Interactions/Weak Force/Leptons

- Quantum Picture of Interactions
- Yukawa Theory
- Boson Propagator
- Feynman Diagrams
- Electromagnetic Interactions
- Renormalization and Gauge Invariance
- Weak and Electroweak Interactions
- Lepton Flavors and Decays
- Lepton Universality
- Neutrinos
- Neutrino Oscillations

Quantum Picture of Interactions

- Quantum Theory views action at a distance through the exchange of quanta associated with the interaction
- These exchanged quanta are virtual and can "violate" the conservation laws for a time defined by the Uncertainty Principle:
  - $\Delta E \, \Delta t \cong \hbar$

Yukawa Theory

- During the 1930's, Yukawa was working on understanding the short range nature of the nuclear force ($R \approx 10^{-15}$m)
- He postulated that this was due to the exchange of massive quanta which obey the Klein-Gordon eqtn:

$$\partial^2\psi / \partial t^2 = c^2(\nabla^2 - m^2c^2/\hbar^2)\,\psi$$

Yukawa Theory

$$\partial^2\psi / \partial t^2 = c^2(\nabla^2 - m^2c^2/\hbar^2)\,\psi$$

for a static potential, this becomes:

$$\nabla^2\psi = (m^2c^2/\hbar^2)\,\psi$$

We can interpret ψ as the potential U(r) and solve for U:

$$U(r) = g_0\, e^{-r/R} / 4\pi\, r,$$

where $R = \hbar/mc$,

and g_0 is a constant (the strength)

Yukawa Theory

The range of the nuclear force was known,

$$R \approx 10^{-15} m$$

Therefore, the mass of this <u>new</u> exchange particle could be predicted:

$$R = \hbar/mc,$$

$$mc^2 = \hbar c/R \approx 200 \text{ MeV-fm}/1 \text{ fm} \approx 200 \text{ MeV}$$

- The pion with mass 140 MeV/c^2 was discovered in 1947!   (the muon was discovered in 1937 and mis-identified as Yukawa's particle, the "mesotron")
- We now realize that this interaction is actually a residual interaction, so Yukawa was a bit fortunate to find a particle with the predicted mass

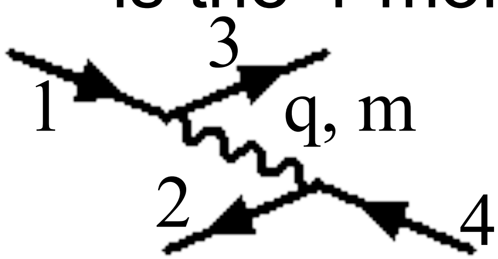
Boson Propagator

The rate for a particular interaction mediated by boson exchange is proportional to the "propagator" squared, where the "propagator" is written as:

$$f(q) = g_0 g \,/\, (q^2 + m^2),$$

where $q^2 = (\Delta p)^2 - (\Delta E)^2$,

is the 4-momentum transfer



$$\Delta p = p_3 - p_1 = p_2 - p_4$$
$$\Delta E = E_3 - E_1 = E_2 - E_4$$

Boson Propagator

- This "propagator" can be derived by taking the Fourier transform of the potential:

$$f(q) = g \int U(r)\, e^{iq\cdot r}\, dV, \quad U(r) = g_0\, e^{-r/R} / 4\pi r$$

  - Therefore, the "propagator" describes the potential in momentum space
- Then, the boson "propagator" is:

$$f(q) = g_0 g / (|q|^2 + m^2)$$

where q is the momentum of the boson,

and m is its mass.

Boson Propagator

The "propagator" can be generalized to four-momentum transfer:

$$f(q) = g_0 g \,/\, (q^2 + m^2),$$

where now $q^2 = (\Delta p)^2 - (\Delta E)^2$,

is the 4-momentum transfer

Rates are proportional to the propagator:

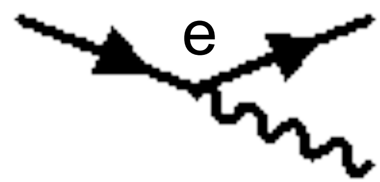
$W = |f|^2 \times$ Phase Space . . .

Feynman Diagrams

- Interactions can be depicted with Feynman diagrams
  - electrons 
  - photons 
  - positrons 
    - (equivalent to electron moving backward in time)

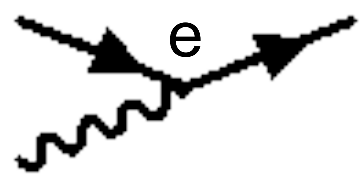
  - electron emits a photon

    $A \sim e$

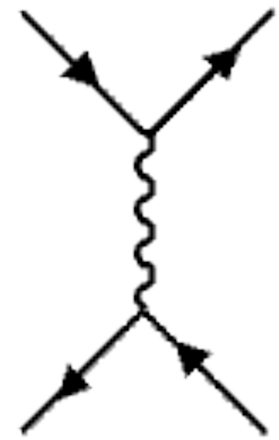


  - electron absorbs a photon

    $A \sim e$



Feynman Diagrams



- Virtual particles
  - lines joining vertices represent virtual particles (undefined mass)

- Vertices are represented by coupling constants, and virtual particles by propagators

Electromagnetic Interactions

- The fine structure constant specifies the strength of the EM interaction between particle and photons:

$$\alpha = e^2 / 4\pi \hbar c = 1 / 137.0360.....$$

- Emission and absorption of a photon represents the basic EM interaction

vertex amplitude = $\sqrt{\alpha} = e$

- cannot occur for free particle

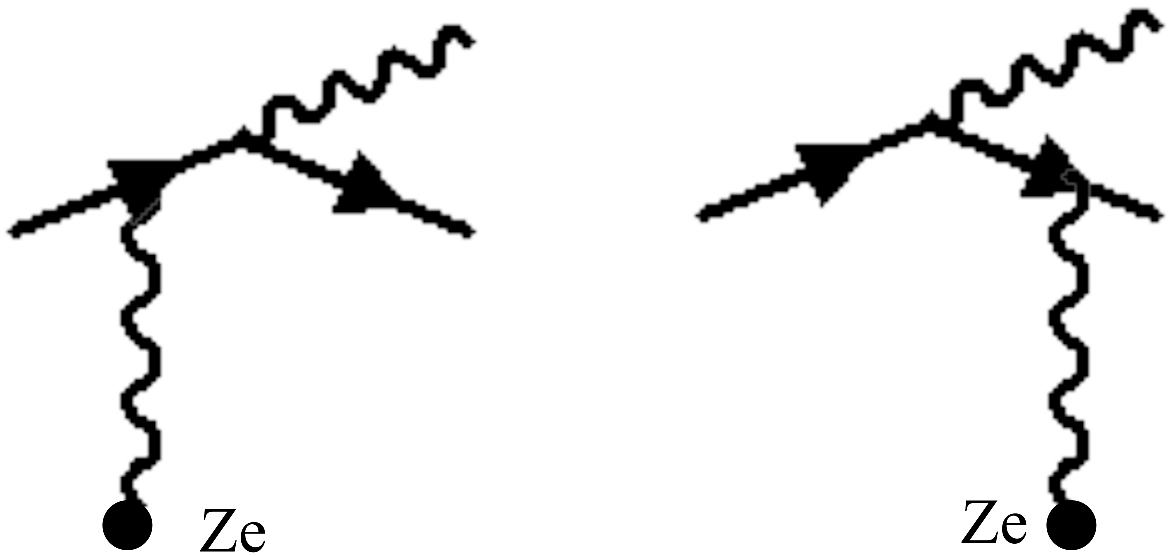
Electromagnetic Interactions

- Coulomb scattering between two electrons:

- Amplitude: α / q^2
- Cross Section = $|\mathrm{Amp}|^2 \sim \alpha^2 / q^4$
  - the Rutherford scattering formula

Electromagnetic Interactions

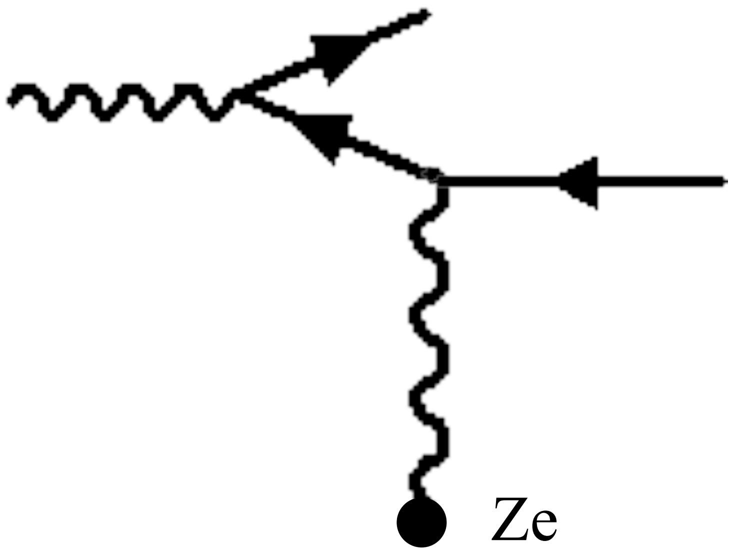
- Bremstrahlung:
  - electron emits photon in field of the nucleus



  - cross section: $\sim \alpha^3 Z^2$

Electromagnetic Interactions

- Pair production ($\gamma \rightarrow e^+e^-$)



cross section:

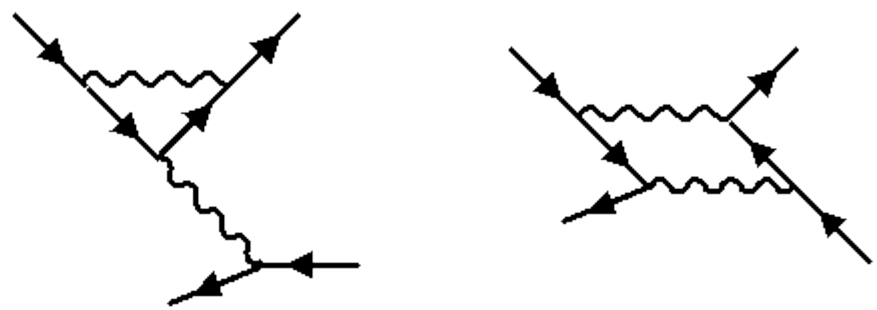
$\sim \alpha^3 Z^2$

- This process is closely related to bremsstrahlung ("crossed diagrams")

Electromagnetic Interactions

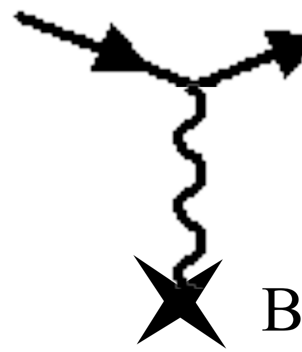
- Higher order processes
  - the diagrams we have seen so far are leading order diagrams, but the rate for a process will be the sum of all orders:
  - For example, Bhabha scattering: $e^+e^- \rightarrow e^+e^-$
    - leading order:

    - higher order:



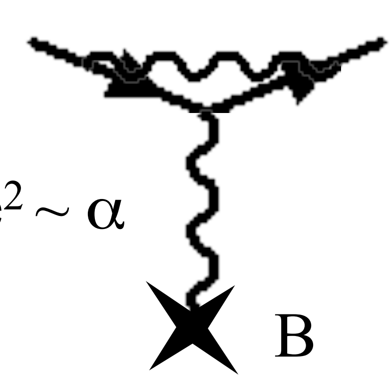
Electromagnetic Interactions

- Example of higher order processes: the electron magnetic moment
  – lowest order:

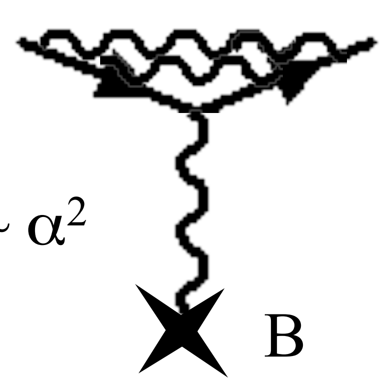


  – higher order:

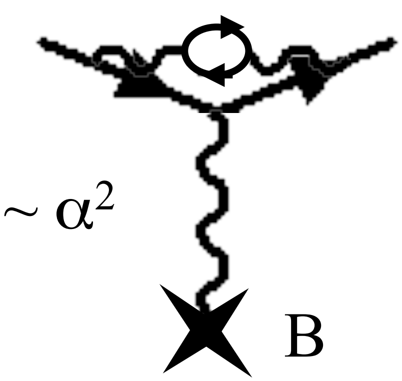
Correction $\sim e^2 \sim \alpha$



$\sim \alpha^2$



$\sim \alpha^2$



Electromagnetic Interactions

- Electron magnetic moment:
  - a Dirac electron has a magnetic moment of

$\mu = g\,\mu_B\, s,\qquad s = 1/2 \qquad g = 2 \qquad \mu_B = e\hbar/mc$

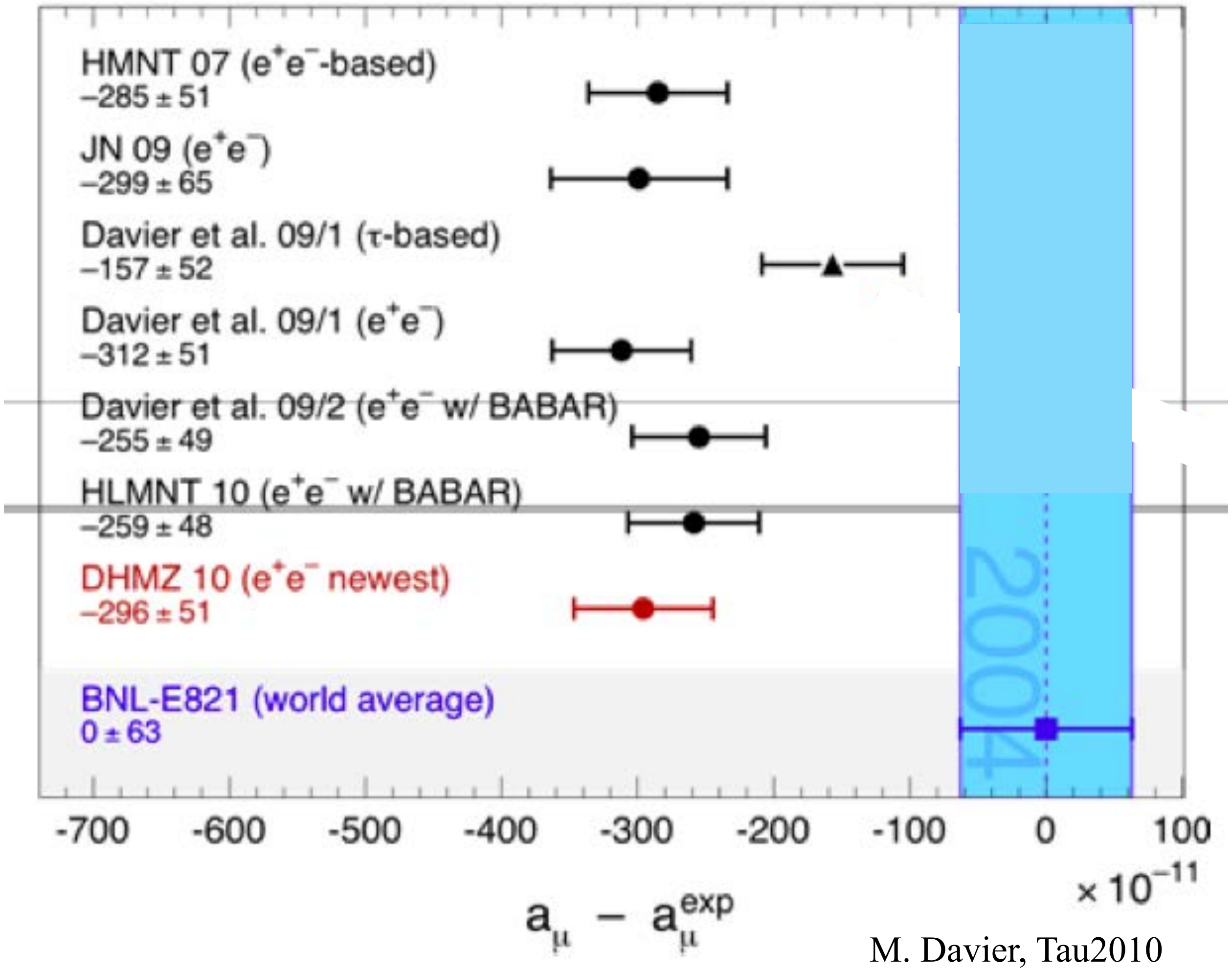
(g-2)/2 is the anomaly due to higher order terms

$$(g-2)^{th}/2 = 0.5\,(\alpha/\pi) - 0.32848\,(\alpha/\pi)^2 + 1.19\,(\alpha/\pi)^3 + ..$$
$$= (115965218.085 \pm 0.076) \times 10^{-11}$$

$$\text{experiment} = (115965218.076 \pm 0.027) \times 10^{-11}$$

this measurement provides very accurate value for the fine structure constant = 1/137.035 999 074(44)

Muon g-2



M. Davier, Tau2010

$\mu = g\, \mu_B\, s,$

$s = 1/2 \quad g = 2$

$\mu_B = eh/mc$

$a = (g-2)/2$

Observed Difference with Experiment:

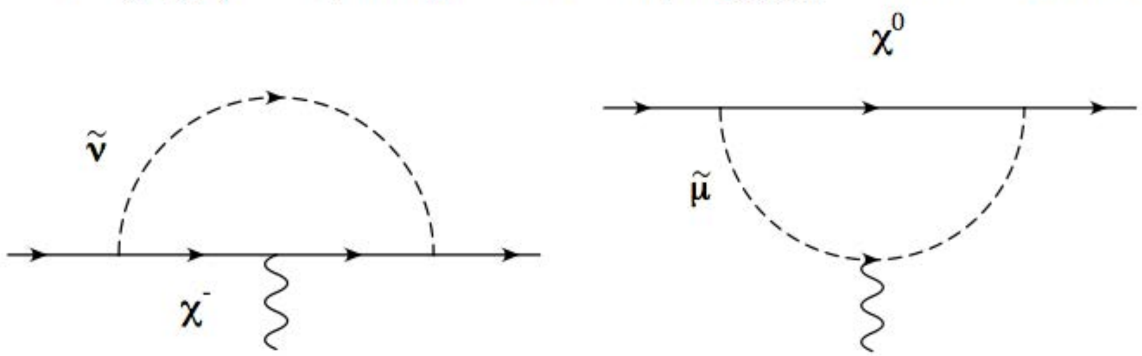
$a_\mu^{exp} - a_\mu^{SM} = (29.6 \pm 8.1) \times 10^{-10}$

➡ 3.6 "standard deviations"

g-2 could mean New Physics ?

Supersymetric particles (Marciano, Munich, 2011)

- Most likely (popular?)
- $a_\mu(\mathrm{SUSY}) = \mathrm{sgn}(\mu)\, 130 \times 10^{-11} \left(\frac{100\mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta$
- $\mathrm{sgn}(\mu) = +$, $\tan\beta = 3 - 40$, $m_{\mathrm{SUSY}} = 100 - 500$ GeV



If SUSY: $\mathrm{sgn}(\mu)+$, dark matter easier, SUSY at LHC likely, EDMS, …

Renormalization and Gauge Invariance

- Electron line represents “bare” electron
- Observable particles are “dressed” by “infinite” number of virtual photons:
  - logarithmically divergent
- These divergences are swept away through renormalization:
  - “Bare” electron mass and charge is always multiplied by divergent integrals. We know this product must be the physical values of the mass and charge, so we set them to be, and the divergences are removed

Renormalization and Gauge Invariance

- In order for a theory to be "renormalizable" it must satisfy local gauge invariance
  - Examples of gauge invariance are familiar in EM and quantum
    - gauge transformations of scalar and vector potential in E&M do not change physical effects
    - wavefunction can change by an arbitrary phase without altering physics

Renormalization and Gauge Invariance

- The coupling constants that appear in the theory are actually not "constants", but "run" with energy.
  – This is due again to virtual processes
  – For example, $\alpha = 1/137$ at very low energy, but $\alpha = 1/128$ at $\sqrt{s} = M_Z$

Leptons Flavors

3 pairs called generations

Leptons spin = 1/2			
Flavor		Mass GeV/c^2	Electric charge
ν_e	electron neutrino	$<1\times10^{-8}$	0
e	electron	0.000511	–1
ν_μ	muon neutrino	<0.0002	0
μ	muon	0.106	–1
ν_τ	tau neutrino	<0.02	0
τ	tau	1.7771	–1

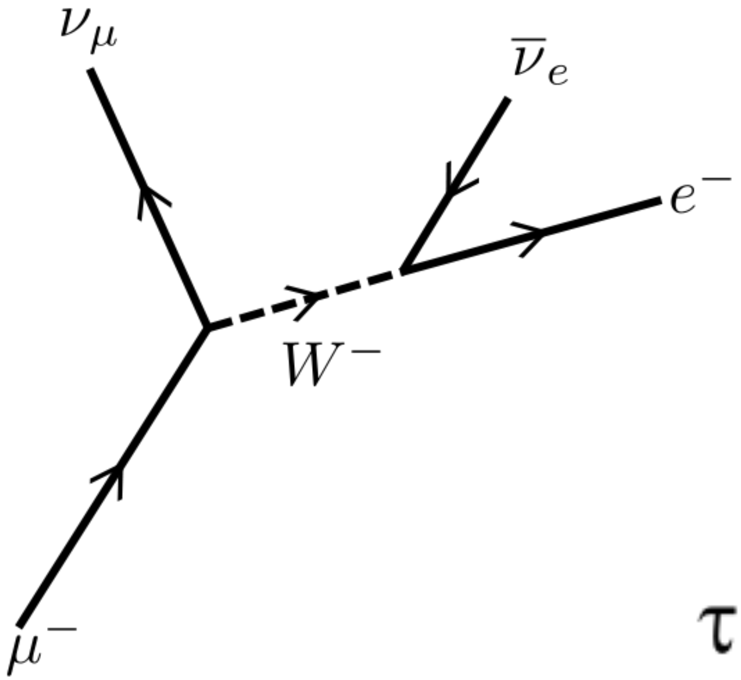
Leptons Flavor Interactions

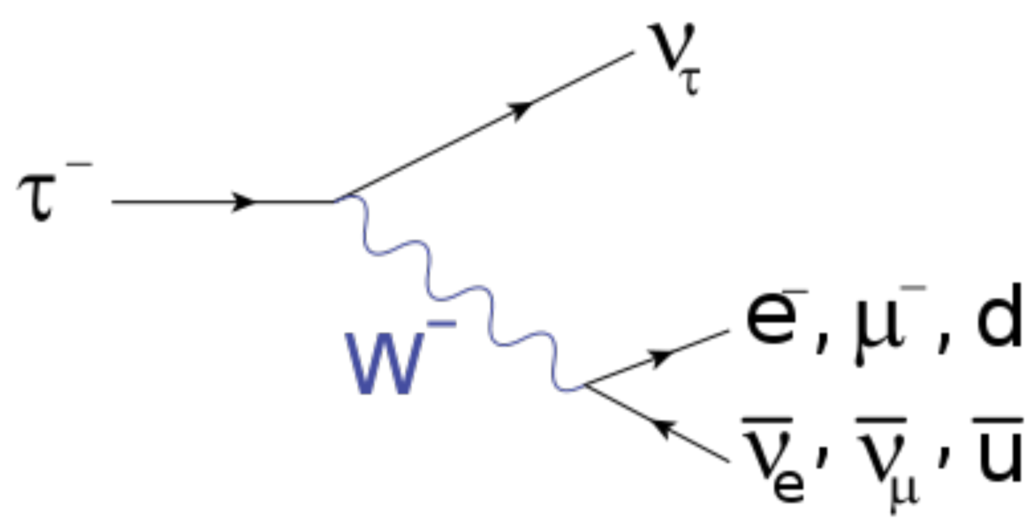
	Strong	EM	Weak	Gravity
e		√	√	√
μ		√	√	√
τ		√	√	√
ν_e			√	√
ν_μ			√	√
ν_τ			√	√

