The Quark Model

- Isospin Symmetry
- Quark model and eightfold way
- Baryon decuplet
- Baryon octet
- Quark spin and color
- Baryon magnetic moments
- Pseudoscalar mesons
- Vector mesons
- Other tests of the quark model
- Mass relations and hyperfine splitting
- EM mass differences and isospin symmetry
- Heavy quark mesons
- The top quarks
- EXTRA – Group Theory of Multiplets

Isospin Symmetry

- The nuclear force of the neutron is nearly the same as the nuclear force of the proton
- 1932 - Heisenberg - think of the proton and the neutron as two charge states of the nucleon
- In analogy to spin, the nucleon has isospin 1/2

$I_3 = 1/2$ proton

$I_3 = -1/2$ neutron

$$Q = (I_3 + 1/2)\, e$$

- Isospin turns out to be a conserved quantity of the strong interaction

Isospin Symmetry

- The notion that the neutron and the proton might be two different states of the same particle (the nucleon) came from the near equality of the n-p, n-n, and p-p nuclear forces (once Coulomb effects were subtracted)

- Within the quark model, we can think of this symmetry as being a symmetry between the u and d quarks:

  p = d u u $\qquad$ I_3 (u) = $\frac{1}{2}$

  n = d d u $\qquad$ I_3 (d) = $-\frac{1}{2}$

Isospin Symmetry

- Example from the meson sector:
  - the pion (an isospin triplet, I=1)

$\pi^+ = u\bar{d}$ $(I_3 = +1)$

$\pi^- = d\bar{u}$ $(I_3 = -1)$

$\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$ $(I_3 = 0)$

The masses of the pions are similar,

$M(\pi^\pm) = 140$ MeV $M(\pi^0) = 135$ MeV,

reflecting the similar masses of the u and d quark

Isospin in Two-nucleon System

- Consider the possible two nucleon systems
  - pp $I_3 = 1 \Rightarrow I = 1$
  - pn $I_3 = 0 \Rightarrow I = 0$ or 1
  - nn $I_3 = -1 \Rightarrow I = 1$

- This is completely analogous to the combination of two spin 1/2 states
  - p is $I_3 = 1/2$
  - n is $I_3 = -1/2$

Isospin in Two-nucleon System

- Combining these doublets yields a triplet plus a singlet

  $2 \otimes 2 = 3 \oplus 1$

- Total wavefunction for the two-nucleon state:

  $\psi(\text{total}) = \phi(\text{space})\ \alpha(\text{spin})\ \chi(\text{isospin})$

- Deuteron = pn

  spin = 1 $\Rightarrow$ α is symmetric

  L = 0 $\Rightarrow$ ϕ is symmetric $[(-1)^L]$

  Therefore, Fermi statistics required χ be antisymmetric

  I = 0, singlet state without related pp or nn states

Isospin in Two-nucleon System

- Consider an application of isospin conservation
  - (i) $p + p \rightarrow d + \pi^+$ (isospin of the final state is 1)
  - (ii) $p + n \rightarrow d + \pi^0$ (isospin of the final state is 1)

  - initial state:
    - (i) $I = 1$
    - (ii) $I = 0$ or 1 (*CG* coeff say 50%, 50%)

  - Therefore $\sigma(ii)/\sigma(i) = 1/2$
  - which is experimentally confirmed

Isospin in Pion-nucleon System

- Consider pion-nucleon scattering:
  - three reactions are of interest for the contributions of the I=1/2 and I=3/2 amplitudes

    (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)

    (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)

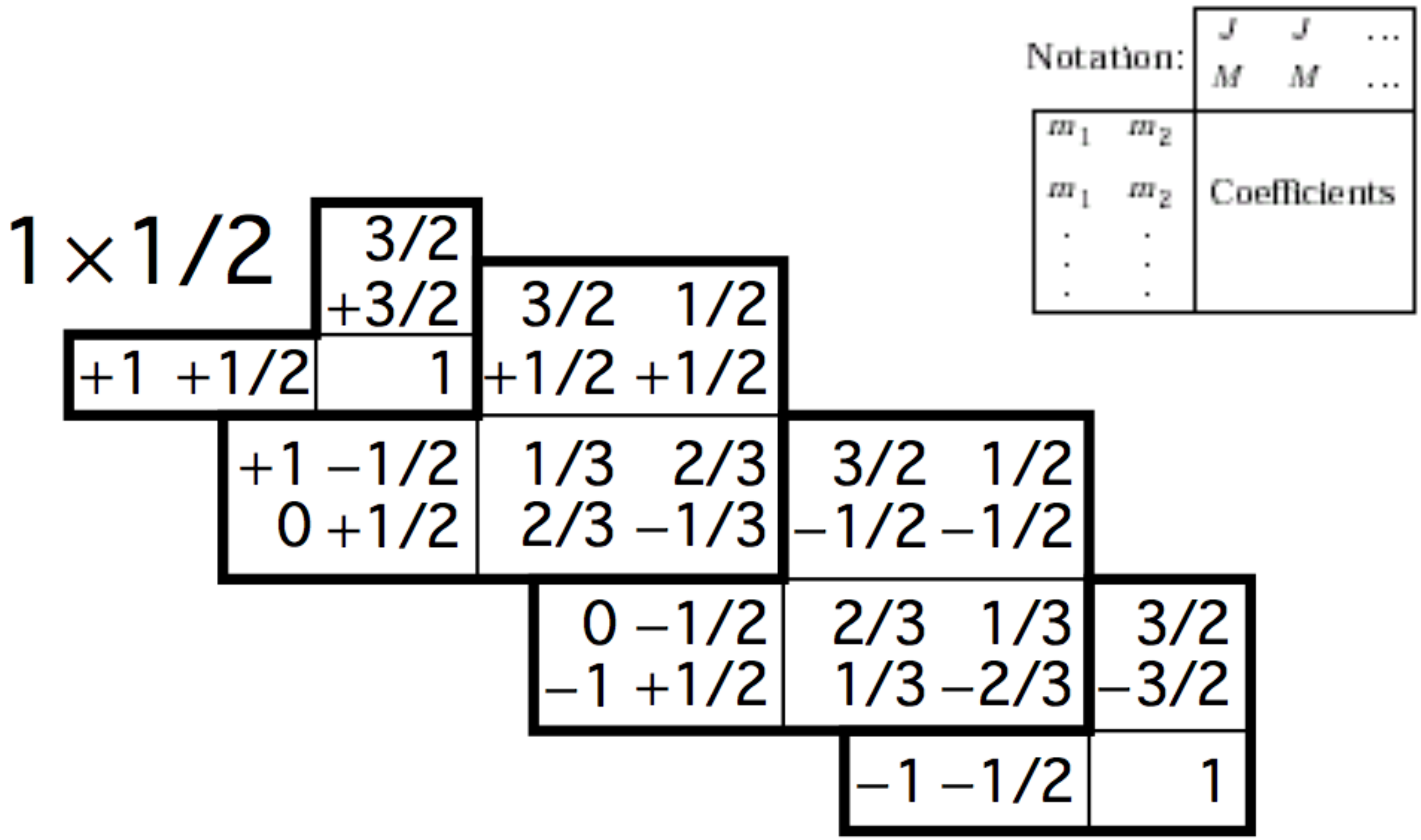
    (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

  - $\sigma \propto |\langle \psi_f | H | \psi_i \rangle|^2 = M_{if}^2$

    $M_1 = \langle \psi_f (1/2) | H_1 | \psi_i (1/2) \rangle$

    $M_3 = \langle \psi_f (3/2) | H_3 | \psi_i (3/2) \rangle$

Clebsch-Gordon Coefficients



Isospin in Pion-nucleon System

– (a) $\pi^+ p \to \pi^+ p$ (elastic scattering)
  (b) $\pi^- p \to \pi^- p$ (elastic scattering)
  (c) $\pi^- p \to \pi^0 n$ (charge exchange)

– (a) is purely I=3/2
  - $\sigma_a = K\,|M_3|^2$

– (b) is a mixture of I = 1/2 and 3/2

$$|\psi_i\rangle = |\psi_f\rangle = \sqrt{1/3}\,|\chi(3/2,-1/2)\rangle - \sqrt{2/3}\,|\chi(1/2,-1/2)\rangle$$

$$\sigma_b = K\,|\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$$

$$= K\,|(1/3)M_3 + (2/3)M_1|^2$$

Isospin in Pion-nucleon System

– (a) $\pi^+ p \to \pi^+ p$ (elastic scattering)
  (b) $\pi^- p \to \pi^- p$ (elastic scattering)
  (c) $\pi^- p \to \pi^0 n$ (charge exchange)

– (c) is a mixture of I = 1/2 and 3/2

$$|\psi_i\rangle = \surd\, 1/3\, |\chi(3/2,-1/2)\rangle - \surd\, 2/3\, |\chi(1/2,-1/2)\rangle$$

$$|\psi_f\rangle = \surd\, 2/3\, |\chi(3/2,-1/2)\rangle + \surd\, 1/3\, |\chi(1/2,-1/2)\rangle$$

$$\sigma_c = K\, |\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$$

$$= K\, |\, \surd(2/9)M_3 - \surd(2/9)M_1|^2$$

Therefore:

$$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$$

Isospin in Pion-nucleon System

$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$

Limiting situations:

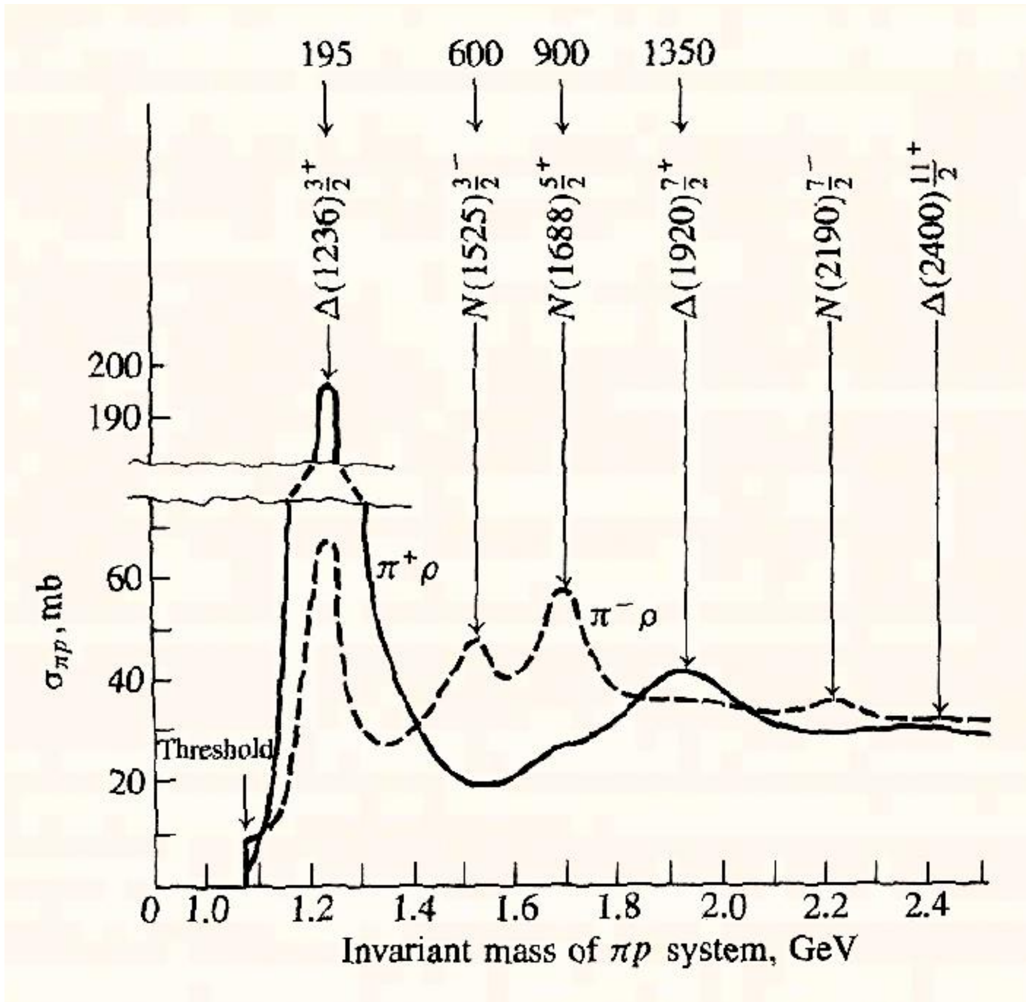
$M_3 >> M_1$

$\sigma_a : \sigma_b : \sigma_c = 9 : 1 : 2$

$M_1 >> M_3$

$\sigma_a : \sigma_b : \sigma_c = 0 : 2 : 1$

Isospin in Pion-nucleon System



Isospin, Strangeness and Hypercharge

- $Q = [I_3 + (B+S)/2]\, e = (I_3 + Y/2)e$
  - B = baryon number
  - S = strangeness

  - Y = B+S = hypercharge

- Example, u and d quark

  u) $I_3 = 1/2$, $B = 1/3$, $S = 0$, $Q = [\,1/2 + 1/6]e = 2/3\ e$

  d) $I_3 = -1/2$, $B = 1/3$, $S = 0$, $Q = [-1/2 + 1/6]e = -1/3\ e$

  Beyond strangeness

  $Y = B + S + C + \tilde{B} + T$

  = baryon # + strangeness + charm + bottom + top

Δ^+ Decays

- Baryon number conservation requires the Δ to decay to to p or n. (Higher mass baryons not accessible)
- Strong interaction favoured 10,000-fold over an EM decay.
- As the strong interaction dominates, use isospin to understand *relative* rates using:

$$|\Delta^+\rangle = |I = \tfrac{3}{2}, I_3 = \tfrac{1}{2}\rangle \quad |\mathrm{p}\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \quad |\mathrm{n}\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$|\pi^+\rangle = |1, 1\rangle \quad |\pi^0\rangle = |1, 0\rangle \quad |\pi^-\rangle = |1, -1\rangle$$

- Decompose $I = \frac{3}{2}$ state into possible combinations $I = \frac{1}{2}$ and $I = 1$

$$\Delta^+ = |\tfrac{3}{2}, \tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle|1, 1\rangle \;+\; \sqrt{\tfrac{2}{3}}\, |\tfrac{1}{2}, \tfrac{1}{2}\rangle|1, 0\rangle$$

$$\tfrac{1}{\sqrt{3}} \quad |\mathrm{n}\rangle \quad |\pi^+\rangle \;+\; \sqrt{\tfrac{2}{3}} \quad |\mathrm{p}\rangle \quad |\pi^0\rangle$$

- And deducing branching ratios:

$$\frac{\mathcal{B}(\Delta^+ \to \pi^0\mathrm{p})}{\mathcal{B}(\Delta^+ \to \pi^+\mathrm{n})} = \frac{|\langle\pi^0\mathrm{p} \mid \Delta^+\rangle|^2}{|\langle\pi^+\mathrm{n} \mid \Delta^+\rangle|^2} = \frac{|\sqrt{2/3}|^2}{|\sqrt{1/3}|^2} = 2$$

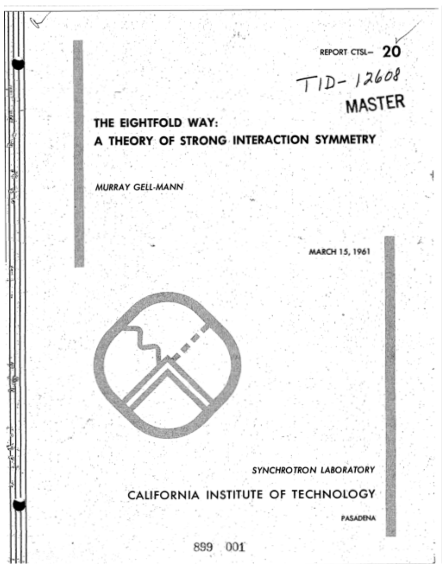
From M. John, http://www-pnp.physics.ox.ac.uk/~mjohn/Lecture3.pdf

Quark Model

- Patterns of observed particles led to proposal in early 1960's that hadrons were composed of quarks
  - u, d, and s (at that time)
- Were quarks real?
  - exhaustive searches for free quarks were unsuccessful
- With the discovery of "confined" quarks in the 1970's it was realized that quarks truly exist, but cannot be freed

