\alpha/G} \sim 90$ GeV
    - Glashow, Weinberg, Salam (1961-8)

Gravitational Interactions

- Gravity is one of the four forces of Nature, but its strength is so small that is not important in accelerator experiments:
  - Force between two equal point masses M:
    - $F_N = G_n M^2 / r^2$
    - compared to EM $F_{EM} = e^2 / r^2$
    - so gravitational constant to be compared to fine structure constant is

      $G_n M^2 / 4\pi\hbar c = 5.3 \times 10^{-40}$ for $M = 1$ GeV/c^2
  - The coupling constant would approach unity for $M > (\hbar c/G_N)^{1/2} = 1.22 \times 10^{19}$ GeV/c^2, the Planck mass

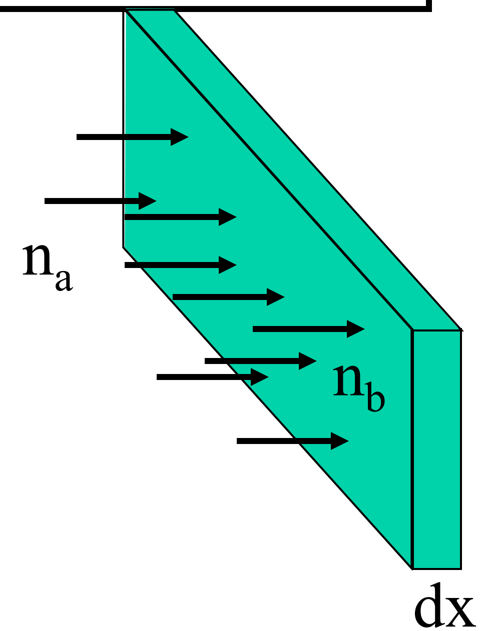
Gravitational Interactions

- Quantum gravitational effects become important at lengths such that the gravitational energy is comparable to the mass
  - $V_N = G_n M_P^2 / r = M_P c^2$
  - $r_P = G_n M_P / c^2 = \hbar / M_P c \approx 2 \times 10^{-20}$ fm $= 2 \times 10^{-35}$ m

The Interactions

	Gravity	Electromag.	Weak	Strong
field boson	graviton	photon	$W^{\pm}$,Z	gluon
spin-parity	2^+	1^-	1^-, 1^+	1^-
mass, GeV	0	0	M_W=80.2 M_Z=91.2	0
range, m	∞	∞	10^{-18}	$\leq 10^{-15}$
source	mass	electric charge	'weak' charge	'color' charge
coupling constant	5×10^{-40}	1/137	1.2×10^{-5}	≤ 1
typical cross-section, m^2		10^{-33}	10^{-39}	10^{-30}
typical lifetime, s		10^{-20}	10^{-10}	10^{-23}

Cross-sections



- Reaction, such as $a + b \rightarrow c + d$
- Flux = $n_a v_i$
  - n_a = density of particles in beam
  - v_i = velocity of particles in beam
- Target
  - n_b = density of particles in target
  - dx = thickness of target
- Reaction rate per target particle (W) = [Flux] • [cross section(σ)]
  - σ = W / Flux
  - 1 barn $\equiv 10^{-28}$ m^2
- Reactions per time per area of beam
  - [Flux] n_b dx [cross section(σ)]

Cross-sections

• barn =		10^{-28} m^2
• millibarn =	mb	= 10^{-31} m^2
• microbarn =	μb	= 10^{-34} m^2
• nanobarn =	nb	= 10^{-37} m^2
• picobarn =	pb	= 10^{-40} m^2
• femtobarn =	fb	= 10^{-43} m^2

Cross-sections

- Fermi's Second Golden Rule (non-rel QM) gives the reaction rate, W:

$$W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

where $M_{if} = \int \psi_f^* U \psi_i \, dV$

and $\rho_f = dN/dE$, is the density of final states

Cross-sections

- The phase space available to a particle (neglecting spin) is:

$$dN = \frac{V}{(2\pi\hbar)^3} p^2 dp d\Omega$$

The volume factor cancels with the normalization of the wavefunctions:

$$\frac{d\sigma}{d\Omega} = \frac{W}{\phi_i} = \frac{W}{v_i} = \frac{2\pi}{\hbar} \frac{|M_{if}|^2}{v_i} \frac{1}{(2\pi\hbar)^3} p_f^2 \frac{dp_f}{dE_0}$$