Leptons Flavors

- The Standard Model includes <u>massless</u> neutrinos
  - left-handed neutrinos and RH antineutrinos
- Lepton flavors are conserved in interactions
  - L_e, L_m, L_t
  - mass(μ) = 106 MeV/c^2
  - mass(e) = 0.5 MeV/c^2
  - However $\mu \rightarrow e\,\gamma$ is forbidden (exp: BR < 1.2×10^{-11})
- Neutrino oscillations are indications that the neutrinos have small masses and that flavor conservation will be violated at a small level

Lepton decays



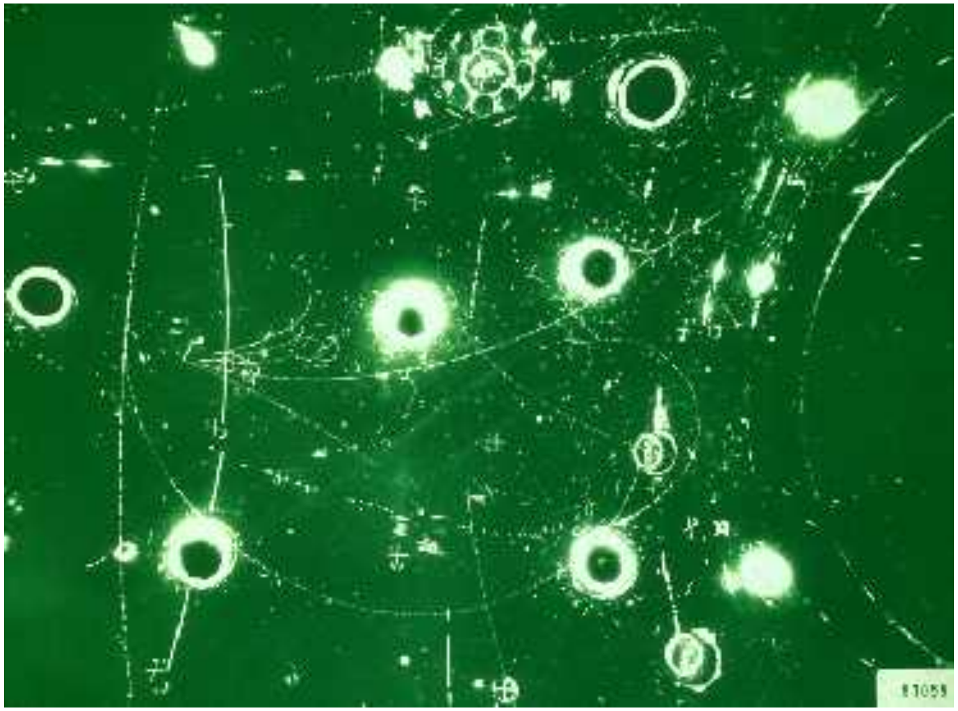


Weak and Electroweak Interactions

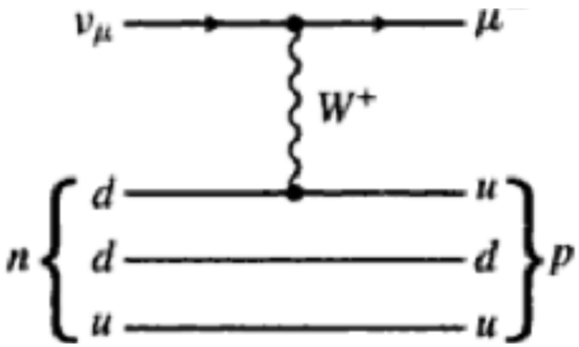
- Nuclear β-decay
- Lepton conservation
- Weak due to very heavy mediating vector bosons:

  $W^{\pm}$, 80 GeV/c^2

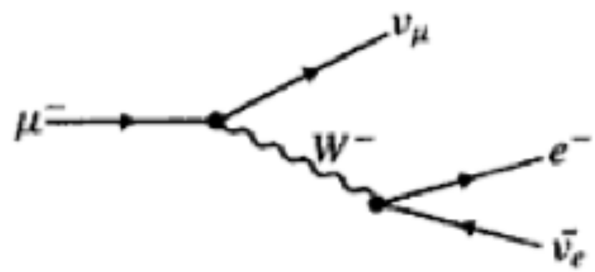
  Z^0 , 91 GeV/c^2
- charge-current
  - β-decay
- neutral-current



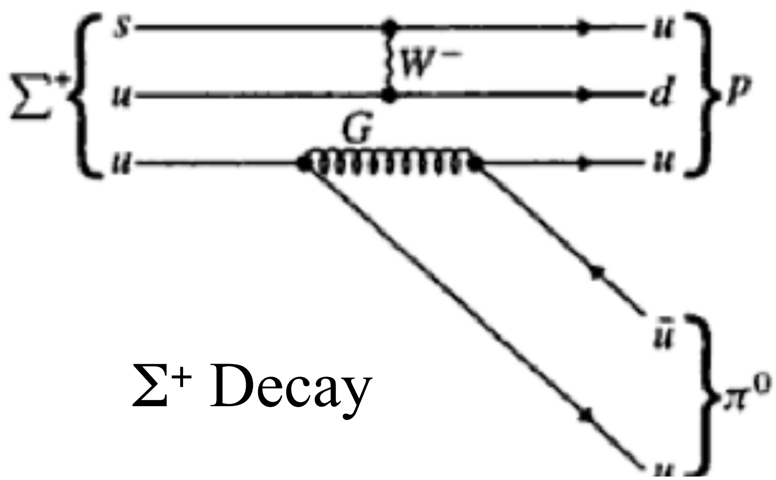
Weak Interactions



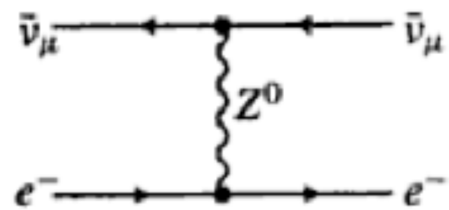
Neutrino Charged Current Interaction



Muon Decay



Σ^+ Decay



Neutral Current

Weak and Electroweak Interactions

- Simplified picture of the Weak Interaction:
  – Propagator:

    $f(q) = g^2 / (q^2 + M_{W,Z}^2)$

  – for $q^2 \ll M^2$, $f(q) = g^2 / M_{W,Z}^2$

  – Fermi's early theory of b-decay postulated an interaction with strength G = 10^{-5}, which we now recognize as $G = g^2 / M_{W,Z}^2 = 10^{-5}\ GeV^{-2}$

  – Once we recognize the origin of the weakness, we can predict the masses of the W and Z

    $M_{W,Z} \sim e/\sqrt{G} \sim \sqrt{4\pi\alpha/G} \sim 90\ GeV$

    - Glashow, Weinberg, Salam (1961-8)

Lepton Decays

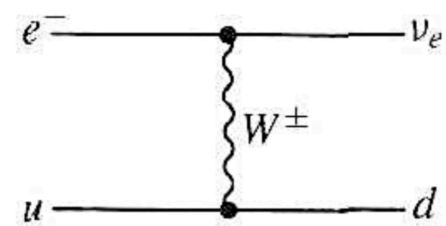
- Muon decay rate
  - $\Gamma(\mu^- \rightarrow e^- \nu_e \nu_\mu) = K\, G_F^2 m_\mu^5$; $\tau_\mu = 2.2 \times 10^{-6}$ sec
- Tau
  - $\Gamma(\tau^- \rightarrow e^- \nu_e \nu_\tau) = K\, G_F^2 m_\tau^5$; $\tau_\tau = 3 \times 10^{-13}$ sec
- Lepton lifetime
  - $\tau_l = 1/\Gamma_{tot} = B(l^- \rightarrow e^- \nu_e \nu_l) \,/\, \Gamma(l^- \rightarrow e^- \nu_e \nu_l)$
  - Since $B(l^- \rightarrow e^- \nu_e \nu_l) = \Gamma(l^- \rightarrow e^- \nu_e \nu_l)/\Gamma_{tot}$
- Ratio
  - $\tau_\tau / \tau_\mu = B(\tau^- \rightarrow e^- \nu_e \nu_\tau)/B(\mu^- \rightarrow e^- \nu_e \nu_\mu) \quad (m_\mu/m_\tau)^5 = 1.36 \times 10^{-7}$

Classification of Weak Interactions

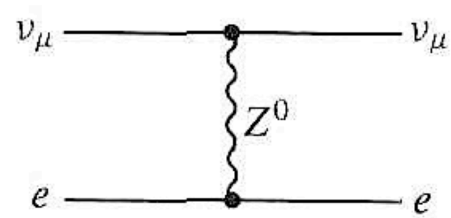
- Weak interactions are mediated by the "intermediate bosons" $W^{\pm}$ and Z^0
- Just as the EM force between two current carrying wires depends on the EM current, the weak interaction is between two weak currents, describing the flow of conserved weak charge, g

$$j \propto \psi^* \psi$$

- Two types of interactions:
  - CC (charged current)



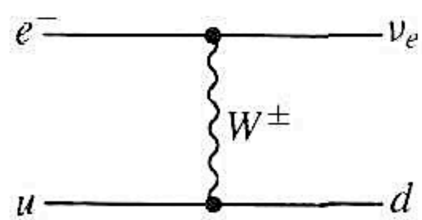
  - NC (neutral current)



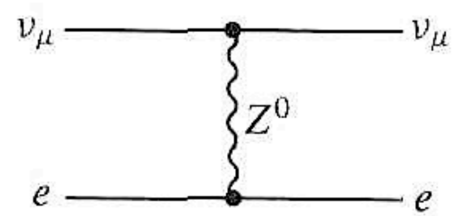
Classification of Weak Interactions

- Weak interactions occur between all types of leptons and quarks, but are often hidden by the stronger EM and strong interactions.