Eightfold Way - 1961

Murray Gell-Mann and Yuval Ne'eman


REPORT CTSL- 20

TID- 12608

MASTER

THE EIGHTFOLD WAY:
A THEORY OF STRONG INTERACTION SYMMETRY

MURRAY GELL-MANN

MARCH 15, 1961

SYNCHROTRON LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA

899 001

$$\Sigma^+ \sim \frac{1}{2}\bar{I}(\lambda_1 - i\lambda_2)\ell \quad \sim D^+\nu$$

$$\Sigma^- \sim \frac{1}{2}\bar{I}(\lambda_1 + i\lambda_2)\ell \quad \sim D^0 e^-$$

$$\Sigma^0 \sim \frac{1}{\sqrt{2}}\bar{I}\lambda_3\ell \quad \sim \frac{D^0\nu - D^+e^-}{\sqrt{2}}$$

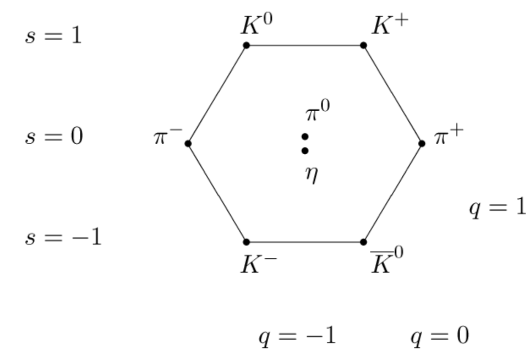
$$p \sim \frac{1}{2}\bar{I}(\lambda_4 - i\lambda_5)\ell \quad \sim S^+\nu$$

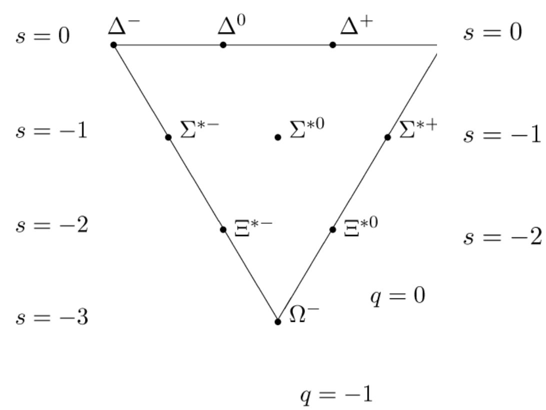
$$n \sim \frac{1}{2}\bar{I}(\lambda_6 - i\lambda_7)\ell \quad \sim S^+e^-$$

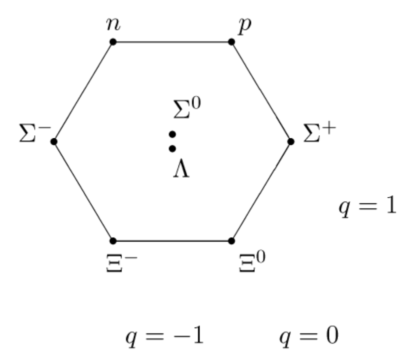
$$\Xi^0 \sim \frac{1}{2}\bar{I}(\lambda_6 + i\lambda_7)\ell \quad \sim D^+\mu^-$$

$$\Xi^- \sim \frac{1}{2}\bar{I}(\lambda_4 + i\lambda_5)\ell \quad \sim D^0\mu^-$$

$$\Lambda \sim \frac{1}{\sqrt{2}}\bar{I}\lambda_8\ell \quad \sim (D^0\nu + D^+e^- - 2S^+\mu^-)/\sqrt{6} \qquad (3.5)$$

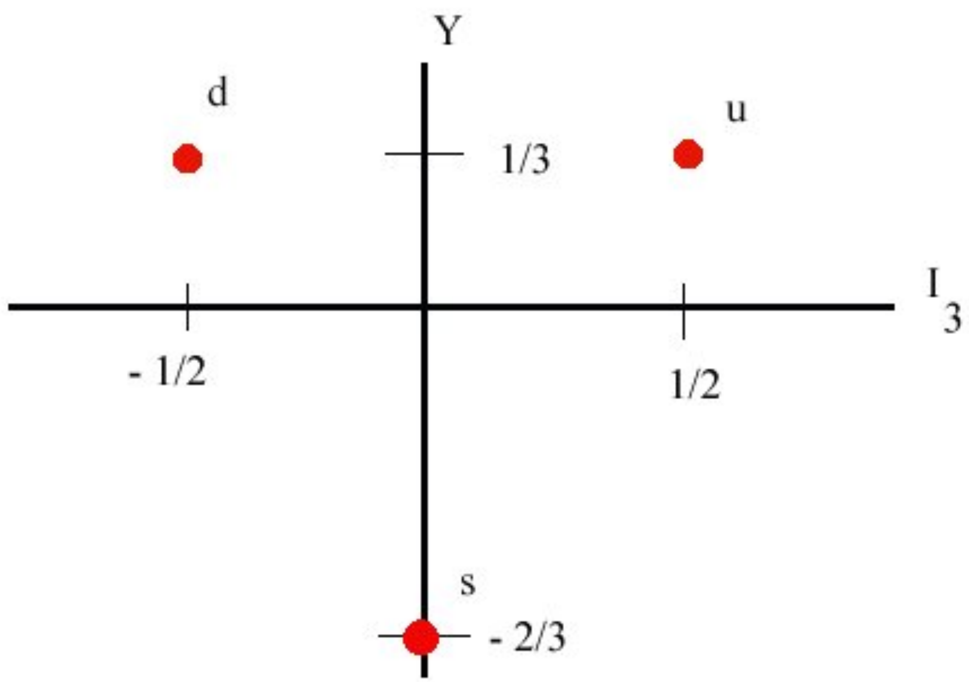






Internal Structure - 1964

- SU(3)
- Murray Gell-Mann and George Zweig

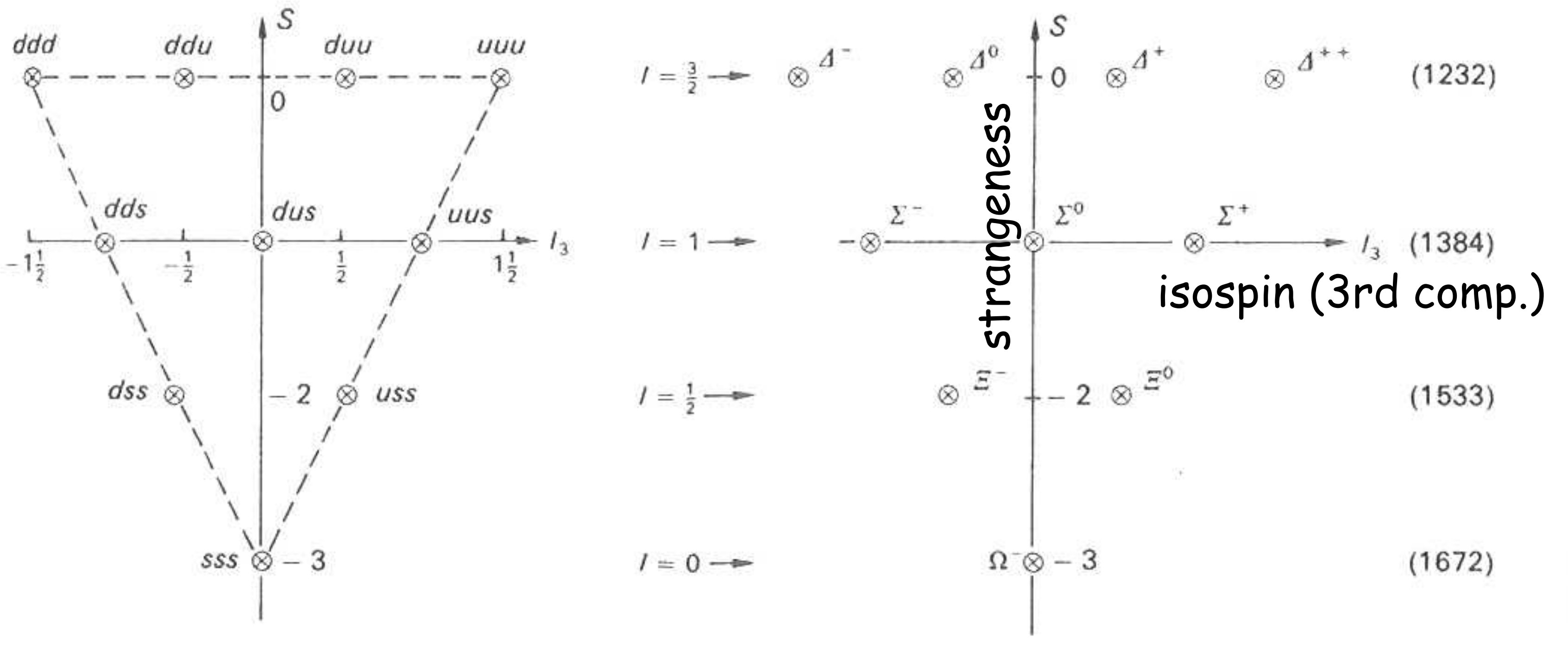


$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A$$

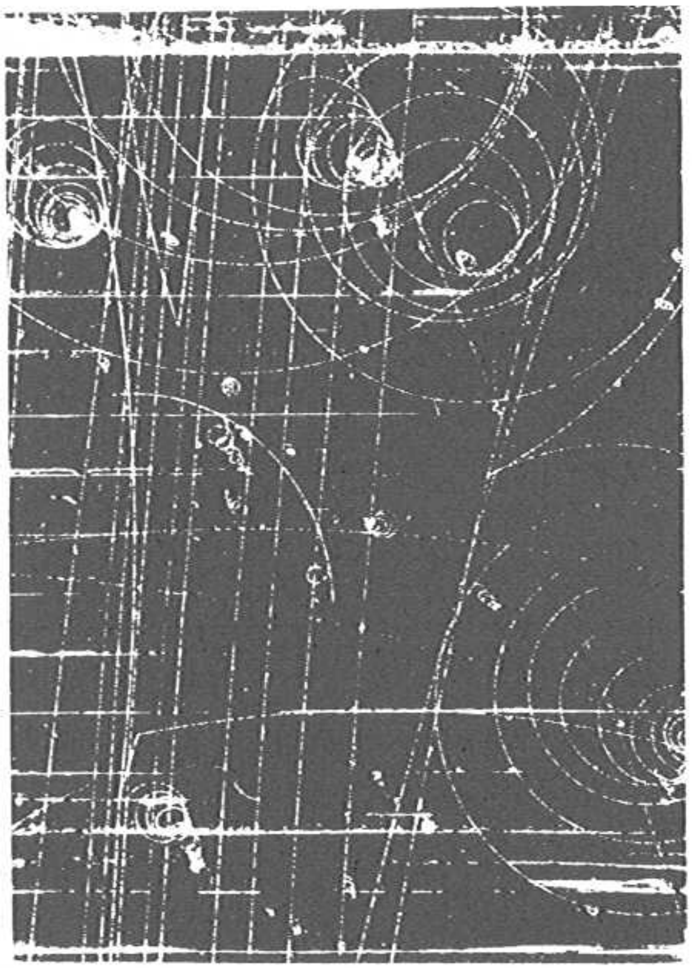
Baryon Decuplet

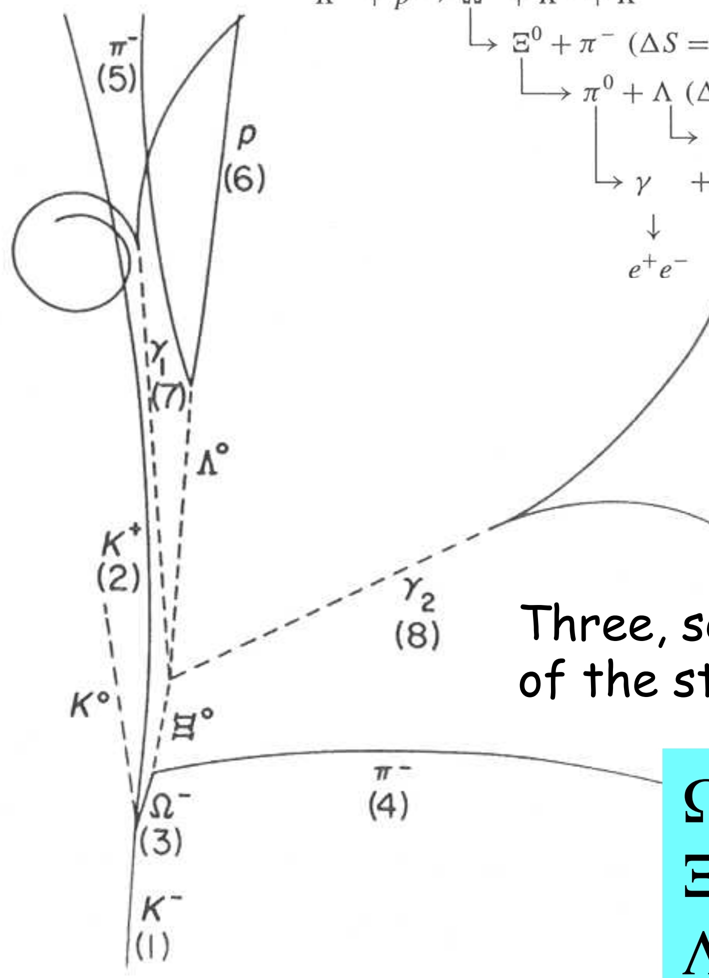
- Lightest spin 3/2 baryons



Before the Ω^- was discovered, it (and its properties) were predicted by this pattern

Discovery of the Ω^-





$K^- + p \rightarrow \Omega^- + K^+ + K^0$
$\quad \hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)
$\quad\quad \hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ weak decay)
$\quad\quad\quad \hookrightarrow \pi^- + p$ ($\Delta S = 1$ weak decay)
$\quad\quad \hookrightarrow \gamma + \gamma$ (electromagnetic decay)
$\quad\quad\quad \downarrow \quad \downarrow$
$\quad\quad\quad e^+e^- \quad e^+e^-$