Cross-sections

- Phase space factor
  - density of states in 1D:
    - N = L p / h
  - in 3D:
    - N = $\int (dx)^3 (dp)^3 / (h)^3 = V / (h)^3 \int p^2\, dp\, d\Omega$

$$dN = \frac{V}{(2\pi\hbar)^3} p^2 dp d\Omega$$

Cross-sections

- Considering $a + b \rightarrow c + d$, energy conservation in the CMS says

$$\sqrt{p_f^2 + m_c^2} + \sqrt{p_f^2 + m_d^2} = E_0$$

$1/(2\,E_c)\,2\,p_f\,dp_f + 1/(2\,E_d)\,2\,p_f\,dp_f = dE_0$

$p_f\,dp_f\,[1/(2E_c) + 1/(2\,E_d)\,] = dE_0$

$dp_f/dE_0 = E_cE_d/[p_f(E_c{+}E_d)] = E_cE_d/(p_fE_0)$

$$\frac{dp_f}{dE_0} = \frac{E_c E_d}{E_0 p_f} = \frac{1}{v_f}$$

Cross-sections

$$\frac{d\sigma}{d\Omega}(a+b\rightarrow c+d)=\frac{1}{4\pi^2\hbar^4}|M_{if}|^2\frac{p_f^2}{v_i v_f}$$

- Now let's consider the spin of the particles
- We must average over the number of initial state spin configurations:

$$g_i = (2s_a+1)(2s_b+1)$$

- and sum over the final states:

$$g_f = (2s_c+1)(2s_d+1)$$

$$\frac{d\sigma}{d\Omega}(a+b\rightarrow c+d)=\frac{1}{4\pi^2\hbar^4}|M_{if}|^2\frac{p_f^2\, g_f}{v_i v_f\, g_i}$$

Cross-sections

- "Crossed reactions"

$$a + b \to c + d$$

$$a + \bar{c} \to \bar{b} + d$$

$$a + \bar{d} \to c + \bar{b}$$

$$a \to \bar{b} + c + d$$

$$c + d \to a + b$$

- The rates are described by the same matrix element but different kinematics

Decays and resonances

- Consider the decay of a particle:
- From uncertainty principle, natural width and lifetime are related:

$$\Gamma = \frac{\hbar}{\tau} = \hbar W = 2\pi |M|^2 \int \rho_f \, d\Omega$$

$$\Gamma = -\hbar \frac{dN_A}{dt} \frac{1}{N_A}$$

Decays and resonances

- Total width:

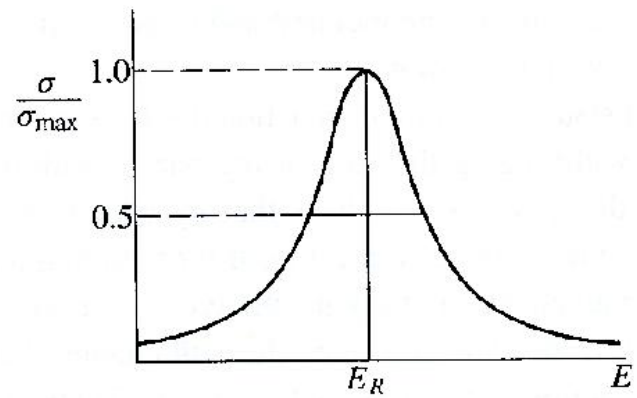
$$\Gamma = \sum_i \Gamma_i$$

- Particle survival with time:

$$N_A(t) = N_A(0) \exp\left(-\frac{\Gamma t}{\hbar}\right)$$

Breit-Wigner Distribution

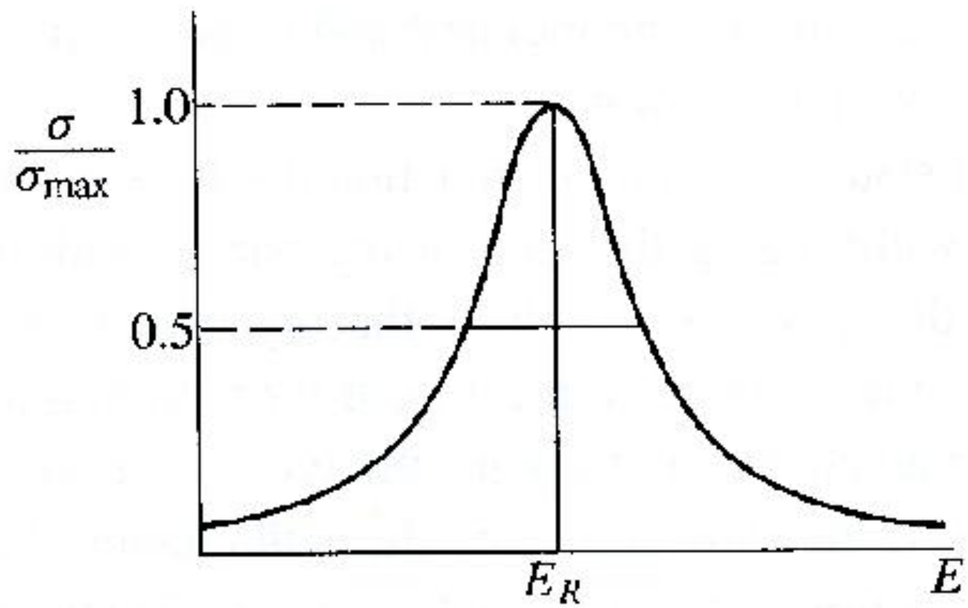
- When two particles collide, they can form an unstable, broad state, known as a “resonance”.
- The cross section which measures the probability of forming the resonant state follows the Breit-Wigner distribution:

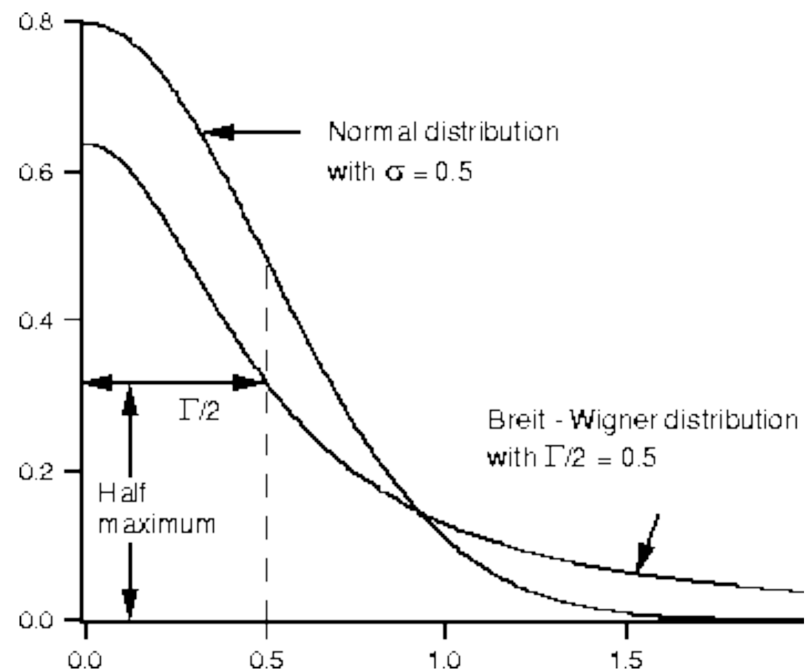


$$\sigma(E) = \sigma_{max} \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$

Breit-Wigner Distribution

$$\sigma(E) = \sigma_{max} \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$





Breit-Wigner Distribution

- $\psi(t) = \psi_0\, e^{-\Gamma/2t}\, e^{-iEo\, t}$
- Fourier Transform:
- $\psi(E) = \int \psi(t)\, e^{iEt}\, dt = \int \psi_0\, e^{-\Gamma/2t}\, e^{-iEo\, t}\, e^{iEt}\, dt$
  $= \int \psi_0\, e^{i[(E - Eo) + i\Gamma/2]t}\, dt$
- $\psi(E) \sim 1/[E-E_0 + i\Gamma/2]$
- $|\psi(E)|^2 \sim 1/[(E-E_0)^2+\Gamma^2/4]$

Breit-Wigner Distribution

- When spin and angular momentum are taken into consideration

$$\sigma = \frac{4\pi ƛ^2(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

where λ is the wavelength of the scattered and scattering particle in their common CMS

Breit-Wigner Distribution

$$\sigma = \frac{4\pi \lambda\!\!\!^{-2}(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

- Elastic scattering

$$\sigma = \frac{4\pi \lambda\!\!\!^{-2}(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma_{el}^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

- General scattering (i $\rightarrow$ j)

$$\sigma = \frac{4\pi \lambda\!\!\!^{-2}(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma_i \Gamma_j/4}{[(E-E_R)^2+\Gamma^2/4]}$$

