Space-Time Symmetries

• Outline
  - Translation and rotation
  - Parity
  - Charge Conjugation
  - Positronium
  - T violation
## Conservation Rules

<table>
<thead>
<tr>
<th>Conserved quantity</th>
<th>strong</th>
<th>EM</th>
<th>weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy-momentum</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>charge</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>baryon number</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>lepton number</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>CPT</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>P (parity)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>C (charge conjugation parity)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>CP (or T)</td>
<td>yes</td>
<td>yes</td>
<td>$10^{-3}$ violation</td>
</tr>
<tr>
<td>I (isospin)</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>
Discrete and Continuous Symmetries

- **Continuous Symmetries**
  - Space-time symmetries
  - Lorentz transformations
  - Poincare transformations
    - combined space-time translation and Lorentz Trans.

- **Discrete symmetries**
  - cannot be built from succession of infinitesimally small transformations - eg. Time reversal, Spatial Inversion

- **Other types of symmetries**
  - Dynamical symmetries - of the equations of motion
  - Internal symmetries - such as spin, charge, color, or isospin
Conservation Laws

- Continuous symmetries lead to additive conservation laws:
  - energy, momentum

- Discrete symmetries lead to multiplicative conservation laws
  - parity, charge conjugation
Translation and Rotation

• Invariance of the energy of an isolated physical system under space translations leads to conservation of linear momentum

• Invariance of the energy of an isolated physical system under spatial rotations leads to conservation of angular momentum

• Noether’s Theorem (Emmy Noether, 1915)
  

Angular Momentum in Quark Model

Lightest baryon and meson have \( L = 0 \)

So for mesons \( J = S(\mathbf{q}, \mathbf{q}) = \vec{S}_q + \vec{S}_{\bar{q}} \) for \( L = 0 \)

\[
S = \frac{1}{2} \pm \frac{1}{2} = 0, 1
\]

In spectroscopic notation

\[
^{2S+1}L_J = \begin{cases} 
1 \text{ } S_0, & (L=0) \\
3 \text{ } S_1, & (L=0) 
\end{cases}
\]

For \( L > 0 \) (\( J = S + L \))

\[
\begin{align*}
L = 1 & \quad J = 0, 1, 2 & \text{for } S = 1 & \quad J = 1 & \text{for } S = 0 \\
L = 2 & \quad J = 1, 2, 3 & \text{for } S = 1 & \quad J = 2 & \text{for } S = 0
\end{align*}
\]

\[
^{2S+1}L_J = \begin{cases} 
1 \text{ } L_L, & (L\geq 1) \\
3 \text{ } L_{L+1}, & (L\geq 1) \\
3 \text{ } L_L, & (L\geq 1) \\
3 \text{ } L_{L-1}, & (L\geq 1)
\end{cases}
\]

Lowest mass states for \( L = 0 \) \( \rightarrow \) \( J = 0, 1 \)

\[
\pi, \rho \quad K, K^* \quad D, D^*
\]

J. Brau

Physics 661, Space-Time Symmetries
Angular Momentum in Quark Model

For baryons \( \mathbf{L} = \mathbf{L}_{12} + \mathbf{L}_3 \)

\[
\mathbf{J} = \mathbf{S}(q_1, q_2, q_3) + \mathbf{L} = \mathbf{S}_{q1} + \mathbf{S}_{q2} + \mathbf{S}_{q3} + \mathbf{L}
\]

\( S = \frac{1}{2}, \frac{3}{2} \) for \( L = 0 \)

In spectroscopic notation

\[
2S+1 \quad L \quad J = 2 \quad S \quad \frac{1}{2}, 4 \quad S \quad \frac{3}{2} \quad (L=0)
\]

\[
2S+1 \quad L \quad J = 2 \quad P \quad \frac{1}{2}, 2 \quad L \quad \frac{3}{2}, 4 \quad L \quad \frac{1}{2}, 4 \quad L \quad \frac{3}{2}, 4 \quad L \quad \frac{5}{2} \quad (L=1)
\]

Lowest mass states for \( L=0 \) -> \( J=1/2, 3/2 \)
(p,n), \( \Delta(1232) \)
Parity