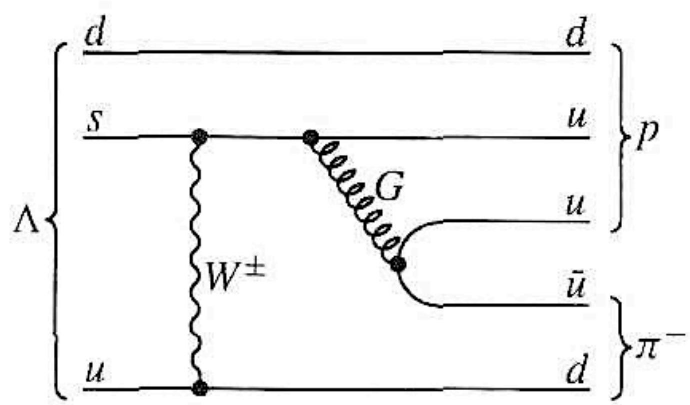
- Semi-leptonic



Leptonic



- Non-leptonic



Lepton universality

- Unit of weak charge
  - all the leptons carry the same weak charge and therefore couple to the W $^{\pm}$ with the same strength
  - The quarks DO NOT carry the same unit of weak charge

- Muon decay

$$\Gamma(\mu \to e \nu_e \bar{\nu}_\mu) = \frac{1}{\tau} \propto G^2 m_\mu^5$$
$$= \frac{G^2 m_\mu^5}{192\pi^3}$$

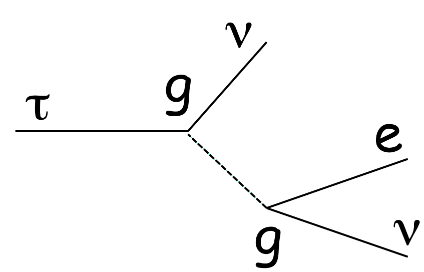
- experimental: $\tau_\mu = 2.197 \times 10^{-6}$ sec

Lepton universality

- Tau decay

$$\Gamma(\tau \to e\nu_e\bar{\nu}_\tau) = B(\tau \to e\,\nu\,\nu)\,\frac{1}{\tau} \propto G^2 m_\tau^5$$

$$= \frac{G^2 m_\tau^5}{192\pi^3}$$



- $B(\tau \to e\,\nu\,\nu) = 17.80 \pm 0.06\%$
- Test universality: since $\Gamma \sim G^2 \sim g^4$

$$g_\tau^4 \propto B(\tau \to e\,\nu\,\nu) / (m_\tau^5\, \tau_\tau)$$

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \to e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

Lepton universality

- Test universality:

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \to e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

With $\tau_\mu = 2.197 \times 10^{-6}$ s, $\tau_\tau = (291.0 \pm 1.5) \times 10^{-15}$ s, $m_\mu = 105.658$ MeV
$m_\tau = 1777.0$ MeV and $B(\tau \to e\,\nu\,\nu) = 17.80 \pm 0.06\%$

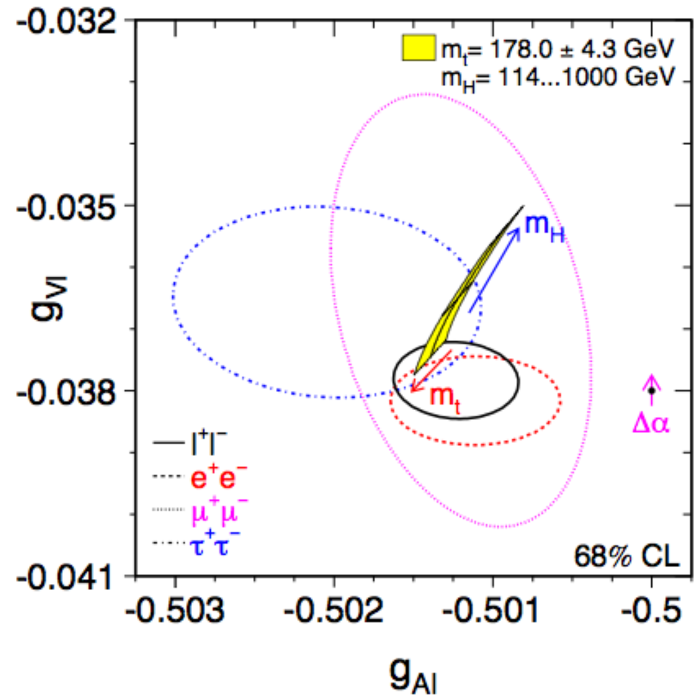
$$\frac{g_\tau}{g_\mu} = 0.999 \pm 0.003$$

$$\frac{g_\mu}{g_e} = 1.001 \pm 0.004$$

Lepton universality

- Lepton universality also holds for the Z couplings:

$$Z^0 \to e^+e^- : \mu^+\mu^- : \tau^+\tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$



- From the muon lifetime we can compute the Fermi constant, G:

$$G/(\hbar c)^3 = 1.1664 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Introduction to Neutrinos

Neutrinos could contribute significantly to the total energy density of the Universe if they have a mass in the eV range

m_{ν_e}	m_{ν_μ}	m_{ν_τ}
< 2 eV	< 190 keV	< 18 MeV

Nuclear and particle physics experiments

$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$ (electron energy spectrum - endpoint)

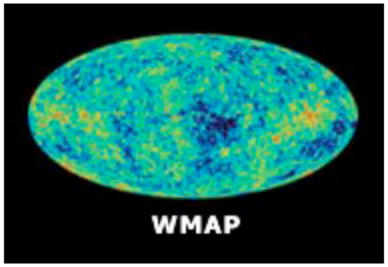
$\pi^+ \rightarrow \mu^+ + \nu_\mu$ ($m^2_{\nu_\mu} = m^2_\pi + m^2_\mu - 2m_\pi\sqrt{m^2_\mu + p^2_\mu}$)

$\tau \rightarrow 5\pi^\pm + \nu_\tau$ $\quad$ $\tau \rightarrow 5\pi^\pm + \pi^0 + \nu_\tau$ (missing E and p)

Astrophysics

Analysis of the Planck cosmic microwave background radiation and large scale structure measurements has put a limit on the sum of the neutrino Masses (2015)

$\Sigma m_\nu < 0.3$ eV



The Nobel Prize in Physics 2015
Takaaki Kajita, Arthur B. McDonald

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The Nobel Prize in Physics 2015

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Takaaki Kajita

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Arthur B. McDonald

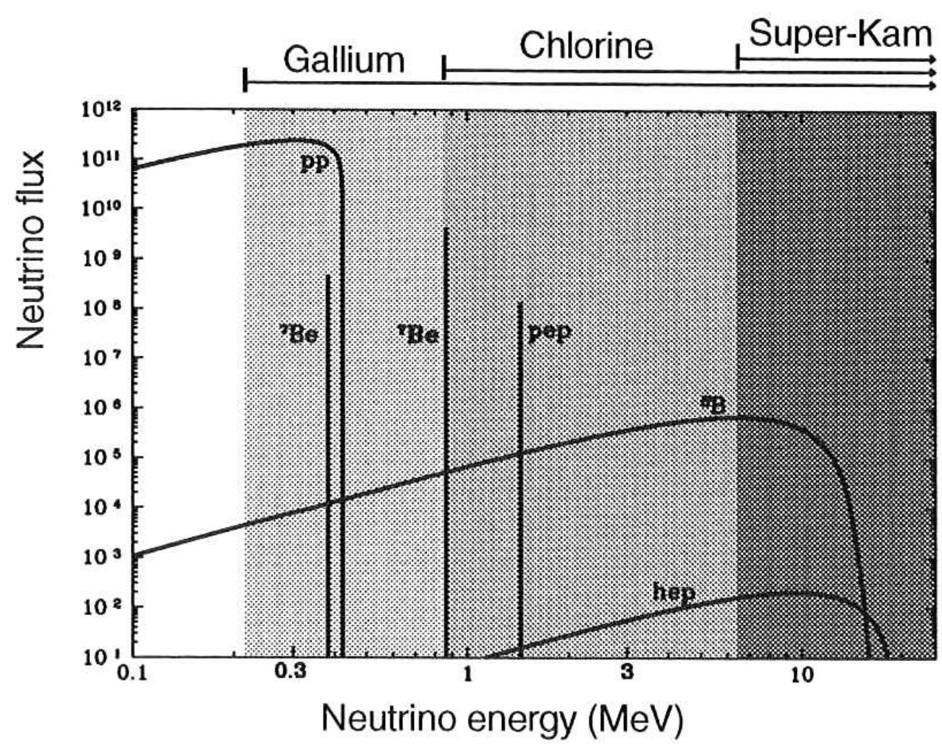
Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

Solar Neutrinos

Solar Neutrinos: The "Standard Solar Model"

	Reaction	Neutrino energy
1	$p + p \rightarrow {}^2H + e^+ + \nu_e$	≤ 0.42 MeV
	or	
	$p + e^- + p \rightarrow {}^2H + \nu_e$	1.442 MeV
2	${}^2H + p \rightarrow {}^3He + \gamma$	
3	${}^3He + {}^3He \rightarrow {}^4He + p + p$	
	or	
	${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	
	or	
	${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	≤ 18.8 MeV
4	${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	0.86 MeV
	${}^7Li + p \rightarrow {}^4He + {}^4He$	
	or	
	${}^7Be + p \rightarrow {}^8B + \gamma$	
	${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$	< 15 MeV
	${}^8Be^* \rightarrow {}^4He + {}^4He$	



Solar Neutrinos

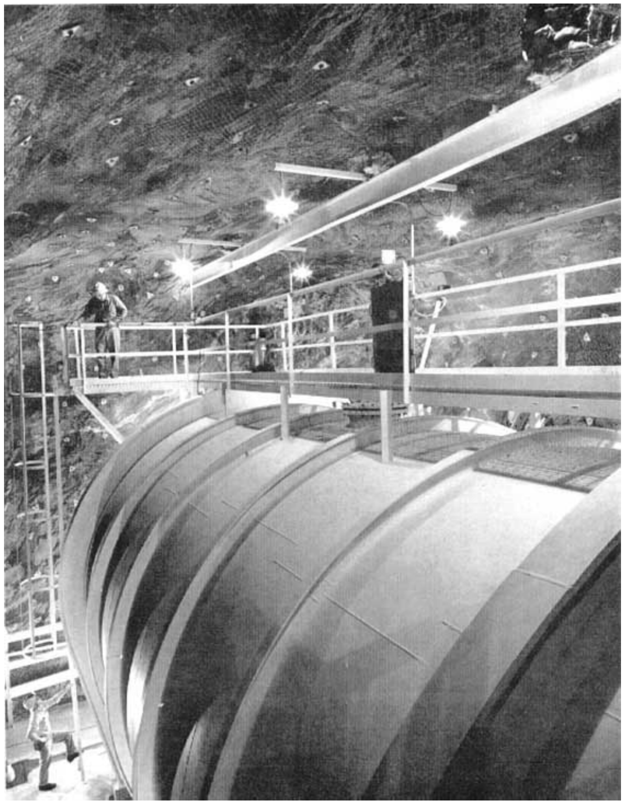
Homestake Mine

beginning in the 1960's, Ray Davis et al pioneered detection of solar neutrinos

615 tons of cleaning fluid, C_2Cl_4

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$$

Argon is chemically extracted and single atoms are counted in subsequent decay



Solar Neutrinos

Homestake Mine

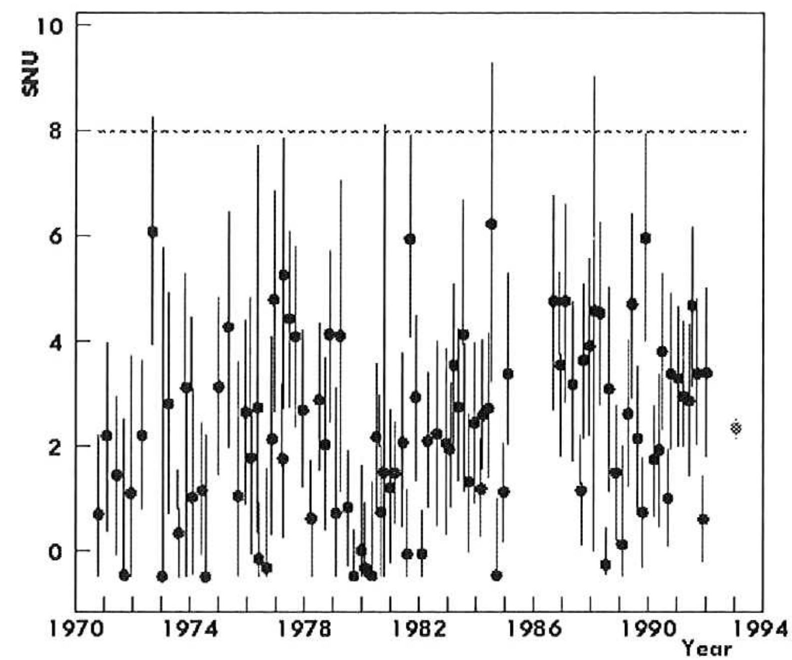
about 15 atoms are counted each month

average rate over 20 years: 2.6 ± 0.2 SNU

(Solar Neutrino Unit: $1\ \mathrm{SNU} = 10^{-36}\ \mathrm{s}^{-1}$,

while Standard Solar Model predicts 7.9 ± 2.6 SNU

This was the origin of the long-standing "Solar Neutrino Problem"