The Δ^{++} pion-proton resonance

$$\pi^+ p \rightarrow \Delta^{++}(1232) \rightarrow \pi^+ p$$

- $J = 3/2$
- $\Gamma = 120$ MeV
- $\sigma_{max} = 8\pi\lambda\!\!\!^{-2}$

since $s_a = 1/2$, $s_b = 0$, and $J = 3/2$:

$$\frac{(2J+1)}{(2s_a+1)(2s_b+1)} = 2$$

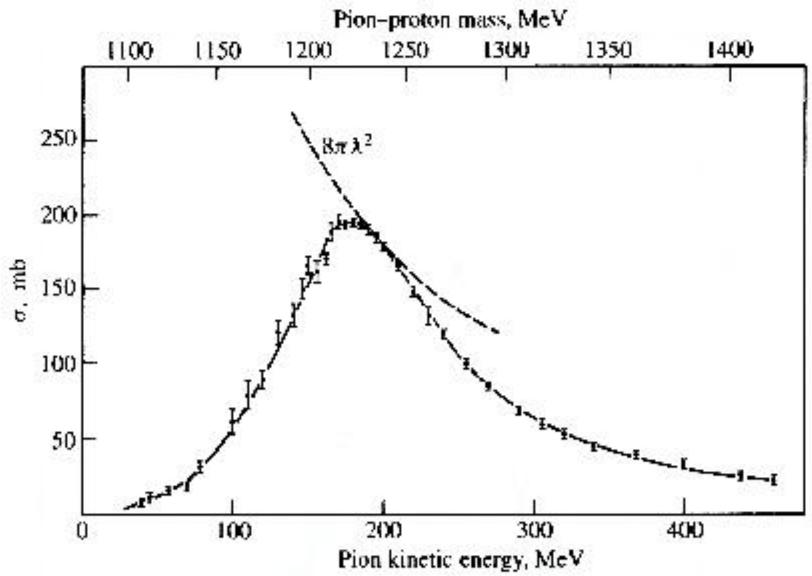
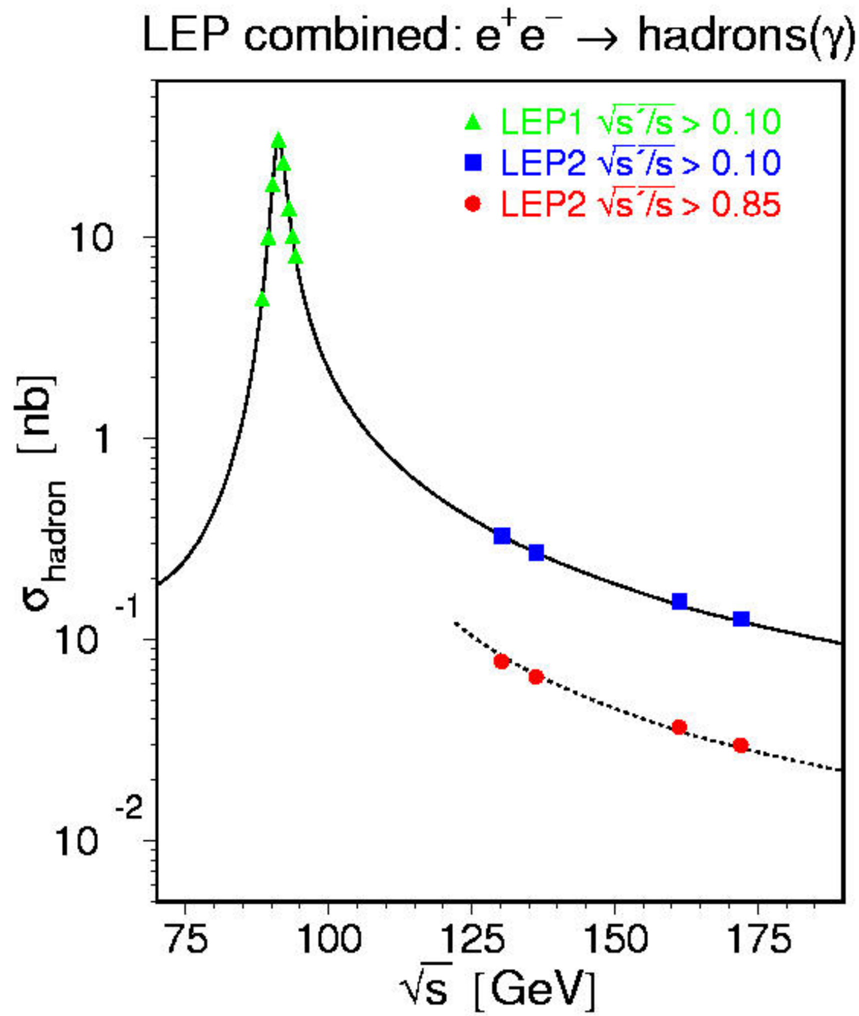