- **Spatial inversion**
  - \((x, y, z) \rightarrow (-x, -y, -z)\)
  - discrete symmetry
- **\(P\ \psi(r) = \psi(-r)\)**
  - \(P\) is the parity operator
- **\(P^2\ \psi(r) = P\ \psi(-r) = \psi(r)\)**
  - therefore \(P^2 = 1\) and the parity of an eigen-system is 1 or -1
Example, the spherical harmonics:

\[ Y_{LM} \]

The spherical harmonics describe a state in a spherically symmetric potential with definite angular momentum.

\[ Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta \]
\[ Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi} \]
\[ Y_2^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \]
\[ Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi} \]
\[ Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi} \]

\[ p = (-1)^L \]
Parity Invariance

**PARITY \((P)\) INVARIANCE**

- \(e\) electric dipole moment
- \(\mu\) electric dipole moment
- \(\text{Re}(d_\tau = \tau\text{ electric dipole moment})\)
- \(\Gamma(\eta \to \pi^+\pi^-)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta \to 2\pi^0)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta \to 4\pi^0)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta' (958) \to \pi^+\pi^-)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta' (958) \to \pi^0\pi^0)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta_c (15) \to \pi^+\pi^-)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta_c (15) \to \pi^0\pi^0)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta_c (15) \to K^+K^-)/\Gamma_{\text{total}}\)
- \(\Gamma(\eta_c (15) \to K^0_S K^0_S)/\Gamma_{\text{total}}\)

- \(p\) electric dipole moment
- \(n\) electric dipole moment
- \(\Lambda\) electric dipole moment

\[<0.87 \times 10^{-28}\text{ e cm, CL = 90%}\]
\[<-0.1 \pm 0.9 \times 10^{-19}\text{ e cm}\]
\[<-0.220\text{ to } 0.45 \times 10^{-16}\text{ e cm, CL = 95%}\]
\[<1.3 \times 10^{-5}, \text{ CL = 90%}\]
\[<3.5 \times 10^{-4}, \text{ CL = 90%}\]
\[<6.9 \times 10^{-7}, \text{ CL = 90%}\]
\[<6 \times 10^{-5}, \text{ CL = 90%}\]
\[<4 \times 10^{-4}, \text{ CL = 90%}\]
\[<1.1 \times 10^{-4}, \text{ CL = 90%}\]
\[<4 \times 10^{-5}, \text{ CL = 90%}\]
\[<6 \times 10^{-4}, \text{ CL = 90%}\]
\[<3.1 \times 10^{-4}, \text{ CL = 90%}\]
\[<0.54 \times 10^{-23}\text{ e cm}\]
\[<0.30 \times 10^{-25}\text{ e cm, CL = 90%}\]
\[<1.5 \times 10^{-16}\text{ e cm, CL = 95%}\]
Parity

- Parity is a multiplicative quantum number

- Composite system:
  - parity is equal to product of the parts
  - eg. two pions in an angular momentum L state
    \[ P = P(\pi_1) \cdot P(\pi_2) \cdot (-1)^L \]

- Intrinsic parity of particles:
  - proton and neutron (+1 by definition, could be -1)
    - parity of fermion and anti-fermion are opposite, and assignment of one is arbitrary
  - pion (-1, measured)
Pion spin and parity

• 1. Spin of the charged pion
  - $p + p \leftrightarrow \pi^+ + d \quad (|M_{if}|^2 = |M_{fi}|^2)$
  - then, “detailed balance” gives:
    \[
    \sigma (pp \rightarrow \pi^+d) \sim (2s_{\pi} + 1) (2s_{d}+1) p_{\pi}^2
    \]
    \[
    \sigma (\pi^+d \rightarrow pp) \sim 1/2 (2s_{p}+1)^2 p_{p}^2
    \]
    (1/2 because protons are identical)
  - Measurements of $\sigma$’s reveals $s_{\pi} = 0$

• 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0 (see following)
Pion spin and parity