Three, sequential decays of the strange quark

$\Omega^- = sss$
$\Xi^0 = ssu$
$\Lambda = sud$

Baryon Decuplet

- Notice the masses

  $M(\Delta) = 1232$

  $M(\Sigma) = 1384$

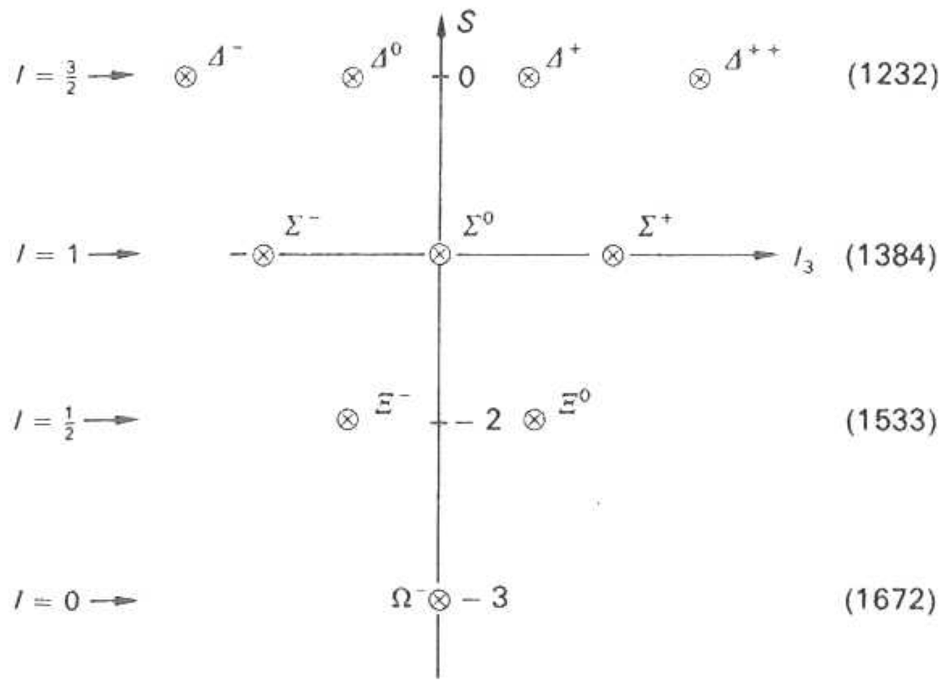
  $= M(\Delta) + 152$

  $M(\Xi) = 1533$

  $= M(\Sigma) + 149$

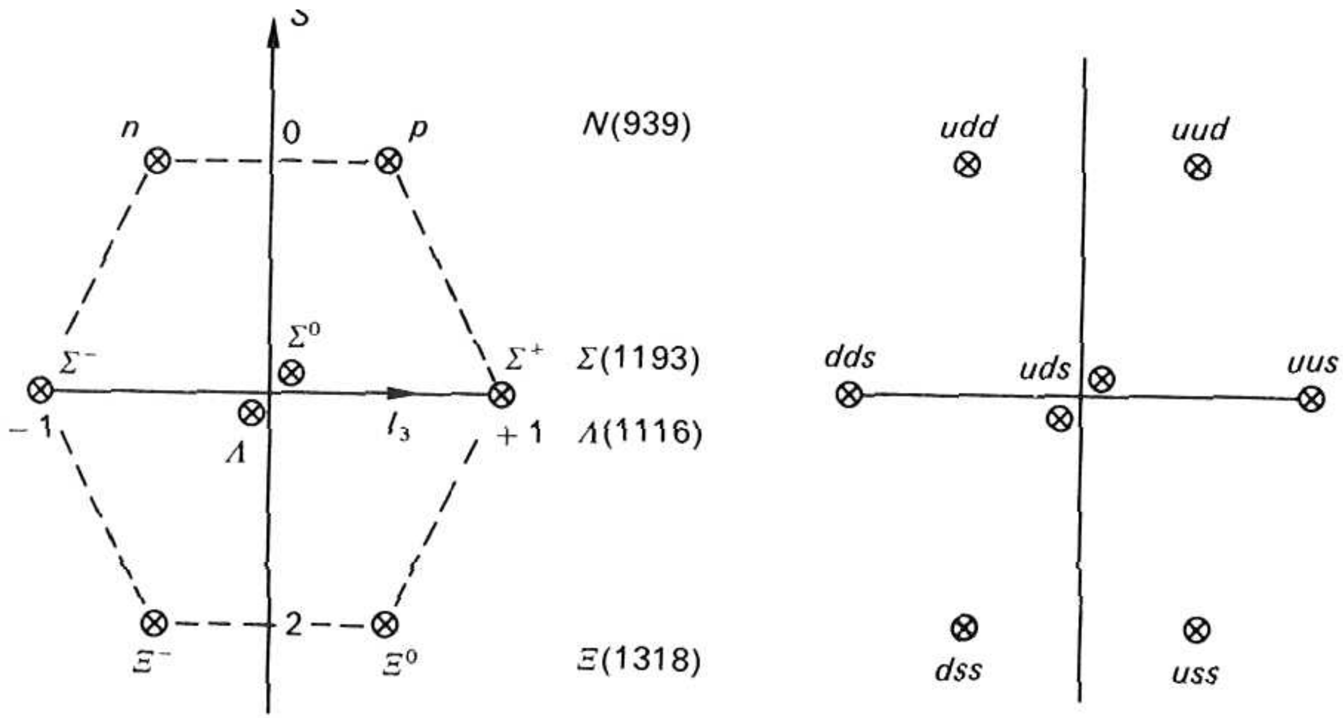
  $M(\Omega) = 1672$

  $= M(\Xi) + 139$



We see an orderly increase of mass with number of strange quarks

Baryon Octet



Mass Splittings

The mass splittings between members of isospin multiplets, as expected, are order of:

$$\frac{\Delta m}{m} \simeq \alpha \simeq 10^{-2}$$

	Δm (MeV/c^2)	m_{av} (MeV/c^2)	$10^3\ \Delta m/m$
$n - p$	1.3	939	1.4
$\Sigma^0 - \Sigma^+$	3.1	1190	2.6
$\Sigma^- - \Sigma^0$	4.9	1195	4.1
$\Xi^- - \Xi^0$	6.5	1318	4.9
$K^0 - K^\pm$	4.0	495	8.1
$\pi^\pm - \pi^0$	4.6	140	33

Note that particle and antiparticle must have identical mass (CTP Theorem). However Σ^+ and Σ^- have different masses, since they are both baryons, rather then baryon and antibaryon.

From L. Spogli, http://webusers.fis.uniroma3.it/~spogli/Isospin.pdf

Quark Spin and Color

- Consider the Δ^{++}
  - spin 3/2   (uuu)
  - therefore   u↑u↑u↑
    - now, this appears to violate the Pauli principle
      - two or more identical fermions cannot exist in the same quantum state
    - resolution, another quantum number (color) and each of the u quarks have a different value:

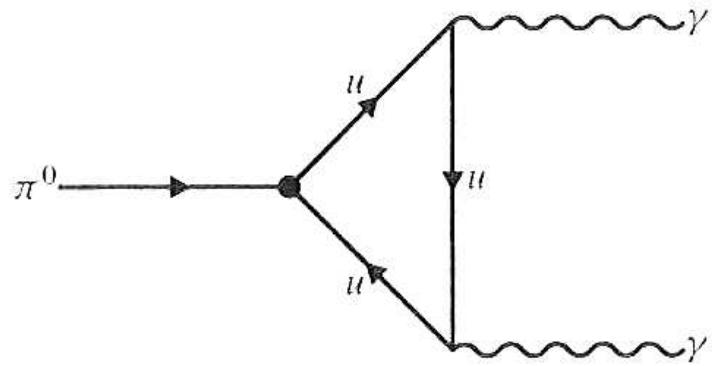
      u↑(red)u↑(green)u↑(blue)

      and we have to aniti-symmetrize the color

      u↑u↑u↑ (rgb-rbg+brg-bgr+gbr-grb)

π^0 Lifetime and Color

- We also know from the rate of decay of the π^0 that there are three colors



$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.73 \text{ eV } (N_c/3)^2$
$\Gamma(\text{observed}) = 7.76 \pm 0.6 \text{ eV}$
$N_c = 2.99 \pm 0.12$

- Also the rate of $e^+e^- \rightarrow$ hadrons tells us $N_c = 3$

π^0 Lifetime and color

- Theory: $\Gamma(\pi^0 \to \gamma\gamma) = 7.73 \text{ eV } (N_c/3)^2$

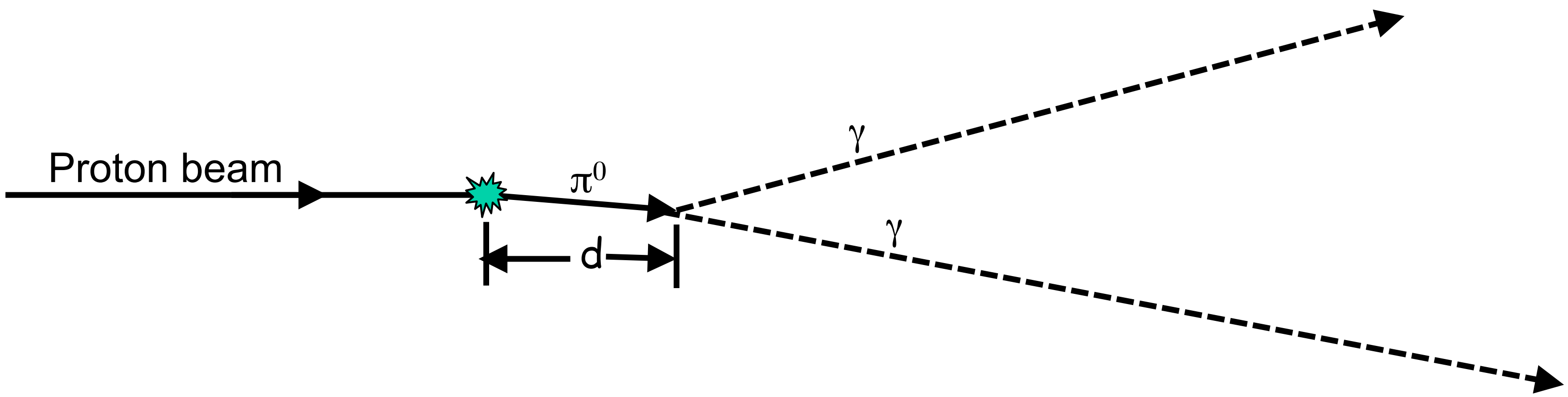
$$\tau = \hbar/\Gamma = 197 \text{ MeV-fm}/(3 \times 10^{23} \text{ fm/s } \Gamma)$$
$$= 6.6 \times 10^{-16} \text{ s } / \Gamma(\text{eV})$$
$$= 8.5 \times 10^{-17} \text{ s } (3/N_c)^2$$

$$d = \gamma\,\beta\,c\,\tau = 2.6 \times 10^{-2}\ \mu\text{m } (p/m)\ (3/N_c)^2$$

Suppose p = 5 GeV, then $d \approx 1\ \mu\text{m } (3/N_c)^2$

π^0 Lifetime Experiment

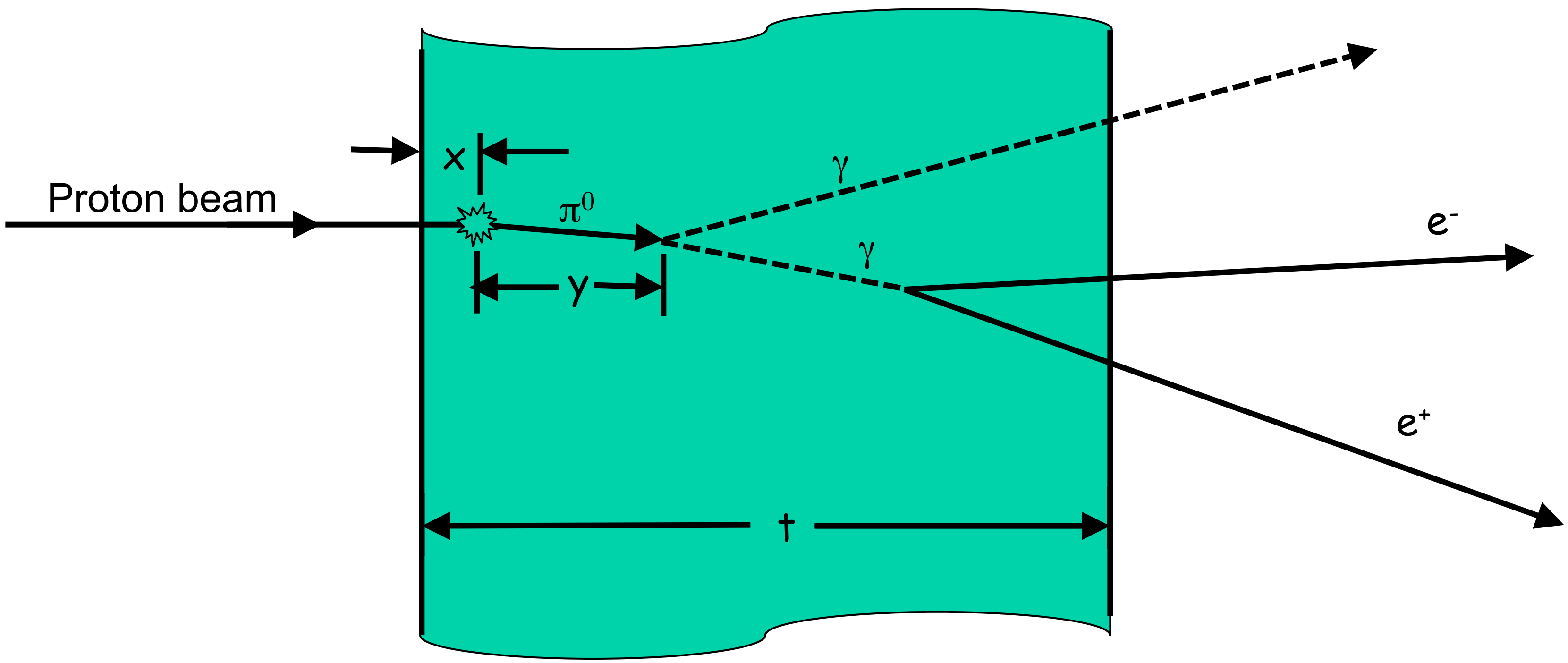
$$p + A \rightarrow \pi^0 + X$$
$$\pi^0 \rightarrow \gamma\gamma$$



$d \sim 1\ \mu m$ for $p = 5$ GeV and $N_c=3$

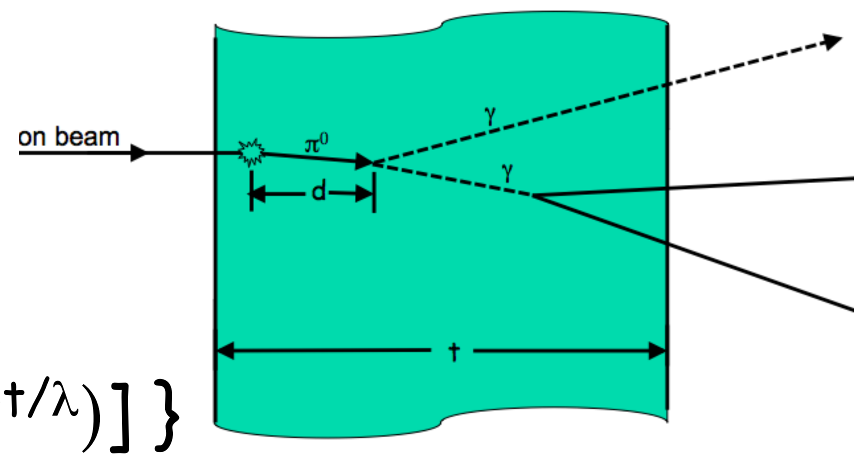
π^0 Lifetime Experiment

Technique - use thin foils for target, convert photons into e^+e^- pairs, and count as function of target thickness (t)



π^0 Lifetime Experiment

1. Production rate of π^0 is K dx
2. Then, π^0s decay with decay length λ
3. Pairs are produced immediately as Dalitz pairs in fraction B of decays, or appear in conversion with prob dy/X_0
4. Prob. that either photon converts in the foil is then $(t-y-x)/X$



Pair Production Rate

$$R(t) = Kt\{ B + 1/X_0[t/2-\lambda+\lambda^2/t(1-e^{-t/\lambda})] \}$$

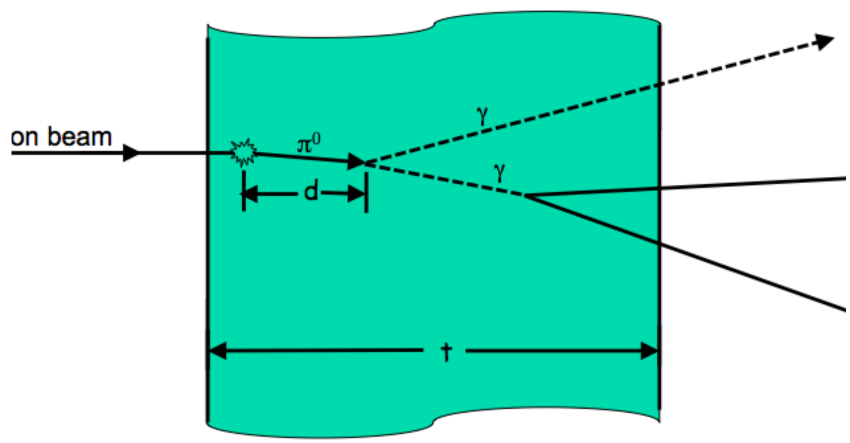
π^0 Lifetime Experiment

$R(t) = Kt\{ B + 1/X_0[t/2-\lambda+\lambda^2/t(1-e^{-t/\lambda})] \}$

Thin foil $\Rightarrow$ $t \ll \lambda$

and $\lambda/x \sim 1\ \mu m/1\ cm < B \sim 10^{-2}$

$R(t) \approx Kt\ (B + t^2/6\lambda X_0)$



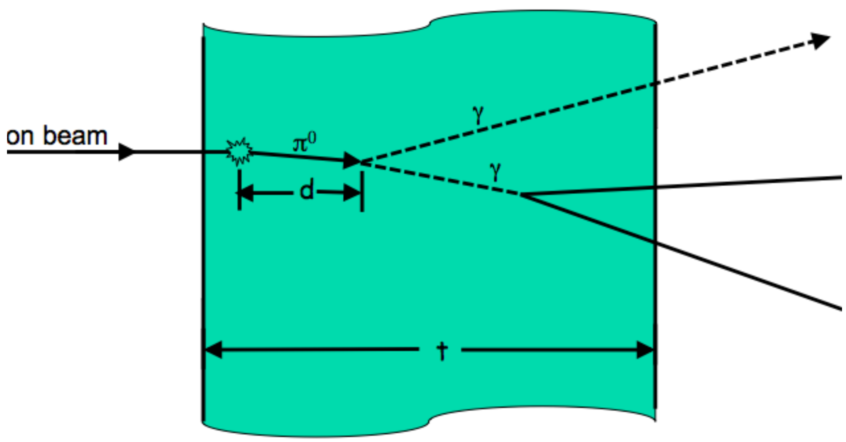
π^0 Lifetime Experiment

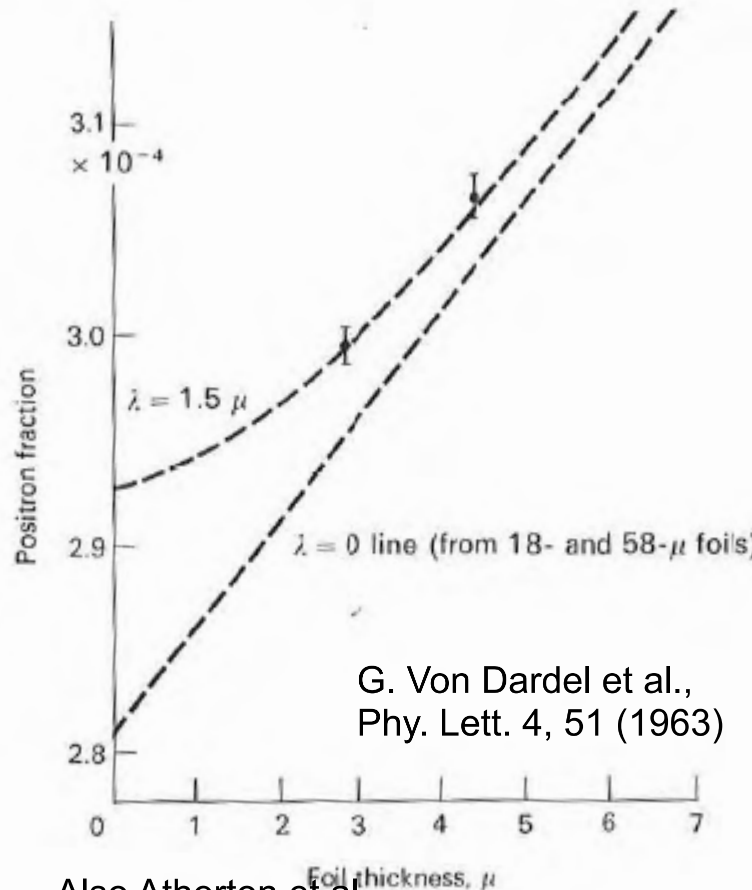
$R(t) \approx Kt\,(B + t^2/6\lambda X_0)$