Solar Neutrinos

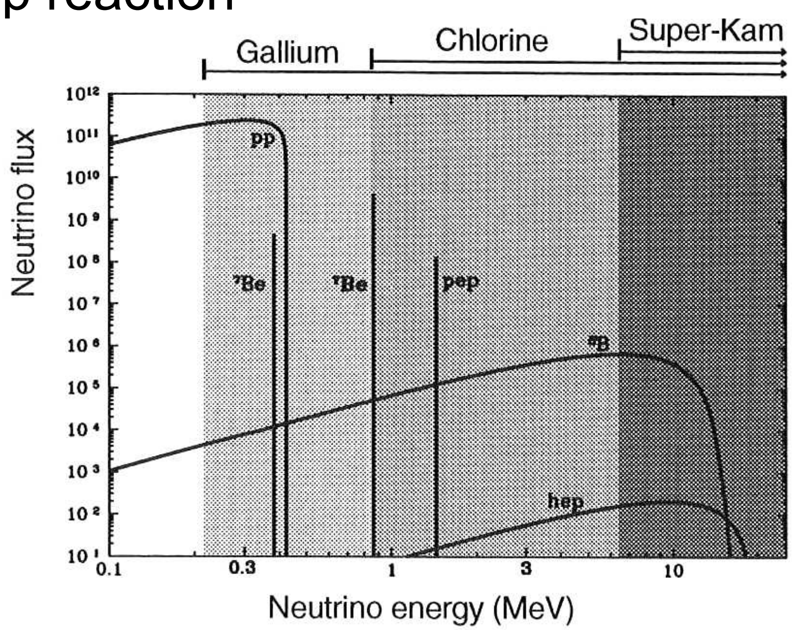
SAGE and GALLEX

$$\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$$

lower energy threshold than for Chlorine
0.233 MeV vs. 0.814 MeV
therefore, sensitive to larger fraction of neutrino flux,
and, in particular, the pp reaction

SAGE (Baksan, Russia)
GALLEX (Gran Sasso, Italy)

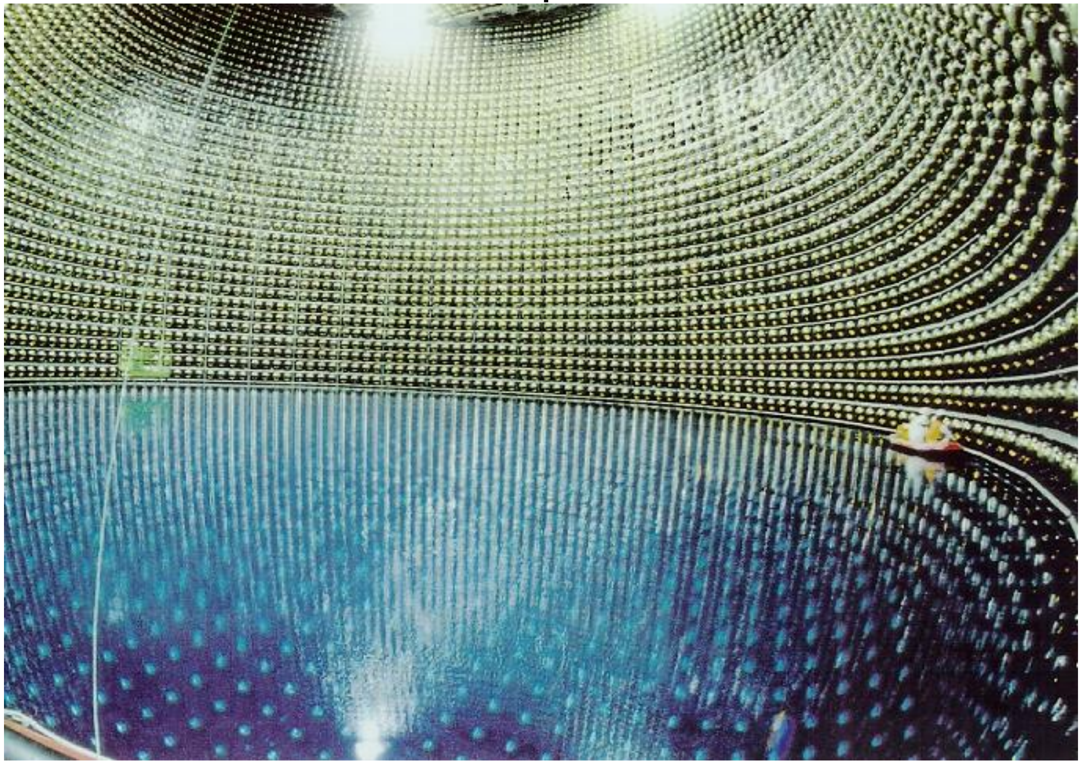
Standard Solar Model: 130 SNU
experiment: 70.3 ± 7 SNU



Solar Neutrinos

Super Kamiokande

- Japanese mine Kamioka
- Large water Cherenkov
  - originally 2.1 ktons
  - Super K ~ 20 ktons



Solar Neutrinos

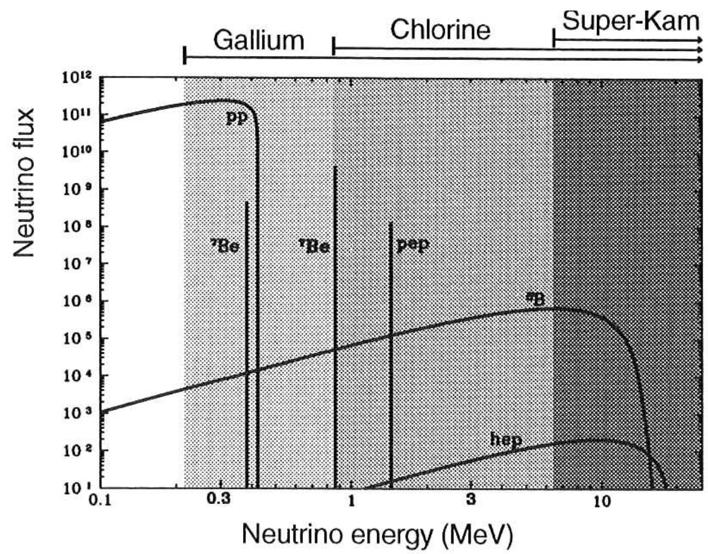
Super K

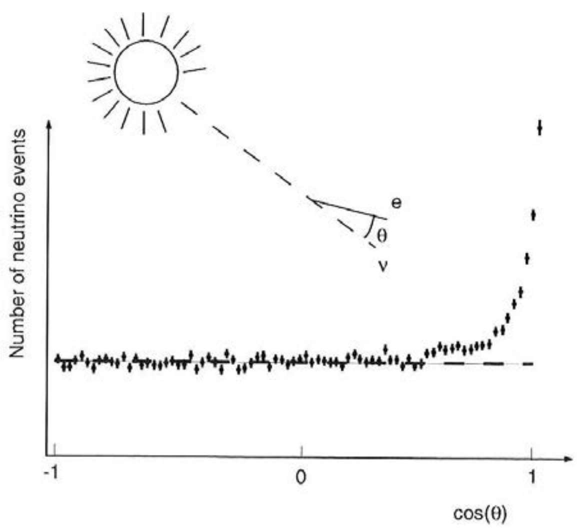
Large threshold (~8 MeV)

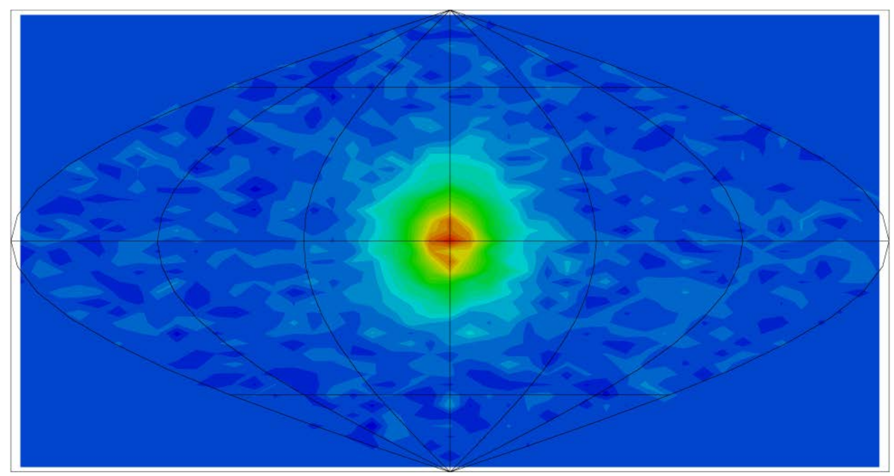
fewer events

-> larger target needed

Direction measurement

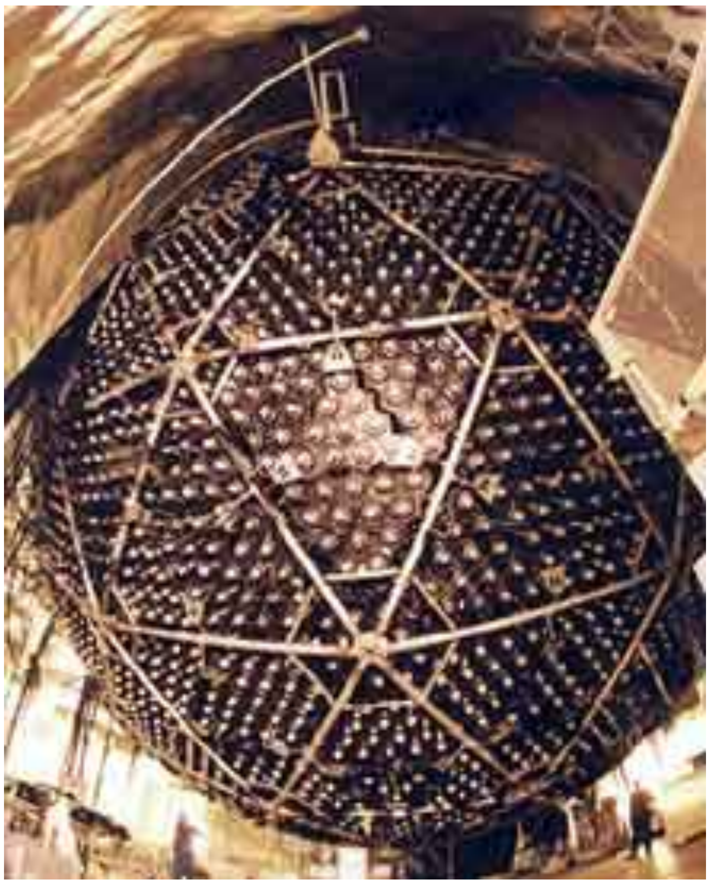






Solar Neutrinos

SNO (Sudbury Neutrino Observatory)
interactions in heavy water
sensitive to neutral current interactions