2. Spin of the neutral pion
   - $\pi^0 \to \gamma\gamma$ shows it must be 0

   - along the flight path of the $\gamma$s in the $\pi^0$ rest frame, the total photon spin ($S_z$) must be 0 or 2
   - If $S_\pi = 1$, then $S_z$ must be 0
   - If $S_\pi = 1$, $S_z = 0$, the two-photon amplitude must behave as $P_{l,m=0}(\cos \theta)$, which is antisymmetric under interchange (=180$^\circ$ rotation)
   - This corresponds to two right or left circularly polarized photons travelling in opposite directions, which must be symmetric by Bose statistics \( \rightarrow \) forbidden \( S_\pi \neq 1 \)
   - Therefore, $S_\pi = 0$ or $S_\pi \geq 2$ (which is ruled out by pion production statistics - all three charges produced equally)
Parity

• Parity of the charged pion
  - the observation of the reaction
    \[ \pi^- + d \rightarrow n + n \]
    is evidence that the charged pion has \( p = -1 \)
    assuming parity is conserved

ARGUMENT

• Capture takes place from an s-state, \( L_i=0 \) (\( S_d=1 \))
  - (X-ray emissions following capture)
• \( J=1 \), since \( S_d=1 \) and \( S_\pi=0 \)
Parity

- Parity of the charged pion
- J=1, since \( S_d = 1 \) and \( S_\pi = 0 \)
- In the two neutron system, \( L + S \) must be even by the antisymmetric requirement on identical fermions
  - \( \Psi = \Psi(\text{space}) \Psi(\text{spin}) \)
  - \( S=1 \) \( \uparrow \uparrow \), \( (\uparrow \downarrow + \downarrow \uparrow)/\sqrt{2} \), \( \downarrow \downarrow \) (symmetric)
  - \( L \) odd for anti-symmetric wave function
  - \( S=0 \) \( (\uparrow \downarrow - \downarrow \uparrow)/\sqrt{2} \) (anti-symmetric)
  - \( L \) even for anti-symmetric wave function
    so for \( J=1, L=1, S=1 \) is only possibility
  - thus, \( (2S+1)J_L = 3P_1 \) state with \( p = (-1)^L = -1 \)
  - since initial state is \( p_d p_\pi (-1)^{L_i} = (+1) p_\pi \), \( p_\pi = -1 \)
Parity

• Parity of the neutral pion

\[ \pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-) \]  (rare decay of the pion)

- the planes of the pairs follow the E vectors of the internally converted photons (\( \pi^0 \rightarrow \gamma\gamma \))
- even system of two photons (bosons)

\[ A \sim (E_1 \cdot E_2) \]  (P=+1)

or \[ A \sim (E_1 \times E_2) \cdot k \]  (P=-1)

- Intensities or rates \( \sim |A|^2 \)

\[ |A|^2 \sim \cos^2 \phi \]

or \[ |A|^2 \sim \sin^2 \phi \]
Parity

- Parity of the neutral pion

\[ \pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-) \]

scalar \((0^+)\) would follow dashed line, while pseudoscalar \((0^-)\) would follow solid line
Parity of particles and antiparticles

- Two-particle systems
  - Fermion-antifermion
    - $p = (-1)^{L+1}$
    - eg, pion which is q-antiq with $L=0$ has $p = -1$
  - Boson-antiboson
    - same parity
    - $p = (-1)^{L}$
    - eg. $\rho \rightarrow \pi^+ \pi^-$
      $1^- \rightarrow 0^- 0^-, L=1$
Parity of particles and antiparticles

• Dirac theory required fermions and antifermions to have opposite intrinsic parity
• This was checked in decays of the spin singlet ground state of positronium
  \[ e^+e^- \left( (2S+1)J_L = 1S_0 \right) \rightarrow 2 \gamma \]
  \[ P = (+1)(-1)(-1)^0 = -1 \]
• This is exactly the case of the \( \pi^0 \) decay
  - the photon polarizations must have a \( \sin^2\phi \) form
Parity of particles and antiparticles

\[ \frac{\text{Rate}(90^0)}{\text{Rate}(0^0)} = 2.04 \pm 0.08 \]

expect 2.00

conclude \( P(e^+) = -P(e^-) \)
Tests of parity conservation

• Strong and EM interactions conserve parity, but weak interactions do not

• LH neutrino observed, but RH neutrino is not
  - this is a maximally violated symmetry
Tests of parity conservation