Fig. 2.11. The $\pi^+ p$ elastic scattering cross-section in the region of the $\Delta^{++}(1232)$ resonance. The central mass is 1232 MeV and the width is $\Gamma = 120$ MeV. Note that the formula (2.28) holds strictly for a narrow resonance. For a broad resonance with width comparable with the central mass, such as the Δ^{++}, the final-state phase-space factor varies appreciably over the width so that, in comparison with Figure 2.10, the resonance curve appears asymmetric.

The Z^0 resonance

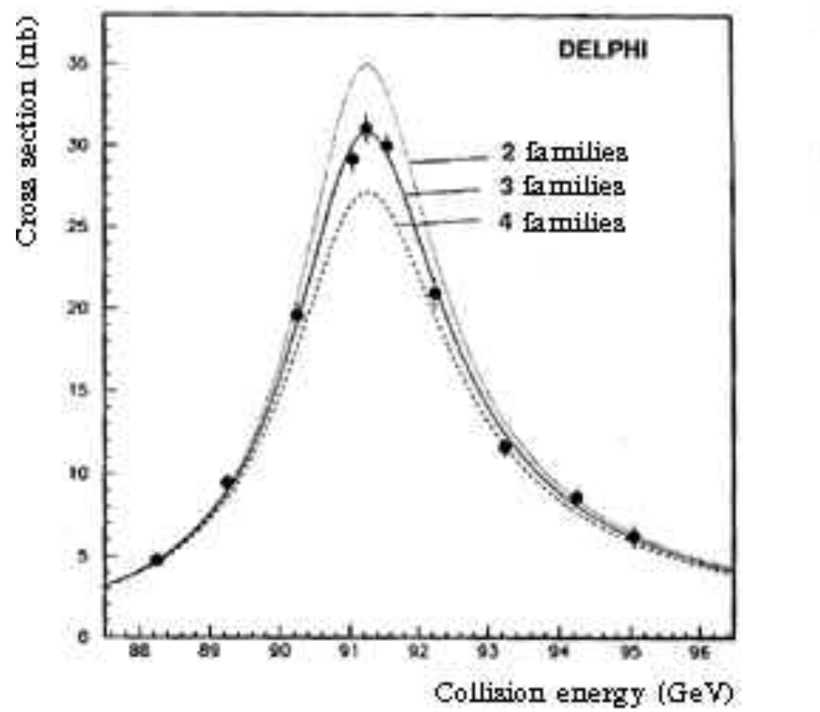
- M_Z = 91 GeV
- Γ_Z = 2.5 GeV

- The Z decays to many different final states:
  - q anti-q
  - $l^+ l^-$
  - ν anti-ν



The Z^0 resonance

- Neutrino counting:
  - the more neutrino families there are, the large the cross section and the broader the resonance
  - $N_\nu = 2.99 \pm 0.01$



Resonances in Astrophysics

- ^{12}C production in stars
  - helium burning red giant stars produce carbon through the triple alpha process:

    $3\,\alpha \rightarrow {}^{12}C$
  - Competing processes
    - $\alpha + \alpha \rightarrow {}^{8}Be$
    - lifetime of ^{8}Be is 10^{-16} sec

      $\alpha + {}^{8}Be \rightarrow {}^{12}C^{*}$ (7.654 MeV resonance with $\Gamma \approx 10$ eV)
    - $^{12}C^{*}$ may decay back to $\alpha + {}^{8}Be$
    - or with small prob (4×10^{-4}) decay to ^{12}C (ground state)
  - ^{12}C production depends crucially on the existence of this resonance (Hoyle, 1953)

Resonances in Astrophysics

- ^{12}C production in stars
  - in principle carbon could be burnt up in stars during a further stage of stellar evolution through the process

$$\alpha + {}^{12}C \rightarrow {}^{16}O$$

  - the absence of an appropriate resonance in oxygen preserves the carbon in the universe