SNO Measurements

Charged Current Reaction (D_2O):

CC $\nu_e + d \rightarrow p + p + e^-$ $E_{thres} = 1.4$ MeV   (only ν_e)

- ν_e energy spectrum (distortion $\Rightarrow$ MSW effect)
- Some directional sensitivity ($1 - 1/3 \cos\theta_e$)

Neutral Current Reaction (D_2O):

NC $\nu_x + d \rightarrow \nu_x + p + n$ $E_{thres} = 2.2$ MeV   (ALL ν types)

- Total solar ^{8}B neutrino flux (active neutrinos)

$$\text{Ratio} = \frac{CC}{NC} = \frac{(\nu_e)\ \text{flux}}{(\nu_e + \nu_\mu + \nu_\tau)\ \text{flux}}$$

Elastic Scattering Reaction (D_2O, H_2O):

ES $\nu_x + e^- \rightarrow \nu_x + e^-$ $E_{thres} = 0$ MeV   (mostly ν_e)

- Low counting rate
- Directional sensitivity (very forward peaked)

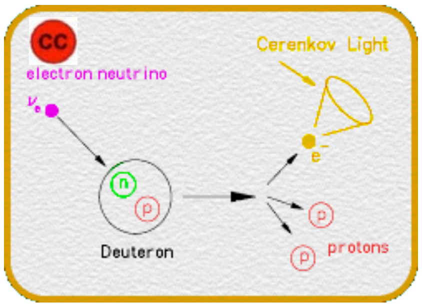
$$\text{Ratio} = \frac{CC}{ES} = \frac{(\nu_e)\ \text{flux}}{0.86\ \nu_e + 0.14(\nu_\mu + \nu_\tau)\ \text{flux}}$$

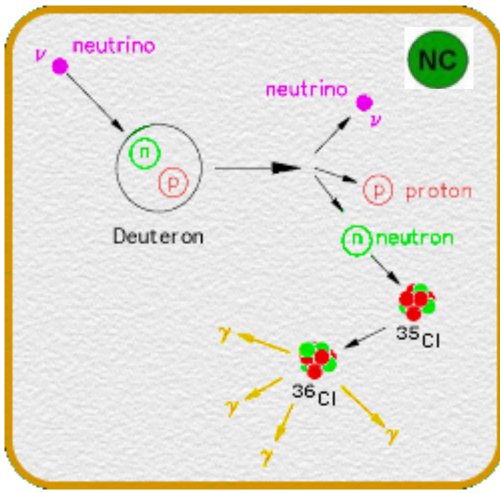
Solar Neutrinos

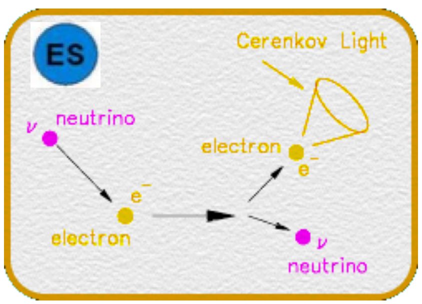
SNO (Sudbury Neutrino Observatory)

interactions in heavy water

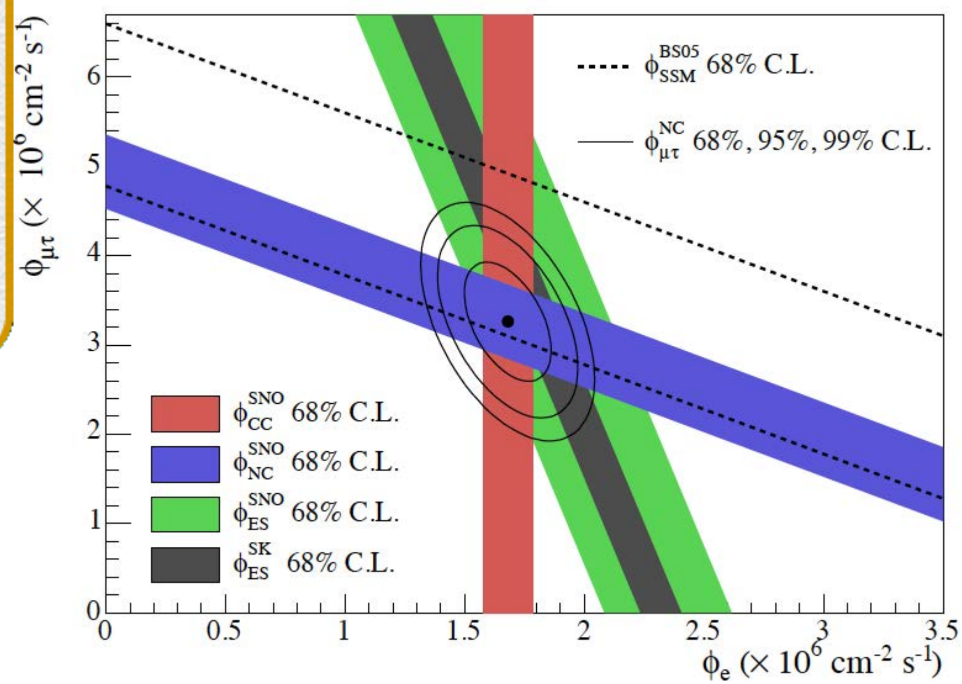
sensitive to neutral current interactions







Flux of ^{8}B neutrinos (non-e vs. e)



Solar Neutrinos

Experiment	measured flux	ratio exp/BP98	threshold energy	Years of running
Homestake	2.56 ± 0.16 ± 0.16	0.33 ± 0.03 ± 0.05	0.814 MeV	1970-1995
Kamiokande	2.80 ± 0.19 ± 0.33	$0.54 \pm 0.08\ ^{+0.10}_{-0.07}$	7.5 MeV	1986-1995
SAGE	75 ± 7 ± 3	0.58 ± 0.06 ± 0.03	0.233 MeV	1990-2006
Gallex	78 ± 6 ± 5	0.60 ± 0.06 ± 0.04	0.233 MeV	1991-1996
Super-Kamiokande	2.35 ± 0.02 ± 0.08	$0.465 \pm 0.005\ ^{+0.016}_{-0.015}$ (BP00)	5.5 (6.5) MeV	1996-
GNO	66 ± 10 ± 3	0.51 ± 0.08 ± 0.03	0.233 MeV	1998-
SNO	$1.68 \pm 0.06 \pm\ ^{+0.08}_{-0.09}$ (CC) 2.35 ± 0.22 ± 0.15 (ES) $4.94 \pm 0.21\ ^{+0.38}_{-0.34}$ (NC)		6.75 MeV	1999-

- The values for Chlorine and Gallium experiments are given in SNU.
- The values for Cerenkov experiments are given in units of 10^{10} counts/m^2 s.
- The errors for the relative values correspond to experimental and theoretical errors, respectively, with the statistical and systematic errors added quadratically. Some of the relative values are based on my own calculation from the published results.
- BP98 Refers to Bahcall and Pinsonneault model of 1998.

from "The Ultimate Neutrino Page"
http://cupp.oulu.fi/neutrino/nd-sol2.html

Neutrino Oscillations

Two possible solutions to the Solar Neutrino Problem:

1. The Standard Solar Model is wrong
   cross sections, temperature, whatever
2. Neutrinos behave differently
   decay, transform, whatever

If the neutrinos are massless, they will not decay
But if the neutrinos have mass they may decay:

eg. $\nu_\alpha \rightarrow \nu_\beta + \gamma$
but the estimate of this rate is very small in the SM

Another possibility, if they have mass, the different flavors may mix
weak-interaction and mass eigenstates may be different

Neutrino Oscillations

$$|\nu_f> = \sum_m c_{fm}|\nu_m>$$

Consider two flavors

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

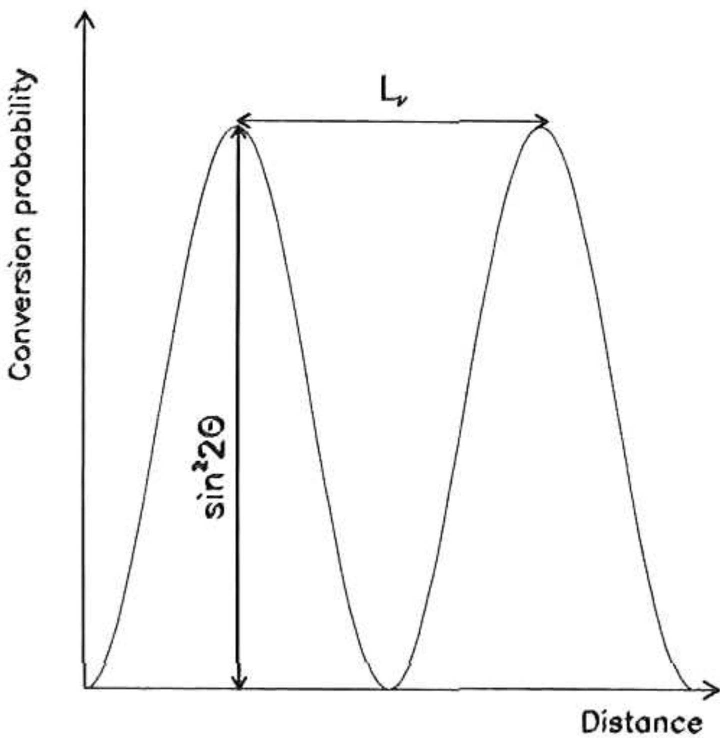
$$|\nu_e(t)> = -\sin\theta e^{-iE_1t}|\nu_1> + \cos\theta e^{-iE_2t}|\nu_2>$$

$$E_i = p + \frac{m_i^2}{2p}$$ since $m_i \ll p$

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\sin^2[\frac{1}{2}(E_2 - E_1)t]$$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2}{4E}t\right]$$

Neutrino Oscillations



$$
\begin{aligned}
A &= \sin^2(2\theta) \\
L_\nu &= \frac{4\pi E\hbar}{\Delta m^2 c^3}
\end{aligned}
$$

$$L_\nu = 2.48 \left(\frac{E}{1\,\mathrm{MeV}}\right)\left(\frac{1\ \mathrm{eV}^2}{\Delta m^2}\right) \quad \text{metres.}$$

Mixing Matrix

$$\nu_{l\mathrm{L}}(x) = \sum_j U_{lj}\, \nu_{j\mathrm{L}}(x), \quad l = e, \mu, \tau,$$

where $\nu_{j\mathrm{L}}(x)$ is the LH component of the field of ν_j possessing a mass m_j and U is a unitary matrix - the neutrino mixing matrix [1,17,18]. The matrix U is often called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or Maki-Nakagawa-Sakata (MNS) mixing matrix. Obviously, Eq. (13.1) implies that the individual lepton charges L_l, $l = e, \mu, \tau$, are not conserved.

$$U = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \end{array}$$

$$\times \, \mathrm{diag}(e^{i\alpha_1/2},\ e^{i\alpha_2/2},\ 1)$$

Mixing Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \overbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{\text{Solar, Reactor}} \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}^{\text{Atmospheric, Accelerator}}$$

$$\times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ \ s_{ij} \equiv \sin\theta_{ij},\ \{\delta, \alpha_1, \alpha_2\} \equiv \text{CP - Violating Phases}$$

Neutrino Oscillations

Experiments:

Reactor

disappearance of electron neutrino
since muon cannot be produced by MeV neutrinos

Accelerator

appearance of electron neutrino,
from muon neutrino beam

Solar

disappearance of electron neutrino

Atmospheric

electron and muon neutrinos produced in atmosphere by cosmic rays

Neutrino Oscillations

Table 13.1: Sensitivity of different oscillation experiments.