- In interactions dominated by the strong or the EM interaction, some parity violation may be observed due to the small contribution of the weak int. to the process
  \[ H = H_{\text{strong}} + H_{\text{EM}} + H_{\text{weak}} \]

- In nuclear transitions, for example, the degree of parity violation will be of the order of the ratio of the weak to the strong couplings \((\sim 10^{-7})\)
Tests of parity conservation

- Examples of nuclear transitions
  - fore-aft asymmetry in gamma emission
    \[ ^{19}\text{F}^* \rightarrow ^{19}\text{F} + \gamma \text{ (110 keV)} \]
    \[ J^P: \ 1/2^- \rightarrow 1/2^+ \]
    \[ \Delta \sim (18 +/- 9) \times 10^{-5} \]

  - very narrow decay
    \[ ^{16}\text{O}^* \rightarrow ^{12}\text{C} + \alpha \]
    \[ J^P: \ 2^- \rightarrow 2^+ \]
    \[ \Gamma = (1.0 +/- 0.3) \times 10^{-10} \text{ eV} \]
    (note \[ ^{16}\text{O}^* \rightarrow ^{16}\text{O} + \gamma, \ \Gamma = 3 \times 10^{-3} \text{ eV} \])
Charge Conjugation Invariance

- Charge conjugation reverses sign of charge and magnetic moment, leaving other coord unchanged
- Good symmetry in strong and EM interactions:
  - eg. mesons in \( p + \bar{p} \rightarrow \pi^+ + \pi^- + \ldots \)
- Only neutral bosons are eigenstates of \( C \)
- The charge conjugation eigenvalue of the \( \gamma \) is -1:
  - EM fields change sign when charge source reverses sign
- Since \( \pi^0 \rightarrow \gamma \gamma, C|\pi^0> = +1 \)
- Consequence of \( C \) of \( \pi^0 \) and \( \gamma \), and \( C \)-cons. in EM int,
  - \( \pi^0 \rightarrow \gamma \gamma \gamma \) forbidden
  - experiment: \( \pi^0 \rightarrow \gamma \gamma \gamma / \pi^0 \rightarrow \gamma \gamma < 3 \times 10^{-8} \)
Charge Conjugation of Multiparticles

- Consider $\pi^+ + \pi^-$ (S=0, so J=L)
  \[ C \left| \pi^+ \pi^- ; L \right> = \left| \pi^- \pi^+ , L \right> = (-1)^L \left| \pi^+ \pi^- ; L \right> \]
  so \[ c (\pi^+ \pi^-) = (-1)^L \]

- Consider fermion/anti-fermion system
  we can see it is $(-1)^{L+S}$ from the following
  1.) for s=0, l=0, an f f system decays to 2 $\gamma$'s, $c= (-1)^2=+1$
  2.) for s=1, l=0, an f f system decays to s=0, l=0
     (spin flip) by emitting 1 $\gamma$, which is $c=-1$
  3.) for l>0, the system decays from l to l-1
     via EM by emitting 1 $\gamma$, which is $c=-1$

These observations show only choice is $c= (-1)^{L+S}$
Charge Conjugation of Multiparticles

- Further considerations on fermion/anti-fermion system
  in this case we must consider $S$, $L$, and $J$
  \[ C |f \bar{f}; S,L,J> = ? \]
  $L$ contributes $(-1)^L$
  What about $S$?
  
  $S=1$ is symmetric and $S=0$ is antisymmetric
  
  1) \[ \uparrow \uparrow, \ (\uparrow \downarrow + \downarrow \uparrow)/\sqrt{2}, \ \downarrow \downarrow \ 0 \ (\uparrow \downarrow - \downarrow \uparrow)/\sqrt{2} \]
  so we have $(-1)^{S+1}$
  It turns out there is another factor of $-1$ from QFT,
  so we get $(-1)^{L+S}$
  (consistent with conclusion on last page)
C & P of particle - antiparticle

- **Parity**
  - Fermion pair \((-1)^{L+1}\)
  - Boson pair \((-1)^L\)