$t = 3, 4, 18, 58\ \mu m$

(platinum)

$p = 5\ GeV/c$



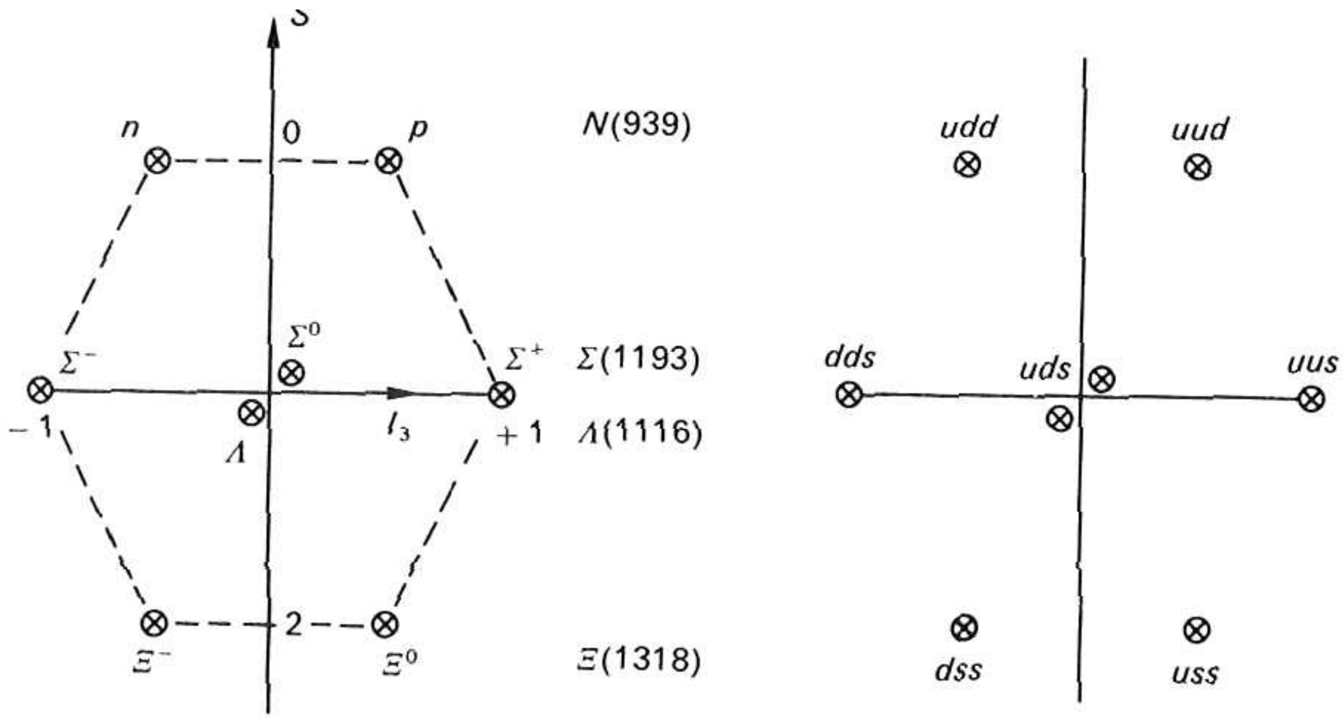


G. Von Dardel et al.,
Phy. Lett. 4, 51 (1963)

Also Atherton et al.,
Phy. Lett. 158B, 81 (1985)

Part. Data Group $\tau = 8.52 \pm 0.18 \times 10^{-17}$ sec ⇒ $N_C = 3$

Baryon Octet



Baryon Octet

- Multiplet including the neutron and proton
  - lightest spin 1/2 baryons
  - wavefunction symmetric under simultaneous interchange of flavor and spin
  - total wave-function must be anti-symmetric
    - (flavor)(spin)(color)(space)
    - color is anti-symmetric for all hadrons because they are color-neutral, singlet states
    - space is symmetric because L=0
    - $\therefore$ (flavor)(spin) is symmetric

Baryon Octet

- Consider the proton (uud)
  - two quarks in the spin singlet state
    - $(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$ (anti-symmetric)
  - these quarks must also be in an anti-symmetric flavor state for an overall symmetric flavor-spin wavefunction
    - $(ud-du)/\sqrt{2}$ (anti-symmetric)
  - Third Quark is spin up

    $(u\uparrow d\downarrow - u\downarrow d\uparrow - d\uparrow u\downarrow + d\downarrow u\uparrow)\, u\uparrow$

Baryon Octet

- Now symmetrize by making cyclic permutation

$$(2u\uparrow u\uparrow d\downarrow + 2d\downarrow u\uparrow u\uparrow + 2u\uparrow d\downarrow u\uparrow$$
$$- u\downarrow d\uparrow u\uparrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow$$
$$- d\uparrow u\downarrow u\uparrow - u\uparrow d\uparrow u\downarrow - d\uparrow u\uparrow u\downarrow) / \sqrt{18}$$

Magnetic Moments of Baryons

- Magnetic moments of the nucleons
  - proton 2.793 nuclear magnetons
  - neutron - 1.913 nuclear magnetons
  - [nuclear magneton = $e\hbar/(2Mc)$]

- Recall the proton wave function:

$$(2u{\uparrow}u{\uparrow}d{\downarrow} + 2d{\downarrow}u{\uparrow}u{\uparrow} + 2u{\uparrow}d{\downarrow}u{\uparrow}$$
$$- u{\downarrow}d{\uparrow}u{\uparrow} - u{\uparrow}u{\downarrow}d{\uparrow} - u{\downarrow}u{\uparrow}d{\uparrow}$$
$$- d{\uparrow}u{\downarrow}u{\uparrow} - u{\uparrow}d{\uparrow}u{\downarrow} - d{\uparrow}u{\uparrow}u{\downarrow}) / \sqrt{18}$$

Magnetic Moments of Baryons

proton: $(2u\uparrow u\uparrow d\downarrow + 2d\downarrow u\uparrow u\uparrow + 2u\uparrow d\downarrow u\uparrow$
$- u\downarrow d\uparrow u\uparrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow$
$- d\uparrow u\downarrow u\uparrow - u\uparrow d\uparrow u\downarrow - d\uparrow u\uparrow u\downarrow) / \sqrt{18}$

$\mu_p = 12/18\ (2\mu_u - \mu_d) + 6/18\ (\mu_d)$
$= 4/3\ \mu_u - 1/3\ \mu_d$
$\mu_n = 4/3\ \mu_d - 1/3\ \mu_u$

$\mu_u = -\ 2\ \mu_d$

$\mu_p = 4/3\ \mu_u - 1/3(-1/2\ \mu_u) = 3/2\ \mu_u$
$\mu_n = 4/3(-1/2\ \mu_u) - 1/3\ \mu_u = -\ \mu_u = -2/3\ \mu_p$

Magnetic Moments of Baryons

proton: $(2u\uparrow u\uparrow d\downarrow + 2d\downarrow u\uparrow u\uparrow + 2u\uparrow d\downarrow u\uparrow$
$- u\downarrow d\uparrow u\uparrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow$
$- d\uparrow u\downarrow u\uparrow - u\uparrow d\uparrow u\downarrow - d\uparrow u\uparrow u\downarrow) / \sqrt{18}$

$\mu_p = 4/3\ \mu_u - 1/3\ \mu_d = 3/2\ \mu_u$

$\mu_n = 4/3\ \mu_d - 1/3\ \mu_u = -\mu_u = -2/3\ \mu_p$

Consider other octet baryons –

Easiest are those with two identical quarks:

Σ^- (dds), Σ^+ (uus), Ξ^-(dss), Ξ^0 (uss)

Magnetic Moments of Baryons

consider Σ^+ (uus) - replace d in proton with s

proton: $(2u\uparrow u\uparrow d\downarrow + 2d\downarrow u\uparrow u\uparrow + 2u\uparrow d\downarrow u\uparrow$

$- u\downarrow d\uparrow u\uparrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow$

$- d\uparrow u\downarrow u\uparrow - u\uparrow d\uparrow u\downarrow - d\uparrow u\uparrow u\downarrow) / \sqrt{18}$

Σ^+: $(2u\uparrow u\uparrow s\downarrow + 2s\downarrow u\uparrow u\uparrow + 2u\uparrow s\downarrow u\uparrow$

$- u\downarrow s\uparrow u\uparrow - u\uparrow u\downarrow s\uparrow - u\downarrow u\uparrow s\uparrow$

$- s\uparrow u\downarrow u\uparrow - u\uparrow s\uparrow u\downarrow - s\uparrow u\uparrow u\downarrow) / \sqrt{18}$

$\mu_p = 4/3\ \mu_u - 1/3\ \mu_d = 3/2\ \mu_u$

$\mu(\Sigma^+) = 4/3\ \mu_u - 1/3\ \mu_s$

Magnetic Moments of Baryons

With similar arguments, we find for the other baryons with two identical quarks:

$$\mu(\Sigma^-) = 4/3\ \mu_d - 1/3\ \mu_s$$

$$\mu(\Xi^0) = 4/3\ \mu_s - 1/3\ \mu_u$$

$$\mu(\Xi^-) = 4/3\ \mu_s - 1/3\ \mu_d$$

Magnetic Moments of Baryons

consider Σ^0 (uds)

get Σ^0 from $I^-(\Sigma^+)$ [$I^-(u) = d/\sqrt{2}$, $I^-(d) = 0$]

$$I^-(\Sigma^+) = I^-(2u\uparrow u\uparrow s\downarrow + 2s\downarrow u\uparrow u\uparrow + 2u\uparrow s\downarrow u\uparrow$$
$$- u\downarrow s\uparrow u\uparrow - u\uparrow u\downarrow s\uparrow - u\downarrow u\uparrow s\uparrow$$
$$- s\uparrow u\downarrow u\uparrow - u\uparrow s\uparrow u\downarrow - s\uparrow u\uparrow u\downarrow) / \sqrt{18}$$

$$= (2d\uparrow u\uparrow s\downarrow + 2u\uparrow d\uparrow s\downarrow + 2s\downarrow d\uparrow u\uparrow + 2s\downarrow u\uparrow d\uparrow$$
$$+ 2d\uparrow s\downarrow u\uparrow + 2u\uparrow s\downarrow d\uparrow - d\downarrow s\uparrow u\uparrow - u\downarrow s\uparrow d\uparrow$$
$$- d\uparrow u\downarrow s\uparrow - u\uparrow d\downarrow s\uparrow - d\downarrow u\uparrow s\uparrow - u\downarrow d\uparrow s\uparrow$$
$$- s\uparrow d\downarrow u\uparrow - s\uparrow u\downarrow d\uparrow - d\uparrow s\uparrow u\downarrow - u\uparrow s\uparrow d\downarrow$$
$$- s\uparrow d\uparrow u\downarrow - s\uparrow u\uparrow d\downarrow) / \sqrt{36}$$

Magnetic Moments of Baryons

$$\Sigma^0 = (2d\uparrow u\uparrow s\downarrow + 2u\uparrow d\uparrow s\downarrow + 2s\downarrow d\uparrow u\uparrow + 2s\downarrow u\uparrow d\uparrow$$
$$+ 2d\uparrow s\downarrow u\uparrow + 2u\uparrow s\downarrow d\uparrow - d\downarrow s\uparrow u\uparrow - u\downarrow s\uparrow d\uparrow$$
$$- d\uparrow u\downarrow s\uparrow - u\uparrow d\downarrow s\uparrow - d\downarrow u\uparrow s\uparrow - u\downarrow d\uparrow s\uparrow$$
$$- s\uparrow d\downarrow u\uparrow - s\uparrow u\downarrow d\uparrow - d\uparrow s\uparrow u\downarrow - u\uparrow s\uparrow d\downarrow$$
$$- s\uparrow d\uparrow u\downarrow - s\uparrow u\uparrow d\downarrow) / \sqrt{36}$$

$$\mu(\Sigma^0) = [6 \times 4\ (\mu_u + \mu_d - \mu_s) + 12 \times \mu_s]/36$$
$$= [2\ \mu_u + 2\ \mu_d - \mu_s]/3$$

Magnetic Moments of Baryons

Baryon	Magnetic moment in quark model	Predicted, n.m.	Observed, n.m.
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	+2.79	+2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	−1.86	−1.913
Λ	μ_s	−0.61	-0.614 ± 0.005
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	+2.68	$+2.46 \pm 0.01$
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	−1.04	-1.16 ± 0.03
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	−1.44	-1.25 ± 0.014
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	−0.51	-0.65 ± 0.01
Ω^-	$3\mu_s$	−1.84	-2.02 ± 0.05

m_n (= m_u = m_d) = 336 MeV

m_s = 509 MeV

Magnetic Moments of Baryons

Dirac:

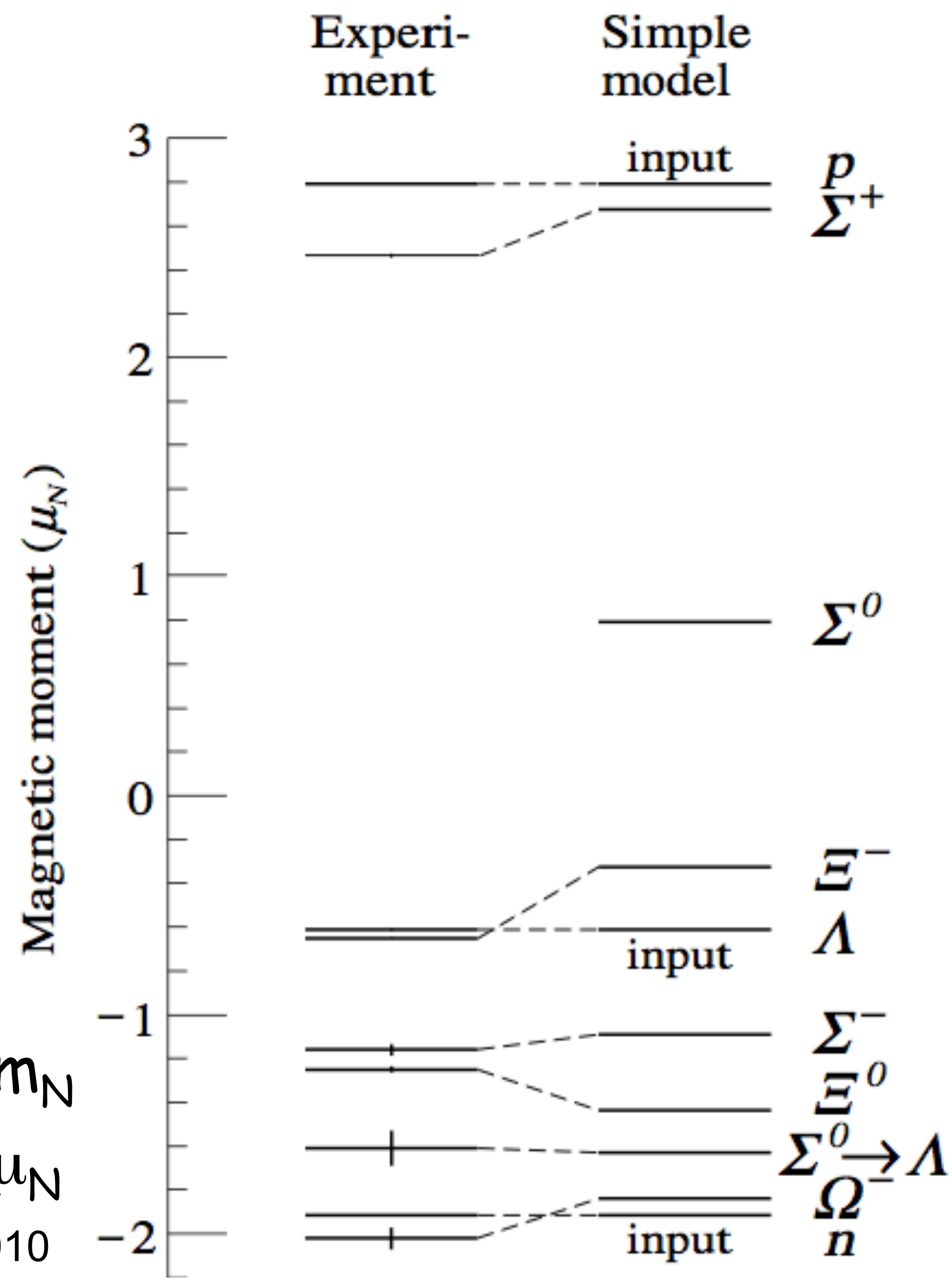
- $\mu = qS/m$

for spin $\frac{1}{2}$:

- $= q/2m$

up: $q_u = 2/3\ e$ $m_u \approx 1/3\ m_N$

so $\mu_u \approx 2\ \mu_N$ & $\mu_d \approx -\mu_N$

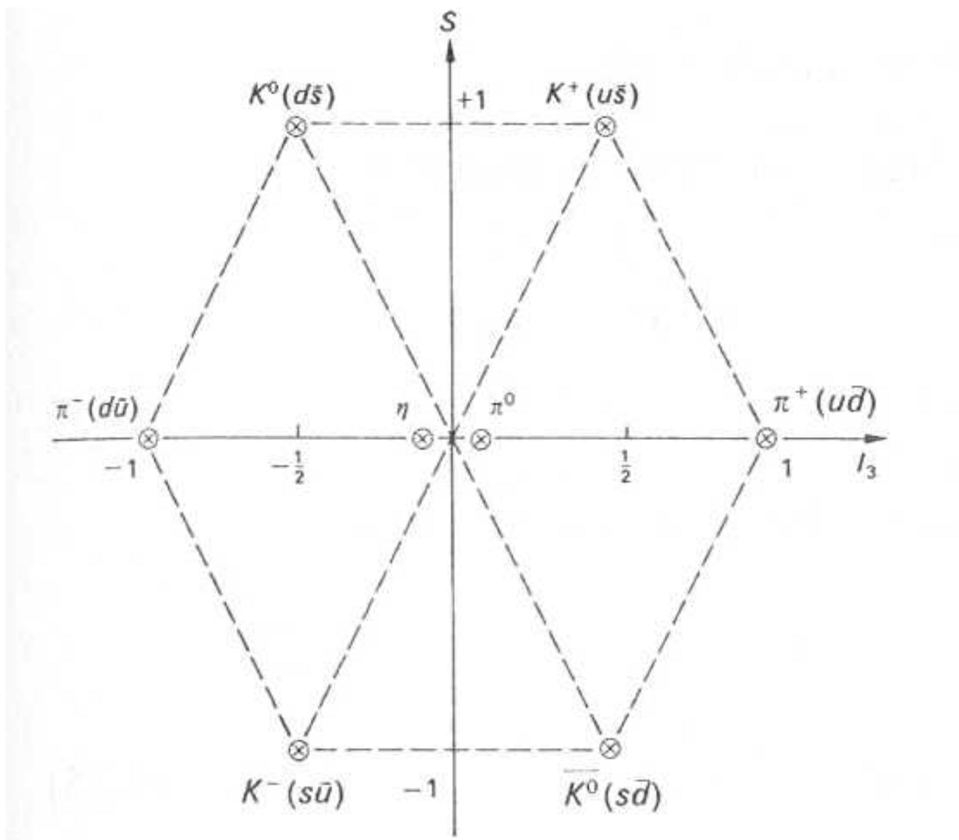


Particle Data Group 2010

$$\mu_u = +1.852\,\mu_N,\ \mu_d = -0.972\,\mu_N,\ \text{and}\ \mu_s = -0.613\,\mu_N$$

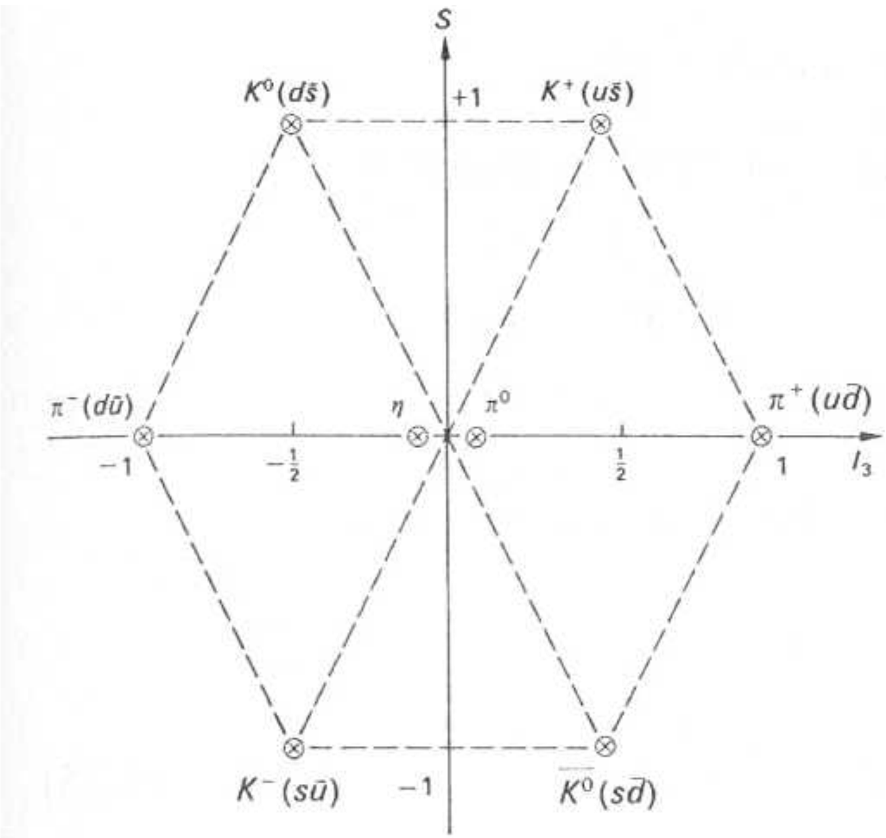
Light Pseudoscalar Mesons

- Combine a quark and an antiquark
  - $3 \otimes \bar{3} = 8 \oplus 1$



Light Pseudoscalar Mesons

- Combine a quark and an antiquark
  - $3 \otimes \bar{3} = 8 \oplus 1$



The strangeless mesons

I	I_3	Wavefunction	Q/e
1	1	$u\bar{d} = \pi^+$	+1
1	−1	$-\bar{u}d = \pi^-$	−1
1	0	$\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}) = \pi^0$	0
0	0	$\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u}) = \eta$	0

This table ignores the strange quarks

Light Pseudoscalar Mesons

- C operation on quarks
  - (Condon-Shortly convention)

	Nucleons		Quarks	
I_3	particle	antiparticle	particle	antiparticle
$+\frac{1}{2}$	p	$+\bar{n}$	u	$+\bar{d}$
$-\frac{1}{2}$	n	$-\bar{p}$	d	$-\bar{u}$

Light Pseudoscalar Mesons

	I	I_3	S	Meson	Quark combination	Decay	Mass, MeV
octet	1	1	0	π^+	$u\bar{d}$	$\pi^\pm \to \mu\nu$	140
	1	-1	0	π^-	$d\bar{u}$		
	1	0	0	π^0	$\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$\pi^0 \to 2\gamma$	135
	$\frac{1}{2}$	$\frac{1}{2}$	$+1$	K^+	$u\bar{s}$	$K^+ \to \mu\nu$	494
	$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	K^0	$d\bar{s}$	$K^0 \to \pi^+\pi^-$	498
	$\frac{1}{2}$	$-\frac{1}{2}$	-1	K^-	$\bar{u}s$	$K^- \to \mu\nu$	494
	$\frac{1}{2}$	$\frac{1}{2}$	-1	$\bar{K}^0$	$\bar{d}s$	$\bar{K}^0 \to \pi^+\pi^-$	498
	0	0	0	η_8	$\frac{1}{\sqrt{6}}(d\bar{d} + u\bar{u} - 2s\bar{s})$	$\eta \to 2\gamma$	549
singlet	0	0	0	η_0	$\frac{1}{\sqrt{3}}(d\bar{d} + u\bar{u} + s\bar{s})$	$\eta' \to \eta\pi\pi$ $\to 2\gamma$	958

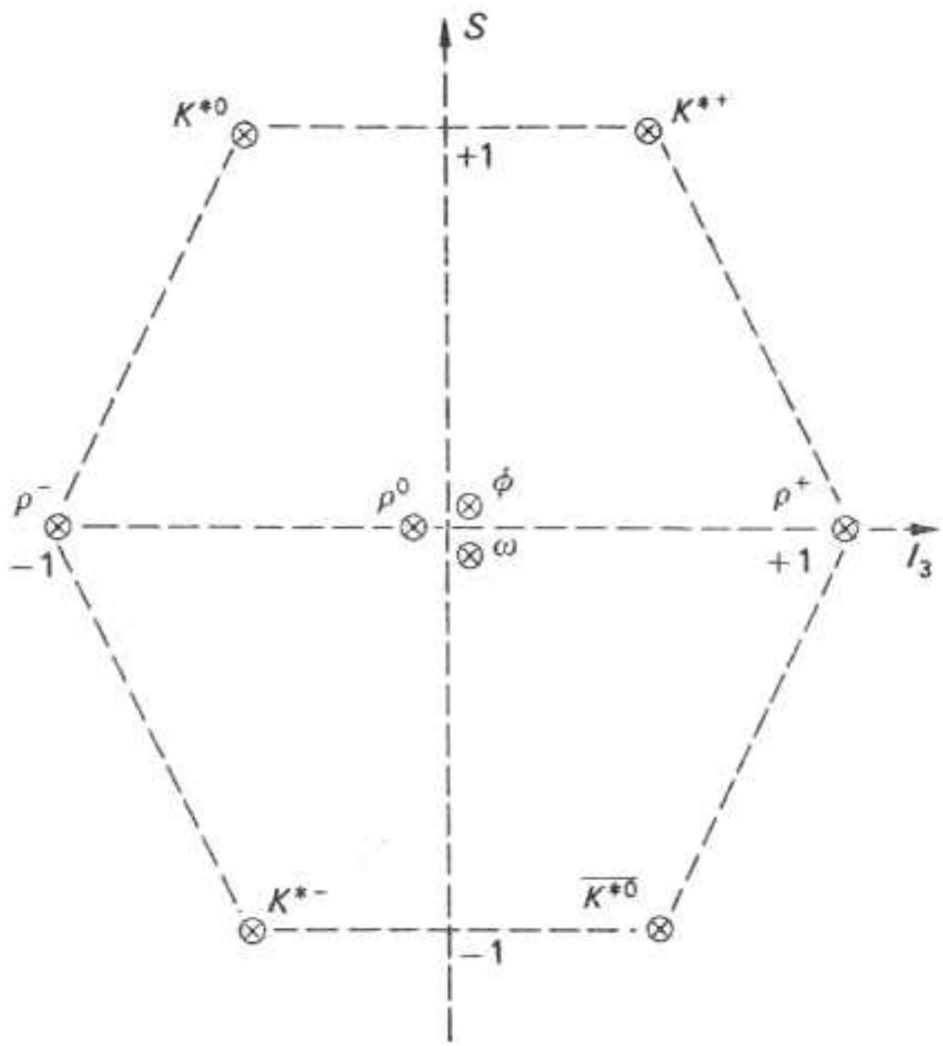
Light Pseudoscalar Mesons

Gell-Mann Okubo Mass Formula

$$4(M_K)^2 = M_\pi^2 + 3\ M_\eta^2$$

$$0.988\ \text{GeV}^2 = 0.924\ \text{GeV}^2$$

The light vector mesons



Mass, MeV	Dominant decay mode
776	$\rho \to 2\pi$
892	$K^* \to K\pi$
783	$\omega \to 3\pi$
1019	$\phi \to K\bar{K}$

The light vector mesons

$\phi(1020)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	K^+K^-	(49.4 ± 0.7) %
Γ_2	$K_L^0 K_S^0$	(33.6 ± 0.6) %
Γ_3	$\rho\pi + \pi^+\pi^-\pi^0$	(15.5 ± 0.6) %
Γ_4	$\rho\pi$	
Γ_5	$\pi^+\pi^-\pi^0$	
Γ_6	$\eta\gamma$	(1.298 ± 0.029) %
Γ_7	$\pi^0\gamma$	$(1.24 \pm 0.10) \times 10^{-3}$
Γ_8	e^+e^-	$(2.96 \pm 0.05) \times 10^{-4}$
Γ_9	$\mu^+\mu^-$	$(2.9 \pm 0.4) \times 10^{-4}$
Γ_{10}	ηe^+e^-	$(1.3 {}^{+0.8}_{-0.6}) \times 10^{-4}$
Γ_{11}	$\pi^+\pi^-$	$(7.3 \pm 1.3) \times 10^{-5}$

$\omega(782)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi^+\pi^-\pi^0$	(88.6 ± 0.7) %
Γ_2	$\pi^0\gamma$	(8.66 ± 0.35) %
Γ_3	$\pi^+\pi^-$	(2.23 ± 0.30) %
Γ_4	neutrals (excluding $\pi^0\gamma$)	$(4.8 {}^{+7.7}_{-3.1}) \times 10^{-3}$
Γ_5	$\eta\gamma$	$(6.5 \pm 1.0) \times 10^{-4}$
Γ_6	$\pi^0 e^+e^-$	$(5.9 \pm 1.9) \times 10^{-4}$
Γ_7	$\pi^0\mu^+\mu^-$	$(9.6 \pm 2.3) \times 10^{-5}$
Γ_8	e^+e^-	$(6.97 \pm 0.13) \times 10^{-5}$

The light vector mesons

- "ideal mixing"