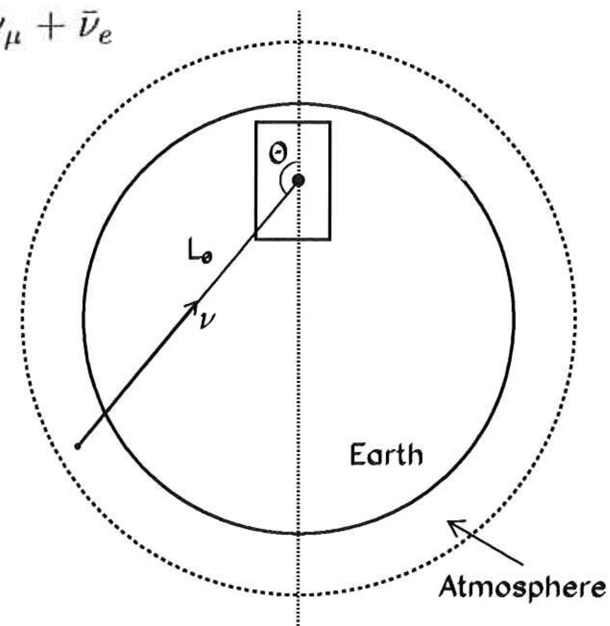
Source	Type of ν	$\overline{E}$[MeV]	L[km]	min(Δm^2)[eV^2]
Reactor	$\overline{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\overline{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \overline{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Atmospheric Neutrinos

$$p/n + N \rightarrow \pi^+/K^+ + \ldots$$
$$\pi^+/K^+ \rightarrow \mu^+ + \nu_\mu$$
$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e,$$
$$p/n + N \rightarrow \pi^-/K^- + \ldots$$
$$\pi^-/K^- \rightarrow \mu^- + \bar{\nu}_\mu$$
$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

$$\boxed{\frac{\varphi_{\nu_\mu} + \varphi_{\bar{\nu}_\mu}}{\varphi_{\nu_e} + \varphi_{\bar{\nu}_e}} = 2}$$

E_ν has broad peak ~ 0.1 GeV

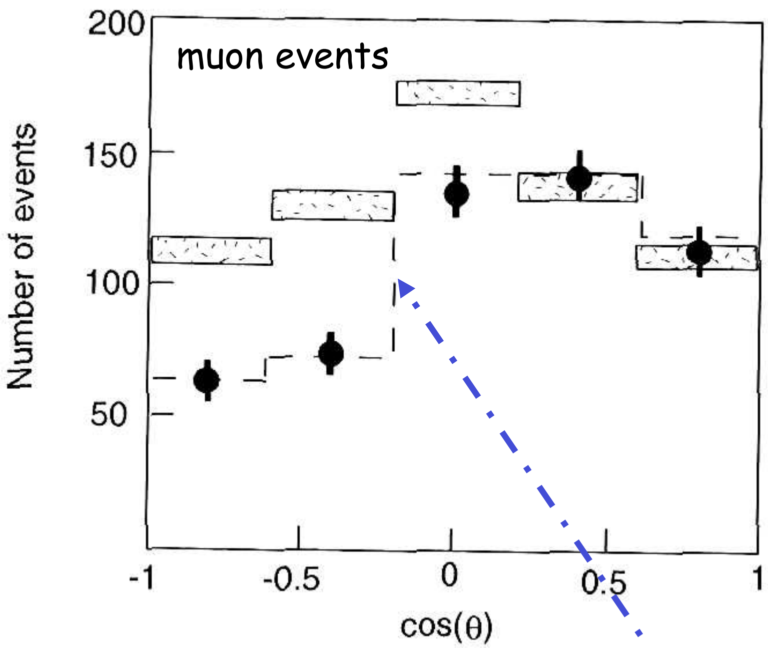


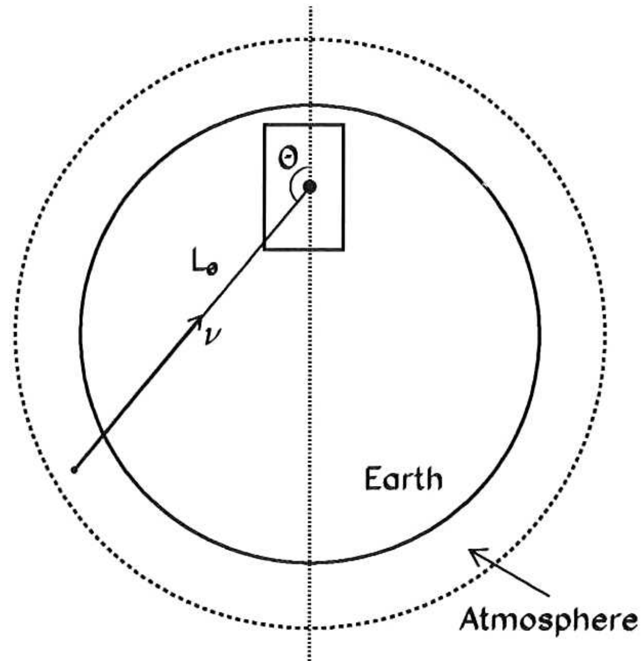
oscillation will depend on azimuth
due to L dependence

$L_{max} \approx 10^4$ km $\Rightarrow$ $\Delta m^2 \sim 10^{-5}$ eV^2

$$L_\nu = 2.48 \left(\frac{E}{1\,\mathrm{MeV}}\right)\left(\frac{1\,\mathrm{eV}^2}{\Delta m^2}\right) \text{ metres.}$$

Atmospheric Neutrinos





$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\ \sin^2\left[\frac{\Delta m^2}{4E} t\right]$$

$\sin^2\theta \sim 1,\ \Delta m^2 \sim 2.2\ 10^{-3}\ eV^2$

$$\boxed{\begin{aligned} A &= \sin^2(2\theta) \\ L_\nu &= \frac{4\pi E\hbar}{\Delta m^2 c^3} \end{aligned}}$$

Neutrino Oscillations

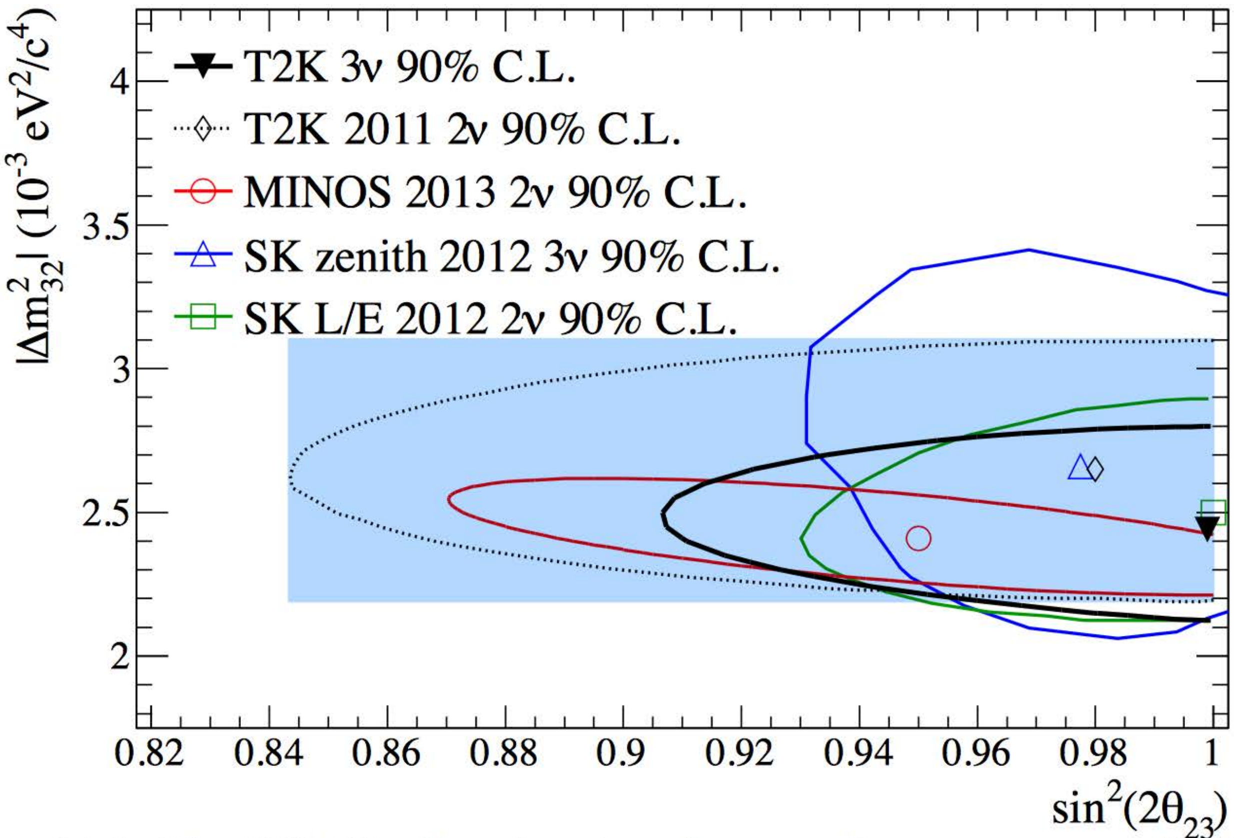
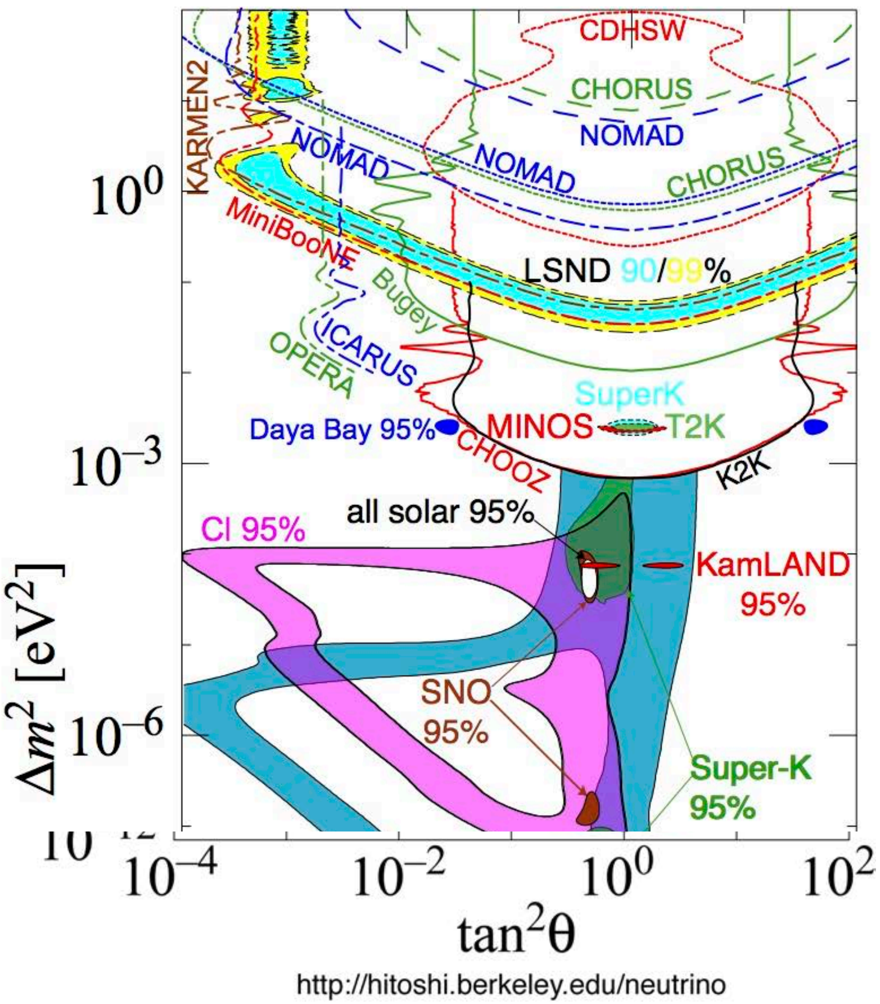
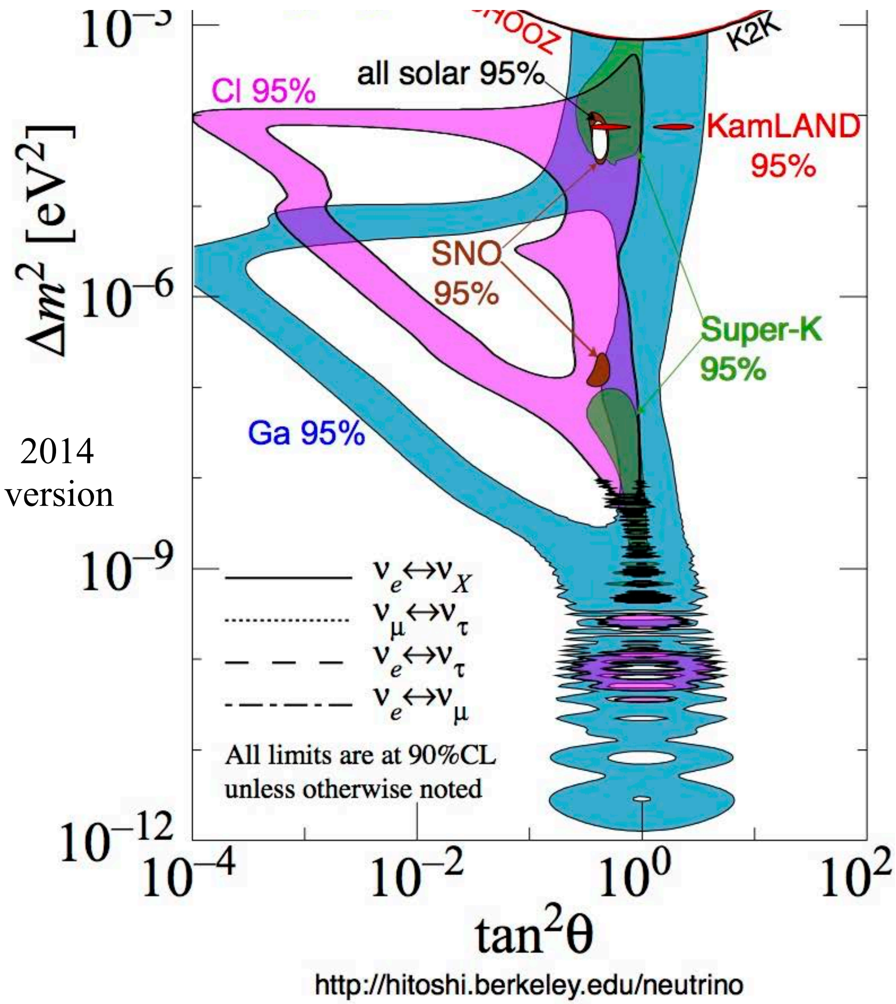