- **Charge conjugation**
  - Fermion pair \((-1)^{L+S}\)
  - Boson pair \((-1)^L\)
**Charge Conjugation Invariance**

**Charge Conjugation (C) Invariance**

\[
\Gamma(\pi^0 \rightarrow 3\gamma)/\Gamma_{\text{total}} < 3.1 \times 10^{-8}, \text{CL = 90%}
\]

\[\eta \text{ C-nonconserving decay parameters}\]
\[
\begin{align*}
\pi^+ \pi^- \pi^0 & \text{ left-right asymmetry} \\
\pi^+ \pi^- \pi^0 & \text{ sextant asymmetry} \\
\pi^+ \pi^- \pi^0 & \text{ quadrant asymmetry} \\
\pi^+ \pi^- \gamma & \text{ left-right asymmetry} \\
\pi^+ \pi^- \gamma & \text{ parameter } \beta \text{ (D-wave)}
\end{align*}
\]

\[
\Gamma(\eta \rightarrow \pi^0 \gamma)/\Gamma_{\text{total}}
\]

\[
\Gamma(\eta \rightarrow 2\pi^0 \gamma)/\Gamma_{\text{total}}
\]

\[
\Gamma(\eta \rightarrow 3\pi^0 \gamma)/\Gamma_{\text{total}}
\]

\[
\Gamma(\eta \rightarrow 3\gamma)/\Gamma_{\text{total}}
\]

\[
\Gamma(\eta \rightarrow \pi^0 e^+ e^-)/\Gamma_{\text{total}} < 4 \times 10^{-5}, \text{CL = 90%}
\]

\[
\Gamma(\eta \rightarrow \pi^0 \mu^+ \mu^-)/\Gamma_{\text{total}} < 5 \times 10^{-6}, \text{CL = 90%}
\]

\[
\Gamma(\omega(782) \rightarrow \eta \pi^0)/\Gamma_{\text{total}} < 2.1 \times 10^{-4}, \text{CL = 90%}
\]

\[
\Gamma(\omega(782) \rightarrow 2\pi^0)/\Gamma_{\text{total}} < 2.1 \times 10^{-4}, \text{CL = 90%}
\]

\[
\Gamma(\omega(782) \rightarrow 3\pi^0)/\Gamma_{\text{total}} < 2.3 \times 10^{-4}, \text{CL = 90%}
\]

\[
\text{asymmetry parameter for } \eta'(958) \rightarrow \pi^+ \pi^- \gamma \text{ decay}
\]

\[
\Gamma(\eta'(958) \rightarrow \pi^0 e^+ e^-)/\Gamma_{\text{total}} < 1.4 \times 10^{-3}, \text{CL = 90%}
\]

\[
\Gamma(\eta'(958) \rightarrow \eta e^+ e^-)/\Gamma_{\text{total}} < 2.4 \times 10^{-3}, \text{CL = 90%}
\]

\[
\Gamma(\eta'(958) \rightarrow 3\gamma)/\Gamma_{\text{total}} < 1.0 \times 10^{-4}, \text{CL = 90%}
\]

\[
\Gamma(\eta'(958) \rightarrow \mu^+ \mu^- \pi^0)/\Gamma_{\text{total}} < 6.0 \times 10^{-5}, \text{CL = 90%}
\]
**Charge Conjugation Invariance**

- **Weak interaction**
  - respects neither $C$ nor $P$, but $CP$ is an approximate symmetry of the weak interaction
Quarkonium and Positronium

**Positronium**

- $2^3S_1$
- $2^3P_2$
- $2^3P_1$
- $2^3P_0$
- $1^3S_1$
- $1^3S_0$
- $2^1S_0$

$\Delta E \times 10^{-4} \text{ eV}$

$M_1$

M1 (8.4 x 10^{-4} eV)