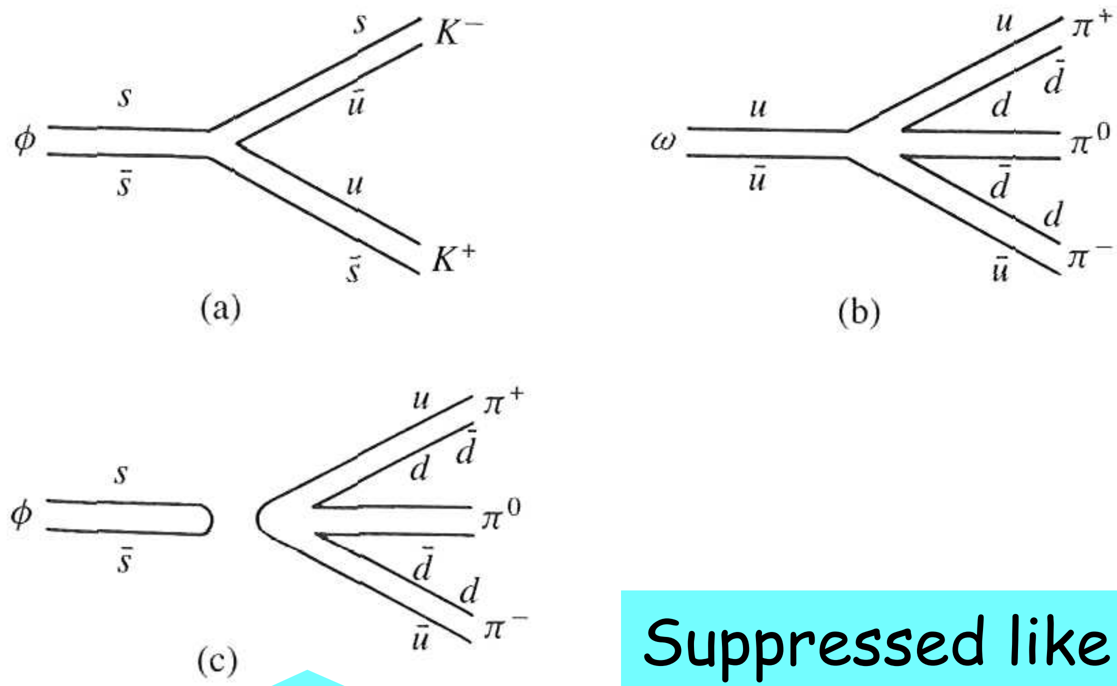
$\phi_8 = (u\bar{u} + d\bar{d} - 2s\bar{s}) / \sqrt{6}$

$\phi_0 = (u\bar{u} + d\bar{d} + s\bar{s}) / \sqrt{3}$

$\phi = (\phi_0 - \sqrt{2}\,\phi_8) / \sqrt{3} = s\bar{s}$

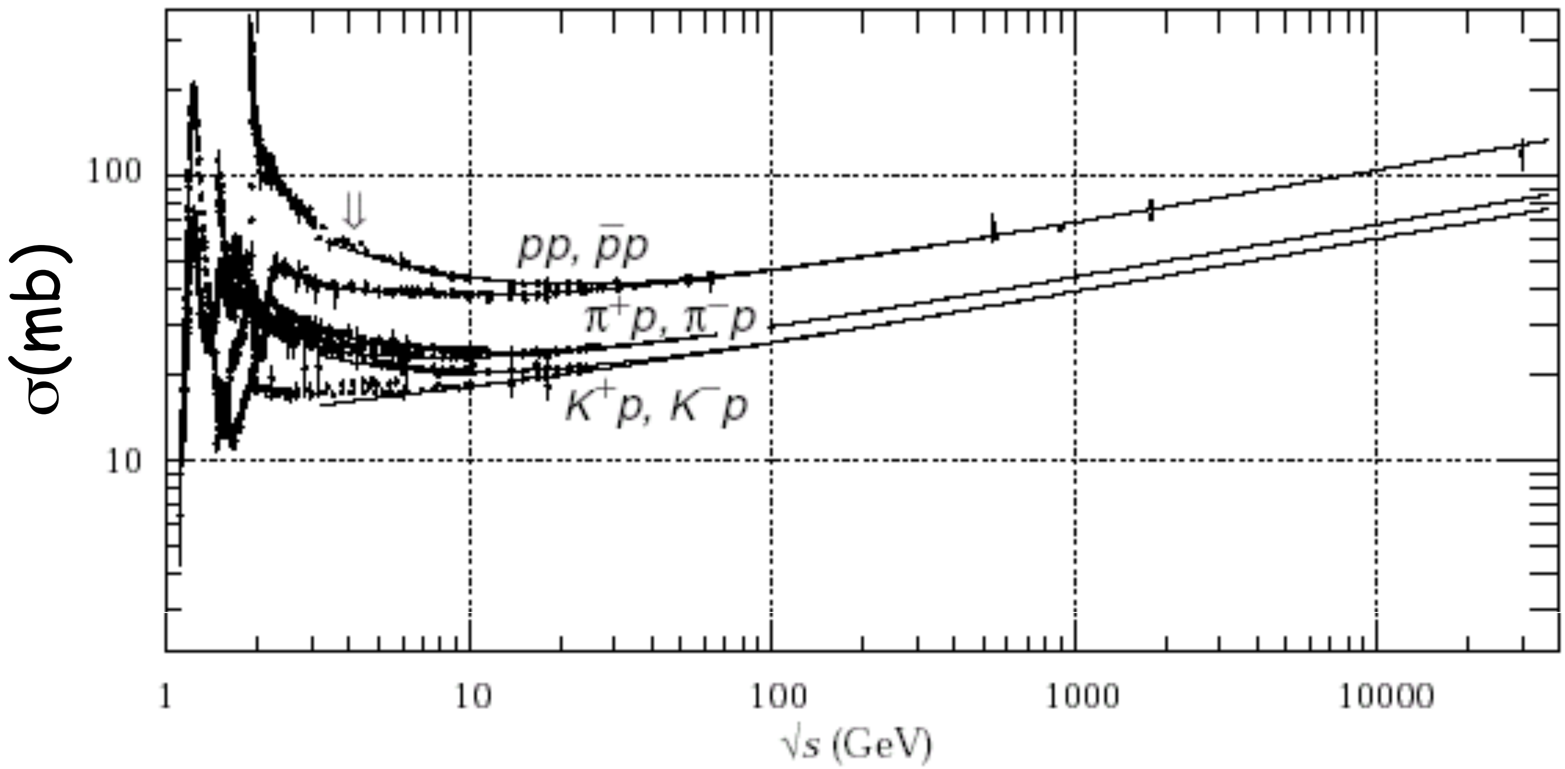
$\omega = (\phi_8 + \sqrt{2}\,\phi_0) / \sqrt{3} = (d\bar{d} + u\bar{u}) / \sqrt{2}$

The light vector mesons



Suppressed like J/ψ, because of unconnected quark lines (OZI rule)

Pion-nucleon cross-sections

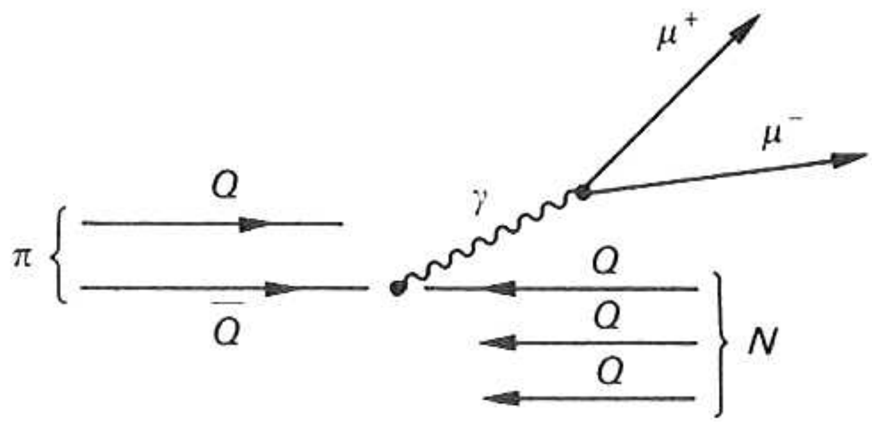


$\sigma(\pi p) = 2/3\ \sigma(pp)$

Additive quark model

Lepton Pair Production on Isoscalar Targets

- Drell-Yan production



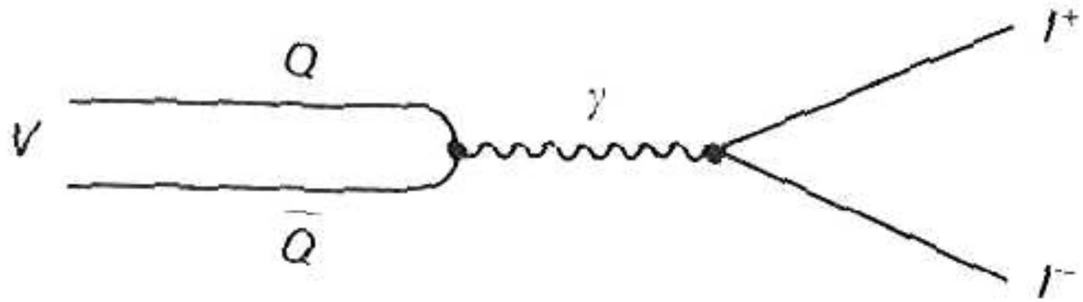
$$\sigma(\pi^- C \to \mu^+\mu^- + \ ...\ ) \sim 18\ Q_u^2 = 18(4/9)$$
$$\sigma(\pi^+ C \to \mu^+\mu^- + \ ...\ ) \sim 18\ Q_d^2 = 18(1/9)$$

since $\pi^- = \bar{u}d$

$\pi^+ = \bar{d}u$

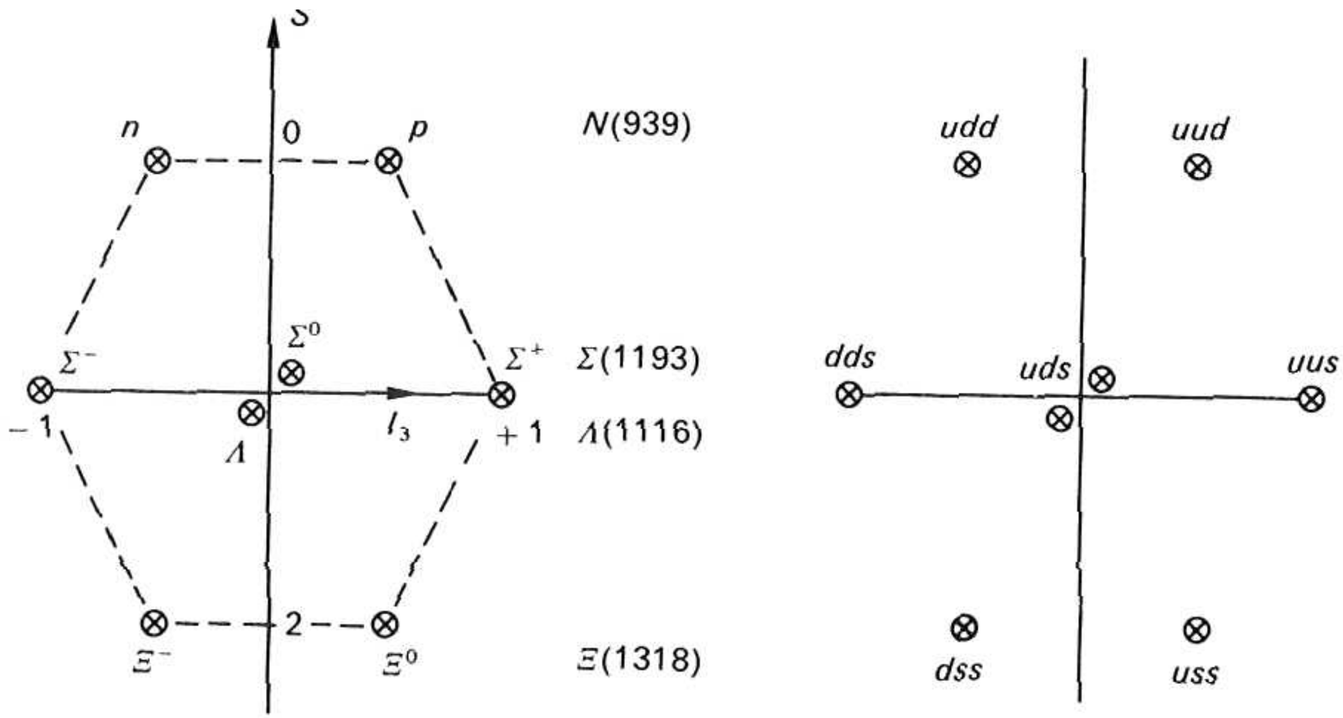
$C = 18u + 18d$

Vector Meson Decay to Leptons



Meson	Quark wavefunction	$\lvert\Sigma a_i Q_i\rvert^2$	$\Gamma_{e^+e^-}$, keV	$\Gamma_{e^+e^-}/\lvert\Sigma a_i Q_i\rvert^2$
ρ	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	$\frac{1}{2}$	6.8 ± 0.3	13.6 ± 0.6
ω	$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	$\frac{1}{18}$	0.60 ± 0.02	10.8 ± 0.4
ϕ	$s\bar{s}$	$\frac{1}{9}$	1.37 ± 0.05	12.3 ± 0.5
ψ	$c\bar{c}$	$\frac{4}{9}$	5.3 ± 0.4	11.9 ± 0.9
Υ	$b\bar{b}$	$\frac{1}{9}$	1.32 ± 0.05	11.9 ± 0.5

Baryon Octet



Baryon Octet

- Notice the masses

  $M(N) = 939$

  $M(\Sigma) = 1193$

  $= M(N) + 254$

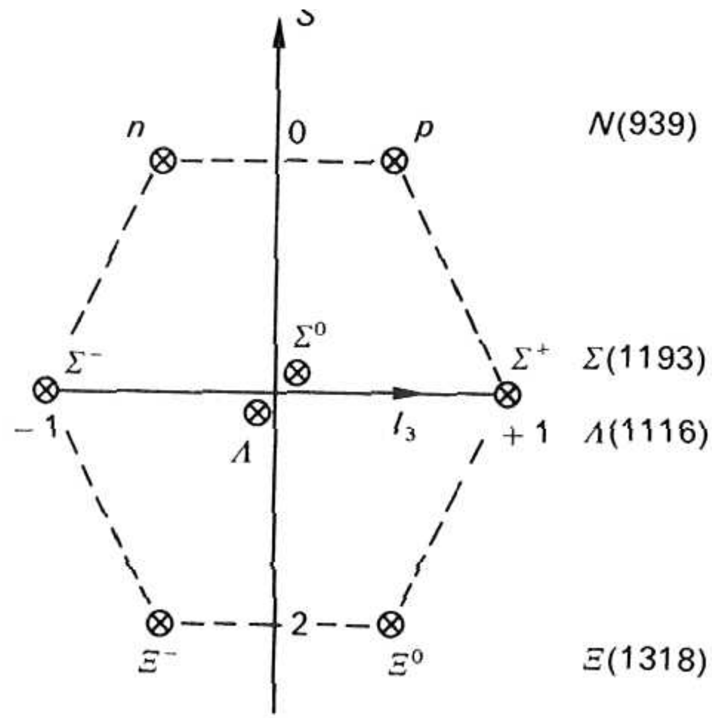
  $M(\Lambda) = 1116$

  $= M(N) + 177$

  $M(\Xi) = 1318$

  $= M(\Sigma) + 125$

  $= M(\Lambda) + 202$



Pattern is more complicated than for decuplet

hyperfine splitting

Mass Relations and Hyperfine Splitting

- $\Delta E(Q\overline{Q}) = 8\pi\alpha_s |\psi(0)|^2 \sigma_i \cdot \sigma_j / 9m_i m_j = 2K \sigma_i \cdot \sigma_j / m_i m_j$
- $\Delta E(QQ) = 4\pi\alpha_s |\psi(0)|^2 \sigma_i \cdot \sigma_j / 9m_i m_j = K \sigma_i \cdot \sigma_j / m_i m_j$

Baryon and mass (MeV)	Quark composition (n denotes u or d)	$\Delta E/K$	Predicted mass, MeV
$N(939)$	$3n$	$-3/m_n^2$	939
$\Lambda(1116)$	$2n, 1s$	$-3/m_n^2$	1114
$\Sigma(1193)$	$2n, 1s$	$1/m_n^2 - 4/(m_n m_s)$	1179
$\Xi(1318)$	$1n, 2s$	$1/m_n^2 - 4/(m_n m_s)$	1327
$\Delta(1232)$	$3n$	$3/m_n^2$	1239
$\Sigma(1384)$	$2n, 1s$	$1/m_n^2 + 2/(m_n m_s)$	1381
$\Xi(1533)$	$1n, 2s$	$1/m_s^2 + 2/(m_n m_s)$	1529
$\Omega(1672)$	$3s$	$3/m_s^2$	1682

due to color field – analogous to, but stronger than, EM hyperfine splitting

Electromagnetic Mass Differences and Isospin Symmetry

$M(\text{hadron}) = M_{bare} + \Delta M_{EM}$

M_{bare} = constituents plus S.I. (incl. hfs)

Expect the ΔM_{EM} for multiplet to be same for like charge particles:

$\Delta M(p) = \Delta M(\Sigma^+)$

$\Delta M(\Sigma^-) = \Delta M(\Xi^-)$

$\Delta M(\Xi^0) = \Delta M(n)$

With bare masses:

$M(p) + M(\Sigma^-) + M(\Xi^0) = M(\Sigma^+) + M(\Xi^-) + M(n)$

$M(p) - M(n) = M(\Sigma^+) - M(\Sigma^-) + M(\Xi^-) - M(\Xi^0)$

-1.3 MeV = -8,0 MeV + 6.4 MeV = -1.6 MeV

$M_d - M_u \approx 2$ MeV

Color

- 3 different color states
  - χ_C = r, g, b
- Color hypercharge
  - Y^C
- Color isospin
  - I_3^C

Hadrons are combinations of quarks with Y^C = 0 and I_3^C = 0

so baryons = r g b

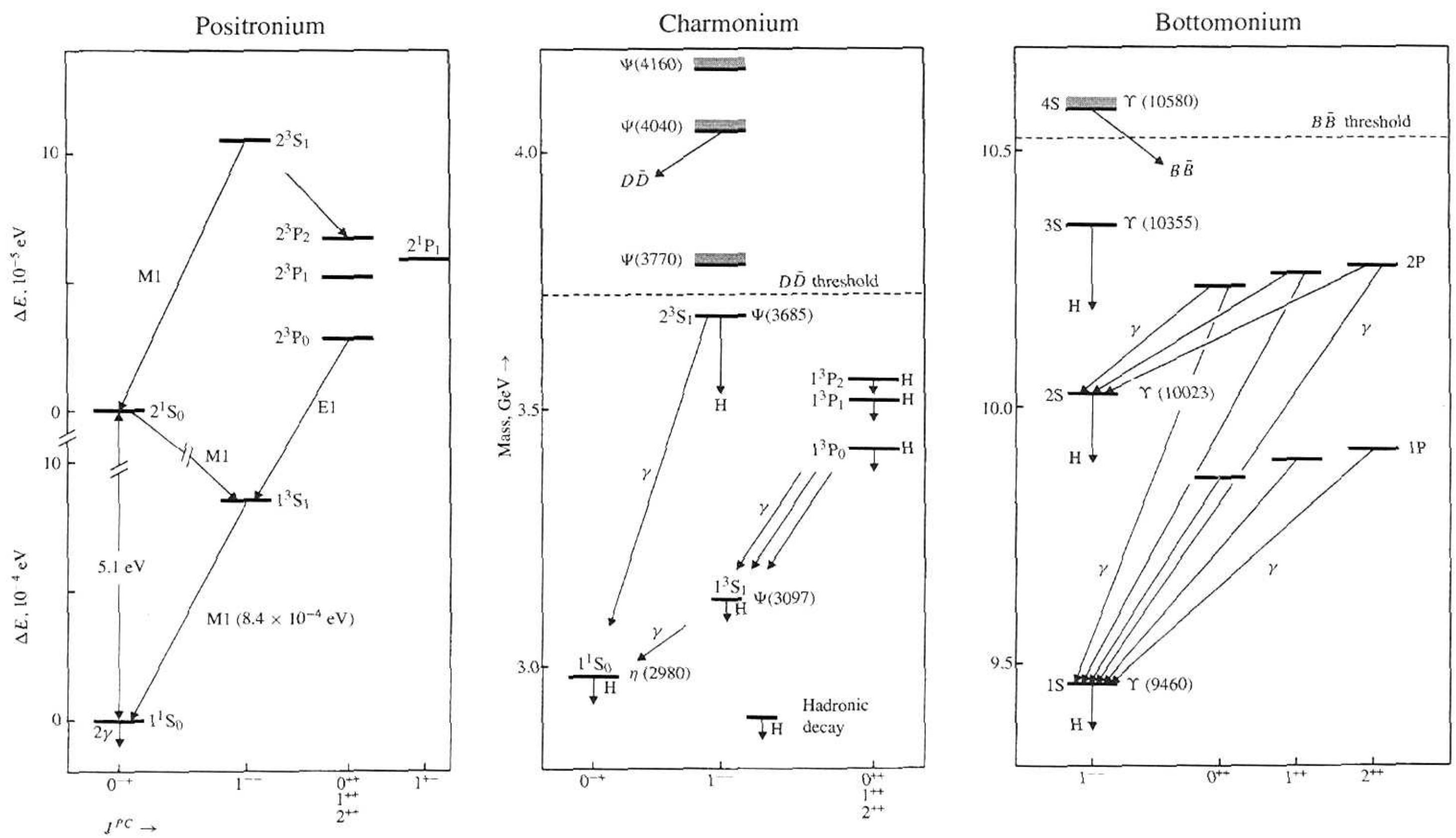
& mesons = r anti-r, etc.

(a) Quarks			(b) Antiquarks		
	I_3^C	Y^C		I_3^C	Y^C
r	1/2	1/3	$\bar{r}$	−1/2	−1/3
g	−1/2	1/3	$\bar{g}$	1/2	−1/3
b	0	−2/3	$\bar{b}$	0	2/3

Quarkonium

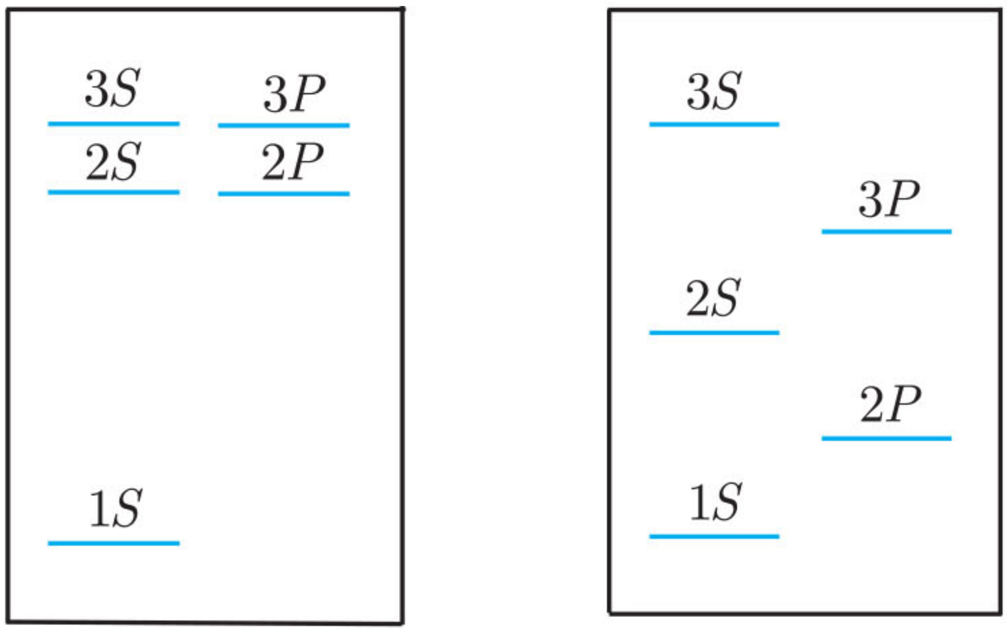
- Similar energy levels to positronium

$n\ ^{2S+1} L_J$



Quarkonium

- But also different!
  - Positronium has Coulomb-like (r^{-1}) energy levels; quarkonium doesn't



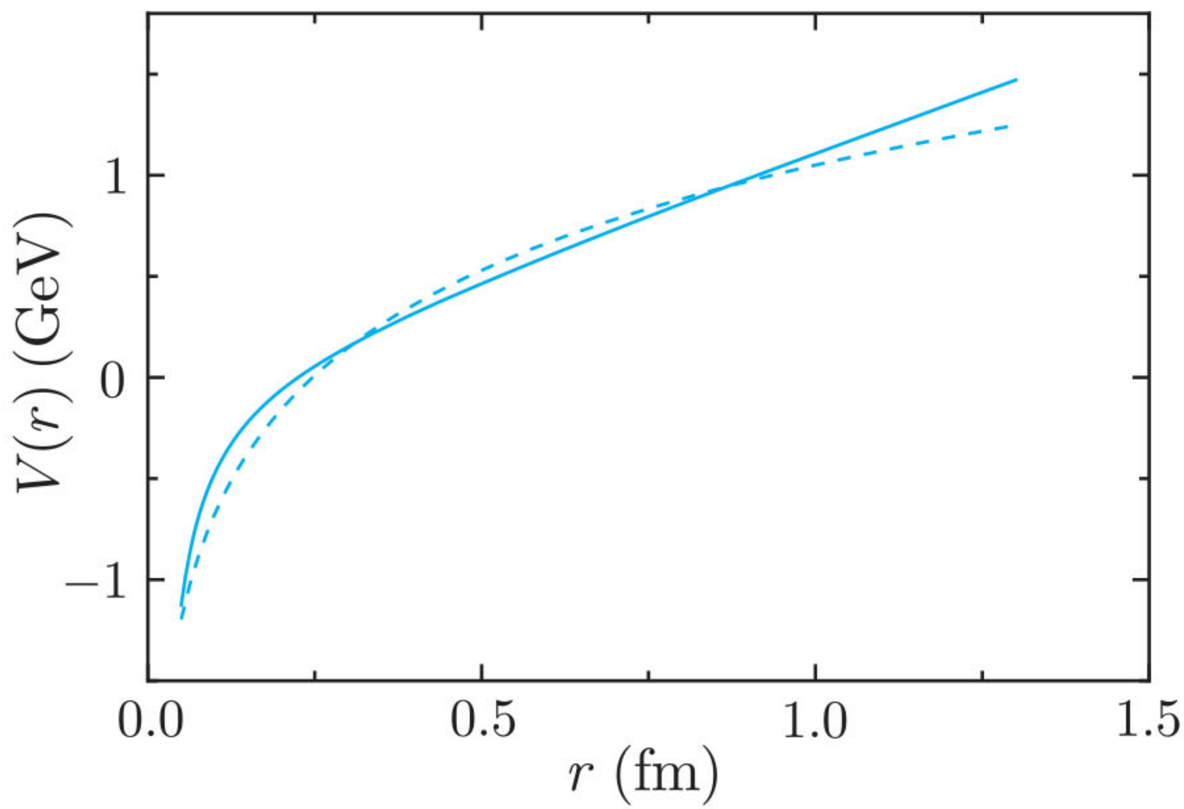
(a) Coulomb (b) Oscillator

Quarkonium

- Different potential:
  - positronium -> Coulomb
  - $V = - \alpha / r$
  - quarkonium -> potential from QCD
  - expected potential of the form
  - $V = -(4/3)\, \alpha_s / r + kr$

Quarkonium

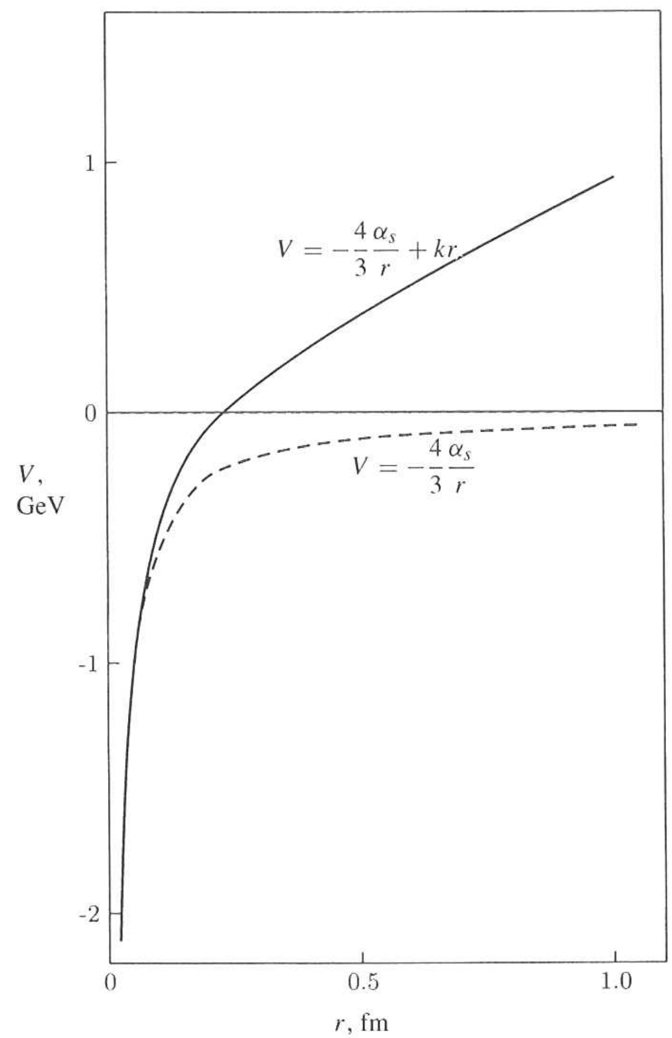
- $V = -a / r + br$ $a = 0.3$
- $b = 0.23\ GeV^2$