Figure 14.6: The 90% CL allowed regions from ν_μ disappearance results: T2K 2013 [23], T2K2011 [22], Super-Kamiokande [138], and MINOS [143]. The MINOS contour was obtained by assuming identical neutrino and antineutrino oscillation parameters. This figure is taken from Ref. 23.

Neutrino Oscillations



2014 version



Reactor Neutrinos

14.6. Measurements of θ_{13}

Reactor $\bar{\nu}_e$ disappearance experiments with $L \sim 1$ km, $\langle E \rangle \sim 3$ MeV are sensitive to $\sim E/L \sim 3 \times 10^{-3}$ eV^2 $\sim |\Delta m^2_{\rm A}|$. At this baseline distance, the reactor $\bar{\nu}_e$ oscillations driven by $\Delta m^2_{\odot}$ are negligible. Therefore, as can be seen from Eq. (14.22) and Eq. (14.24), θ_{13} can be directly measured. A reactor neutrino oscillation experiment at the Chooz

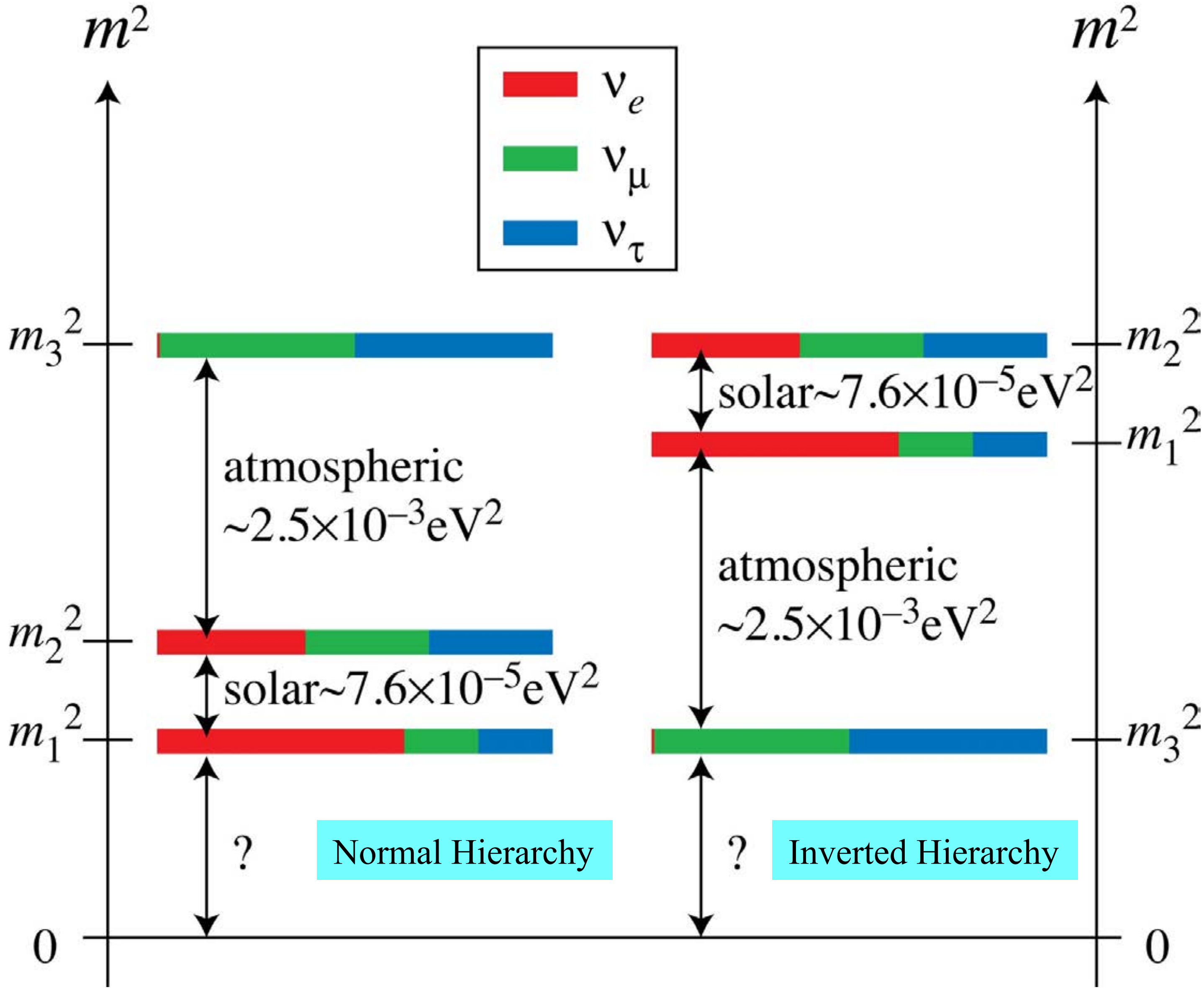
$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) \cong 1 - 2|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \left(1 - \cos \frac{\Delta m^2_{31}}{2p} L\right) , \qquad (14.22)$$

$$\sin^2\theta_{13} = |U_{e3}|^2, \quad s^2_{23} \equiv \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2},$$
$$c^2_{23} \equiv \cos^2\theta_{23} = \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2}. \qquad (14.24)$$

Neutrino Oscillations

Table 14.7: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters, derived from a global fit of the current neutrino oscillation data (from [174]) . The values (values in brackets) correspond to $m_1 < m_2 < m_3$ $(m_3 < m_1 < m_2)$. The definition of Δm^2 used is: $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0$, if $m_1 < m_2 < m_3$, and $\Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit ($\pm 1\sigma$)	3σ
Δm_{21}^2 [10^{-5} eV ^{2}]	$7.54^{+0.26}_{-0.22}$	$6.99 - 8.18$
$\lvert\Delta m^2\rvert$ [10^{-3} eV ^{2}]	2.43 ± 0.06 (2.38 ± 0.06)	$2.23 - 2.61$ $(2.19 - 2.56)$
$\sin^2\theta_{12}$	0.308 ± 0.017	$0.259 - 0.359$
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	$0.437^{+0.033}_{-0.023}$	$0.374 - 0.628$
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	$0.455^{+0.039}_{-0.031}$,	$0.380 - 0.641$
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	$0.0234^{+0.0020}_{-0.0019}$	$0.0176 - 0.0295$
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	$0.0240^{+0.0019}_{-0.0022}$	$0.0178 - 0.0298$
δ/π (2σ range quoted)	$1.39^{+0.38}_{-0.27}$ $(1.31^{+0.29}_{-0.33})$	$(0.00 - 0.16) \oplus (0.86 - 2.00)$
		$((0.00 - 0.02) \oplus (0.70 - 2.00))$


m^2
m^2
ν_e
ν_μ
ν_τ
$m_3{}^2$
$m_2{}^2$
$m_1{}^2$
atmospheric
$\sim 2.5\times10^{-3}\mathrm{eV}^2$
solar$\sim 7.6\times10^{-5}\mathrm{eV}^2$
?
Normal Hierarchy
$m_2{}^2$
$m_1{}^2$
$m_3{}^2$
solar$\sim 7.6\times10^{-5}\mathrm{eV}^2$
atmospheric
$\sim 2.5\times10^{-3}\mathrm{eV}^2$
?
Inverted Hierarchy
0
0