$J^{PC} \rightarrow$

**Charmonium**

- $\Psi(4160)$
- $\Psi(4040)$
- $\Psi(3770)$
- $D\bar{D}$ threshold

- $2^3S_1$
- $1^3S_1$
- $1^3P_2$
- $1^3P_1$
- $1^3P_0$

$\Delta E \times 10^{-4} \text{ eV}$

**Bottomonium**

- $4S \rightarrow \Upsilon(10580)$
- $b\bar{b}$ threshold

- $3S \rightarrow \Upsilon(10355)$
- $2S \rightarrow \Upsilon(10023)$

- $1S \rightarrow \Upsilon(9460)$

**References**

J. Brau

Physics 661, The Quark Model
Positronium

- Bound state of electron and positron
- \( V(r) = -\alpha/r \)
- \( E_n = -m_r \alpha^2 / 2n^2 = -13.6 \text{ ev} / 2n^2 \)

\[
E_1 = -6.8 \text{ eV} \quad E_2 = -1.7 \text{ eV} \quad \Delta E = 5.1 \text{ eV}
\]

- Each principal level \( n \) state has
  - \( S = 0 \) or 1
  - \( L = 0, \ldots, n-1 \) (so \( L=0 \) for \( n=1 \), \( L=0 \) or 1 for \( n=2 \))

- Parity and \( C \)-parity of each level
  - \( P = P_e - P_e^+ (-1)^L = (-1)^{L+1} \)
  - \( C = (-1)^{L+S} \)
Positronium

- Principal energy levels from non-relativistic Schrödinger equation in a Coulomb potential

\[ E_n = -\frac{\alpha^2 \mu c^2}{2n^2}, \quad \mu = m/2 \]

- Relativistic corrections:
  - spin-orbit
    - \( S, P, D \)
  - spin-spin
    - \( ^3S_1, ^1S_0, \ldots \)

  - these are about the same size in positronium:
    - \( \Delta E \sim \frac{\alpha^4 mc^2}{n^3} \)
Positronium

• Fine structure
  - Orthopositronium/parapositronium splitting
  - \( E(n=1, \, ^3S_1) - E(n=1, \, ^1S_0) = 8.45 \times 10^{-4} \text{ eV} \)

• There are two contributions
  1) spin-spin - magnetic moment of positron gives rise to B field which interacts with the magnetic moment of the electron
     - Depends on relative alignment of spins
  2) one-photon exchange
     shifts only \( ^3S_1 \) state

• Total shift for \( n=1 \)
  - \( \frac{7}{6} \alpha^2 (13.6 \text{ eV}) = 0.000845 \text{ eV} \)
Positronium

\[ J^{PC} = 0^{-+}, 1^{--}, 0^{++}, 1^{++}, 2^{++}, 1^{+-} \]

5.1 eV

\[ \begin{align*}
1 \times 10^{-4} \text{ eV} \\
8 \times 10^{-4} \text{ eV} \\
\end{align*} \]
Positronium

- **Lifetimes:**
  - Two photon \((C=(-1)^2 = 1)\)
    - two photons means rate is to order \(\alpha^2\)
    - overlap of wavefunctions at origin to annihilate
      - \(|\psi(0)|^2 = 1/ \pi a^3\)
      - \(a = 2 \hbar/(2\pi mc \alpha)\)
    - exact result:
      - \(\Gamma = \alpha^5 m/2\)
  - Three photon \((C=(-1)^3 = -1)\)
    - rate at higher order
      - \(\Gamma = 2(\pi^2-9) \alpha^6 m/(9\pi)\)
Positronium

- $e^+e^- \rightarrow \gamma\gamma$
  - $\tau = 1.25 \times 10^{-10}$ sec
  - singlet state
  - even ang. momentum $\rightarrow J=0 \rightarrow C = (-1)^{L+S} = (-1)^0 = +1$
  - $C = (-1)^n_\gamma \rightarrow C = +1$

- $e^+e^- \rightarrow \gamma\gamma\gamma$
  - $\tau = 1.4 \times 10^{-7}$ sec
  - triplet state $\rightarrow J=1 \rightarrow C = (-1)^{L+S} = (-1)^{0+1} = -1$
  - $C = (-1)^n_\gamma \rightarrow C = -1$
C & P of e+e- system

• Interchange of particles
  - Spin symmetry \((-1)^{S+1}\)
    \[ \alpha(s=0, s_3=0) = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \]
  - Spatial symmetry \((-1)^{L+1}\)
    • Recall opposite intrinsic parities of e+ and e-

- So - total symmetry is \((-1)^{L+S}\)

• Interchange of space and spin
  is equivalent to Charge Conjugation
Positronium

- Excellent agreement on Theory and Experimental results for lifetimes and energy levels

<table>
<thead>
<tr>
<th>Table 4.4. Positronium lifetimes and level spacings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow 2\gamma$ rate</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow 3\gamma$ rate</td>
</tr>
<tr>
<td>$(\Delta E/h)(1^3S_1-1^1S_0)$</td>
</tr>
<tr>
<td>$(\Delta E/h)(2^3S_1-2^3P_2)$</td>
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</tbody>
</table>
Time reversal

- $t \rightarrow t' = -t$
- Symmetry of strong and EM interactions, but violated by weak interaction
## Time reversal violation

<table>
<thead>
<tr>
<th></th>
<th>Effect of $T$</th>
<th>Effect of $P$</th>
</tr>
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<tbody>
<tr>
<td>position</td>
<td>$r$</td>
<td>$r$</td>
</tr>
<tr>
<td></td>
<td>$-r$</td>
<td></td>
</tr>
<tr>
<td>momentum</td>
<td>$p$</td>
<td>$-p$</td>
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<tr>
<td></td>
<td>$-p$</td>
<td>$-p$</td>
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<tr>
<td>spin</td>
<td>$\sigma$</td>
<td>$-\sigma$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td></td>
<td>$(r \times p)$</td>
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</tr>
<tr>
<td>electric field</td>
<td>$E$ ($= -\nabla V$)</td>
<td>$E$</td>
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<tr>
<td></td>
<td>$-E$</td>
<td>$-E$</td>
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<tr>
<td>magnetic field</td>
<td>$B$</td>
<td>$-B$</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
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<tr>
<td>mag. dip. mom.</td>
<td>$\sigma \cdot B$</td>
<td>$\sigma \cdot B$</td>
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<td>$\sigma \cdot B$</td>
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<td>$\sigma \cdot E$</td>
<td>$-\sigma \cdot E$</td>
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<td>$-\sigma \cdot E$</td>
<td>$-\sigma \cdot E$</td>
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<td>long. pol.</td>
<td>$\sigma \cdot p$</td>
<td>$\sigma \cdot p$</td>
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<td></td>
<td>$-\sigma \cdot p$</td>
<td>$-\sigma \cdot p$</td>
</tr>
<tr>
<td>trans. pol.</td>
<td>$\sigma \cdot (p_1 \times p_2)$</td>
<td>$-\sigma \cdot (p_1 \times p_2)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma \cdot (p_1 \times p_2)$</td>
<td>$\sigma \cdot (p_1 \times p_2)$</td>
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</tbody>
</table>
**Time Reversal Invariance**

**TIME REVERSAL ($T$) INVARIANCE**

- $e$ electric dipole moment
- $\mu$ electric dipole moment
- $\mu$ decay parameters
  - transverse $e^+$ polarization normal to plane of $\mu$ spin, $e^+$ momentum
    - $\alpha'/A$
    - $\beta'/A$
- $\text{Re}(d_\tau = \tau$ electric dipole moment)
- $P_T$ in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$
- $P_T$ in $K^+ \rightarrow \mu^+ \nu_\mu \gamma$
- $\text{Im}(\xi)$ in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ decay (from transverse $\mu$ pol.)
- asymmetry $A_T$ in $K^0 - \bar{K}^0$ mixing
  - $\text{Im}(\xi)$ in $K^0_{\mu 3}$ decay (from transverse $\mu$ pol.)
    - $A_T(D^\pm \rightarrow \bar{K}^0_S K^\pm \pi^+ \pi^-)$
    - $A_T(D^0 \rightarrow K^+ K^- \pi^+ \pi^-)$
    - $A_T(D^{+}_s \rightarrow \bar{K}^0_S K^\pm \pi^+ \pi^-)$
    - $\Delta S^T_T \left( S^-_{\ell^-,\bar{K}^0_S} - S^+_{\ell^+,\bar{K}^0_S} \right)$
    - $\Delta S^T_T \left( S^+_{\ell^-,\bar{K}^0_S} - S^-_{\ell^+,\bar{K}^0_S} \right)$
    - $\Delta C^T_T \left( C^-_{\ell^-,\bar{K}^0_S} - C^+_{\ell^+,\bar{K}^0_S} \right)$

Values:
- $<0.87 \times 10^{-28}$ e cm, CL = 90%
- $(-0.1 \pm 0.9) \times 10^{-19}$ e cm
- $(-2 \pm 8) \times 10^{-3}$
- $(-10 \pm 20) \times 10^{-3}$
- $(2 \pm 7) \times 10^{-3}$
- $-0.220$ to $0.45 \times 10^{-16}$ e cm, CL = 95%
- $(-1.7 \pm 2.5) \times 10^{-3}$
- $(-0.6 \pm 1.9) \times 10^{-2}$
- $-0.006 \pm 0.008$
- $(6.6 \pm 1.6) \times 10^{-3}$
- $-0.007 \pm 0.026$
- $[b] (-12 \pm 11) \times 10^{-3}$
- $[b] (1.7 \pm 2.7) \times 10^{-3}$
- $[b] (-14 \pm 8) \times 10^{-3}$
- $1.17 \pm 0.21$
- $-1.37 \pm 0.15$
- $0.10 \pm 0.16$
T Violation

- Transverse polarization

<table>
<thead>
<tr>
<th>Effect of T</th>
<th>Effect of P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p_1 \times p_2)$</td>
<td>$-\sigma(p_1 \times p_2)$</td>
</tr>
</tbody>
</table>

- $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$
- $n \rightarrow p + e^- + \overline{\nu}_e$

- T-violating contributions below $10^{-3}$
T Violation

- Detailed Balance

\[ p + ^{27}\text{Al} \leftrightarrow \alpha + ^{24}\text{Mg} \]

curve and points

T violation

\(< 5 \times 10^{-4}\)
Neutron Electric Dipole Moment

- Most sensitive tests of T violation come from searches for the neutron and electron dipole moments
  - electron $< 0.87 \times 10^{-28}$ e-cm
  - neutron $< 0.29 \times 10^{-25}$ e-cm

- Note, the existence of these electric dipole moments would also violate parity
Neutron Electric Dipole Moment

• How big might we expect the EDM to be?
  - $\text{EDM} = (\text{charge} \times \text{length}) \times \text{T-violating parameter}$
  
  length $\sim 1/\text{Mass}$
  
  assume weak interaction $\sim g^2 / M_W^2$
  
  $\sim e^2 M_N / M_W^2 \sim 4\pi/137 \times (0.94 \text{ GeV}) / (80\text{GeV})^2$
  
  $\sim 10^{-5} \text{ GeV}^{-1}$ [200 MeV-fm]
  
  $\sim 10^{-6} \text{ fm} \sim 10^{-19} \text{ cm}$
  
  T-violating parameter $\sim 10^{-3}$ (like $K^0$)

Then:

$\text{EDM} \sim e \times (10^{-19} \text{ cm}) (10^{-3})$

$\sim 10^{-22} \text{ e-cm}$

Experiment: Neutron EDM $< 0.29 \times 10^{-25} \text{ e-cm}$
Neutron Electric Dipole Moment

(a)

Magnetic ‘bottle’ storage cell

Shutter

RF spin flip coil

N S

Polarising foil

Detector

$^3\text{He} + \text{Ar gas counter}$

690 blade turbine

Ultra-cold neutrons

Ni guide tube

Cold source

Reactor

D$_2$
Neutron Electric Dipole Moment
T Violation

TIME REVERSAL ($T$) INVARIANCE

- $e$ electric dipole moment
  - $<0.87 \times 10^{-28}$ e cm, CL = 90%
- $\mu$ electric dipole moment
  - $(-0.1 \pm 0.9) \times 10^{-19}$ e cm
- $p$ electric dipole moment
  - $<0.54 \times 10^{-23}$ e cm
- $n$ electric dipole moment
  - $<0.29 \times 10^{-25}$ e cm, CL = 90%