Quarkonium

- $V = -(4/3)\, \alpha_s / r + kr$

- $\alpha_s = 0.2$
- $k \approx 1$ GeV fm^{-1}

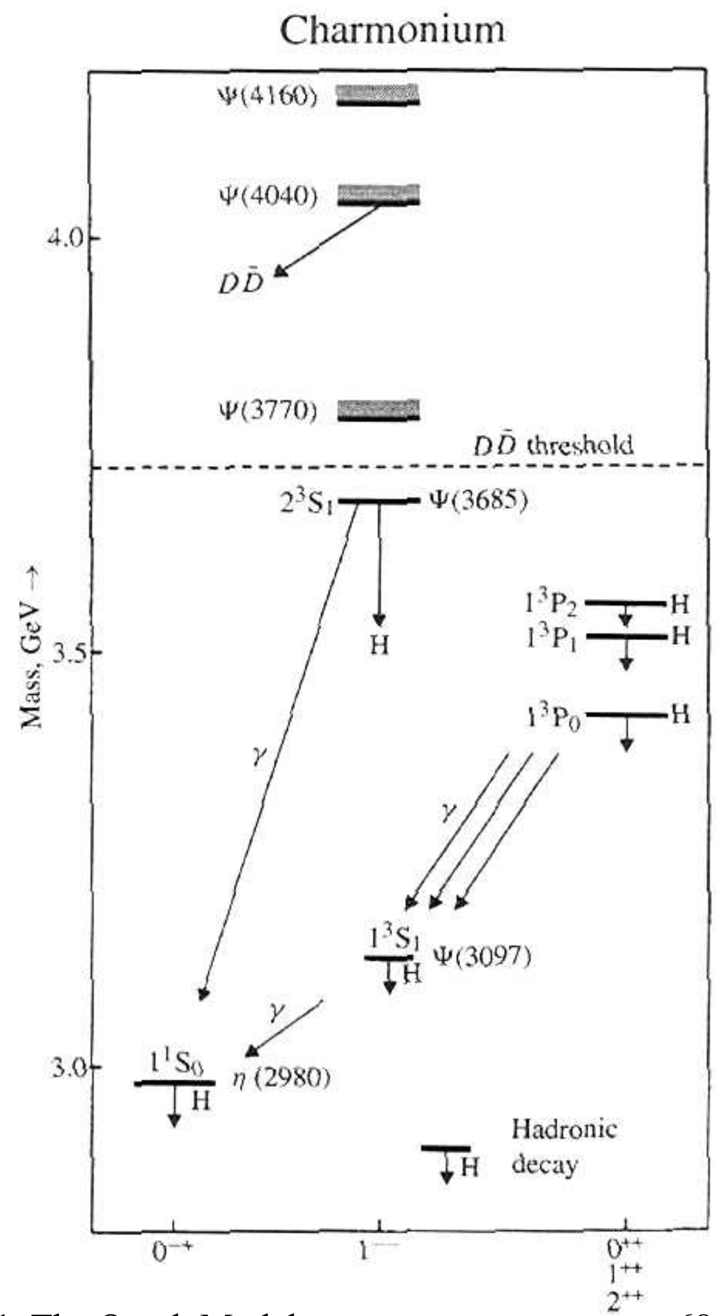


Quarkonium

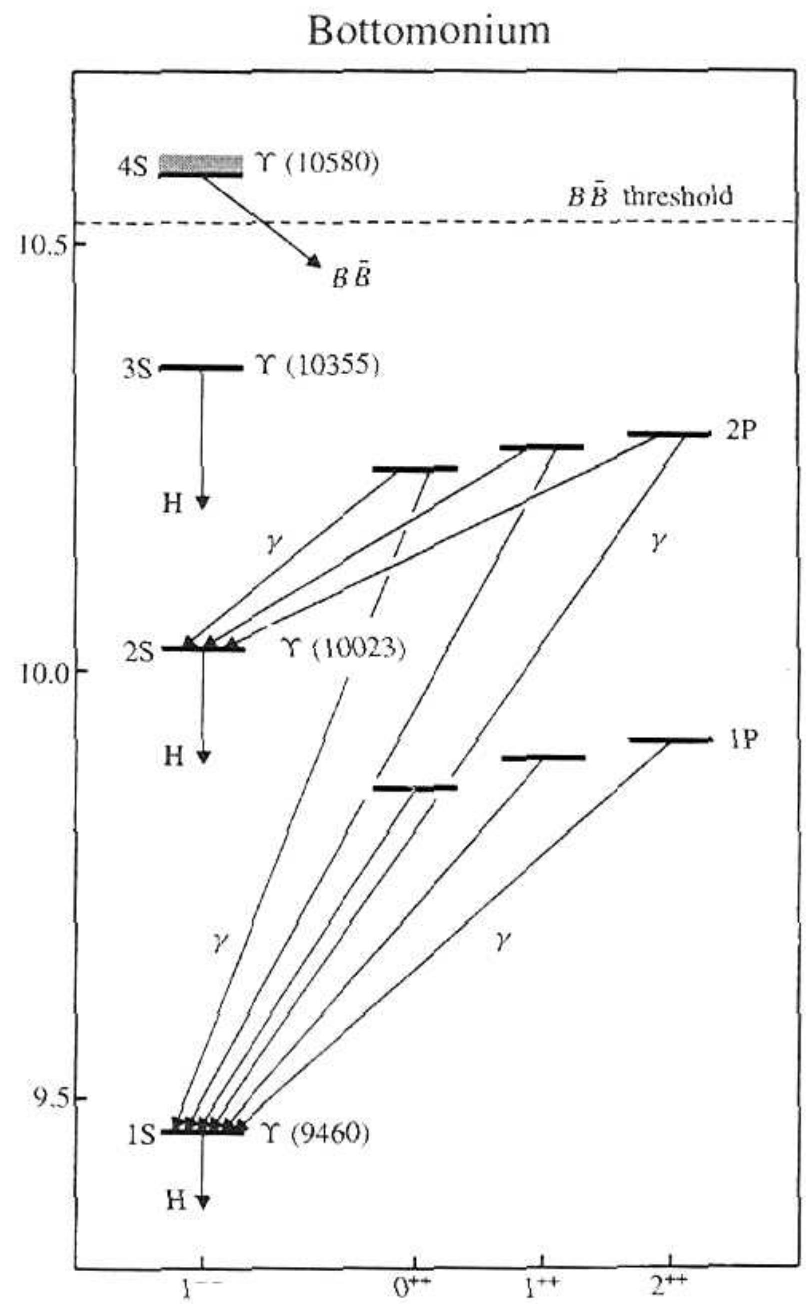
- For the heavy quarks, a non-relativistic approximation is valid:
  - p/m ~ 0.13

- Fine structure is of order α_s, and therefore coarser than for positronium, as observed

Charmonium


Charmonium
Ψ(4160)
Ψ(4040)
4.0
$D\bar{D}$
Ψ(3770)
$D\bar{D}$ threshold
2^3S_1
Ψ(3685)
H
Mass, GeV →
3.5
1^3P_2
H
1^3P_1
H
1^3P_0
H
γ
γ
1^3S_1
Ψ(3097)
H
γ
3.0
1^1S_0
η (2980)
H
Hadronic
decay
H
0^{-+}
1^{--}
0^{++}
1^{++}
2^{++}

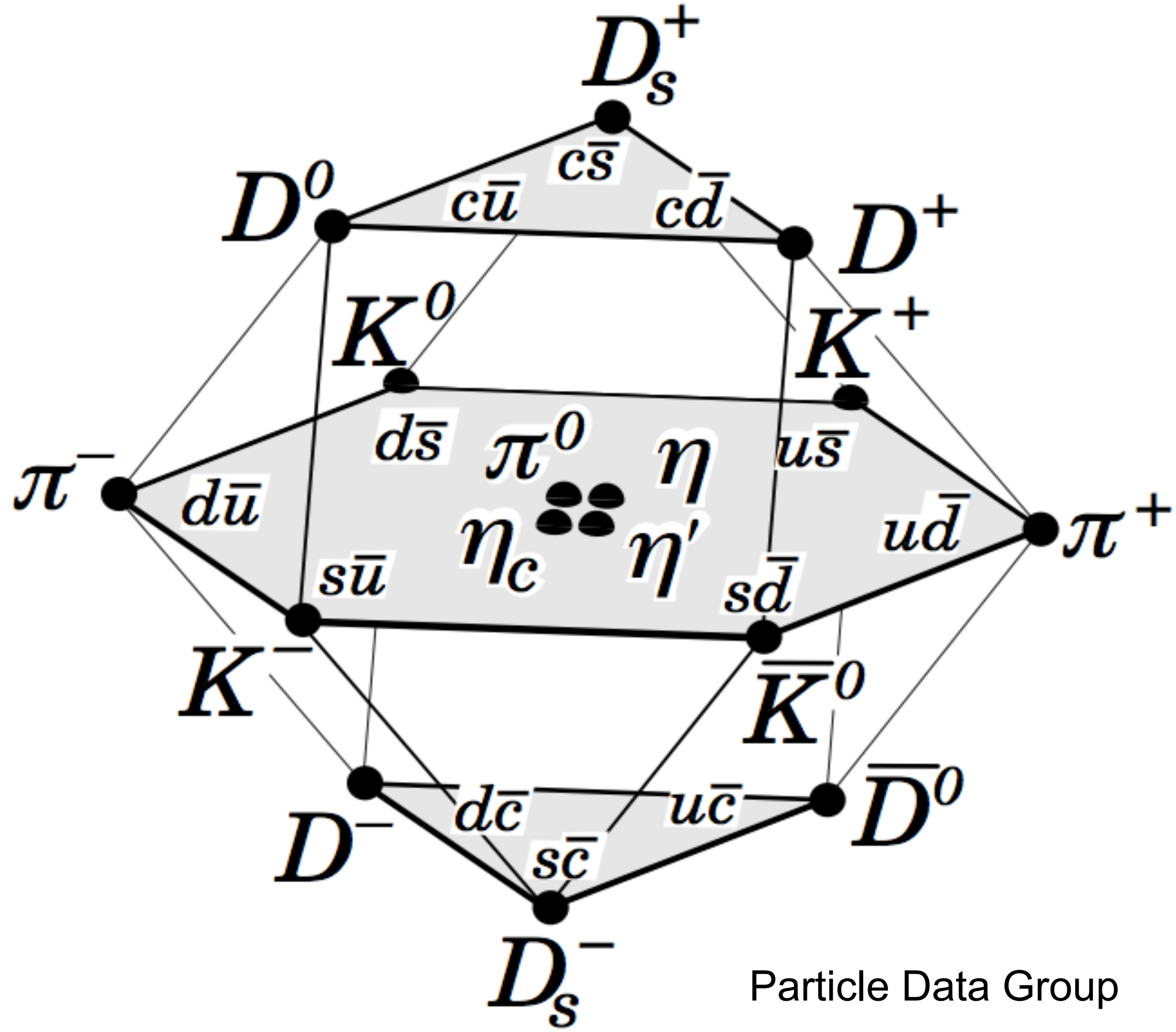
Bottomonium



Mesons Built of Light and Heavy Quarks

Pseudoscalar mesons in SU(4)

- D^+ (=$c\bar{d}$)
- D^0 (=$c\bar{u}$)
- D^+_s (=$c\bar{s}$)

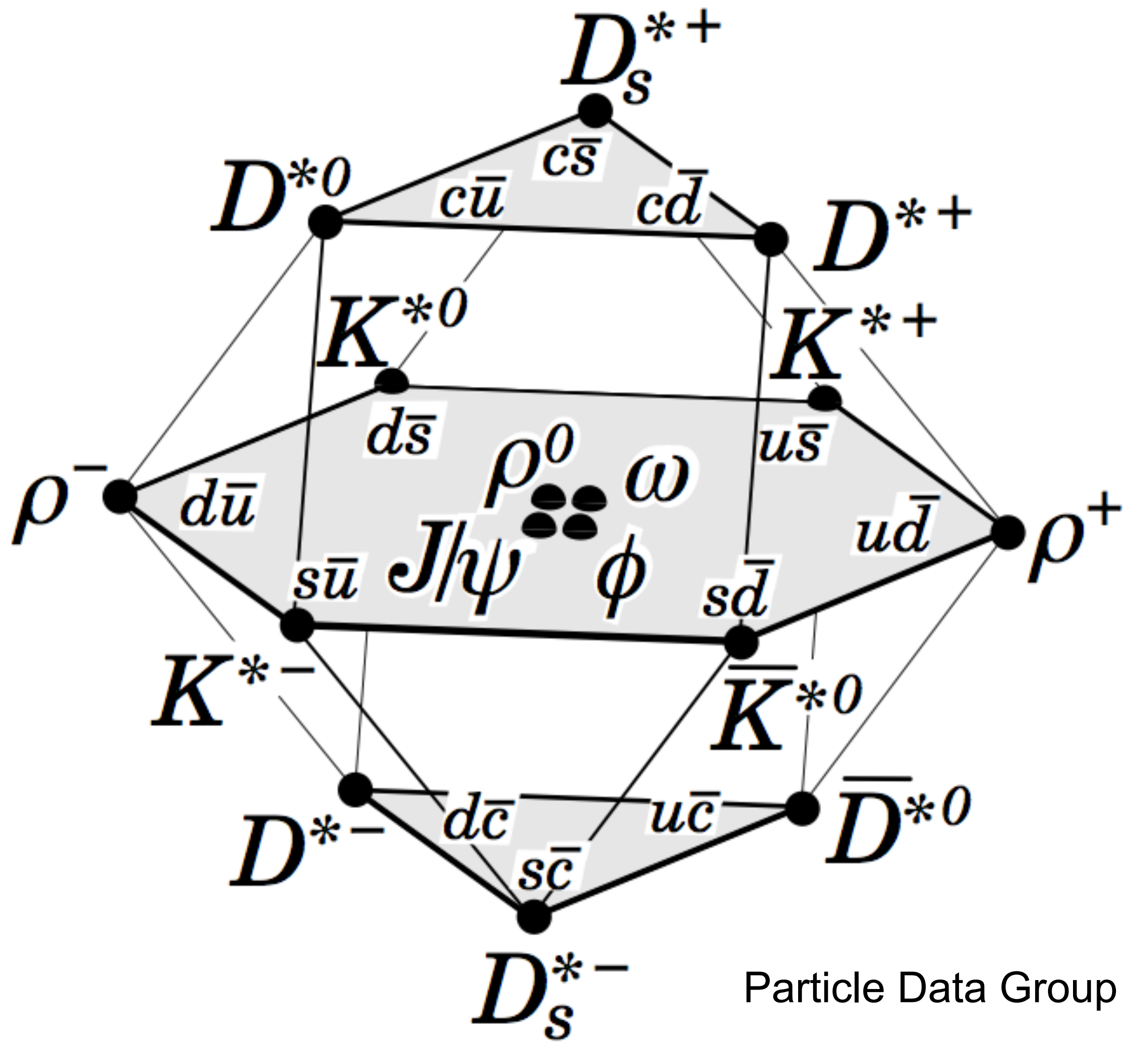


Particle Data Group

Mesons Built of Light and Heavy Quarks

Vector mesons in SU(4)

- D^{*+} (=cd)
- D^{*0} (=cu)
- D^{*+}_{s} (=cs)



Particle Data Group

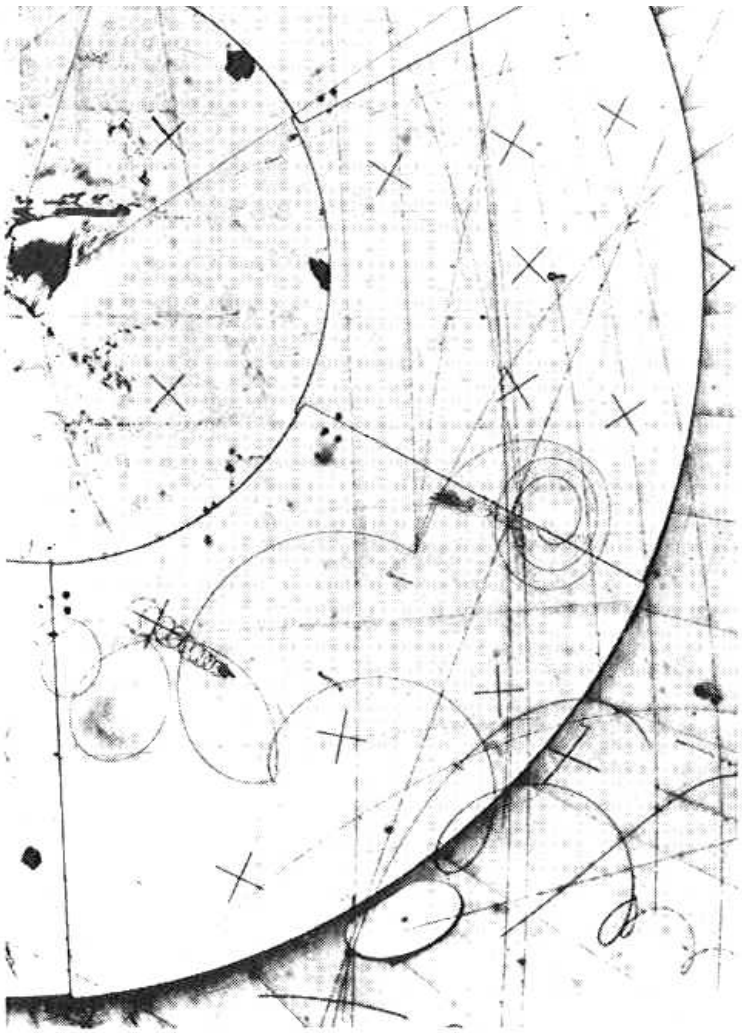
Mesons Built of Light and Heavy Quarks

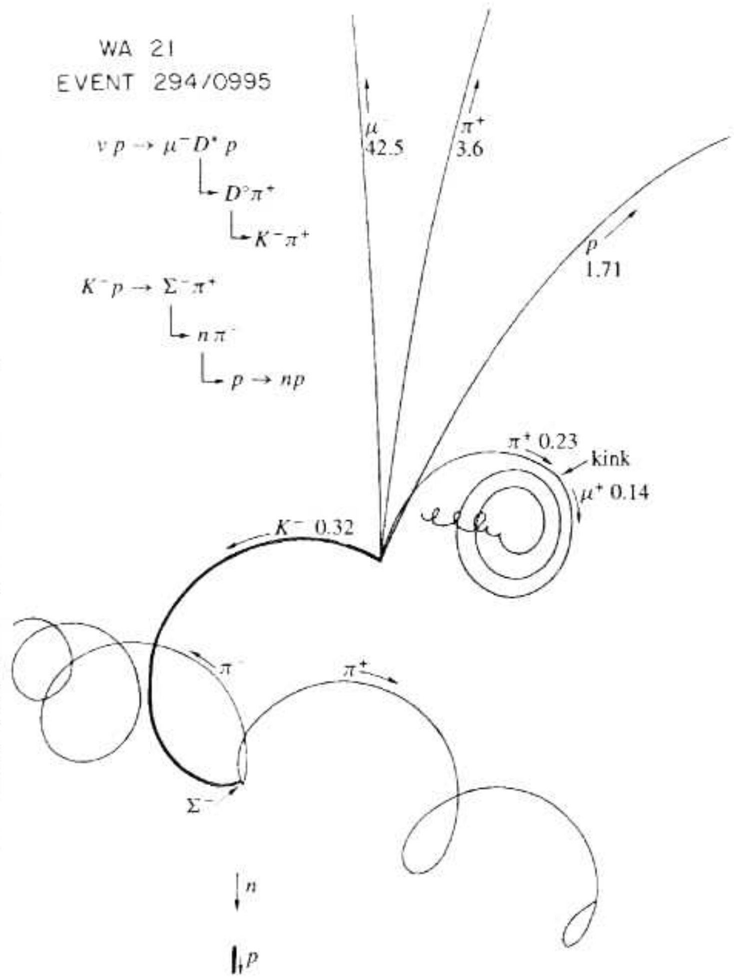
Charmed pseudoscalar mesons decay via $\Delta C = \pm 1$ weak interactions with lifetimes of ~ 10^{-12} sec largely to final states containing strange particles

eg. $D^0 \rightarrow K^- \pi^+$

or $c \rightarrow s$

Mesons Built of Light and Heavy Quarks





Mesons Built of Light and Heavy Quarks

- In heavy quark-light quark system, heavy quark has small effect on energy levels ~ Λ/M ~ 0.2 Gev/M
- Hyperfine splitting smaller than for light quark mesons - ~1/M

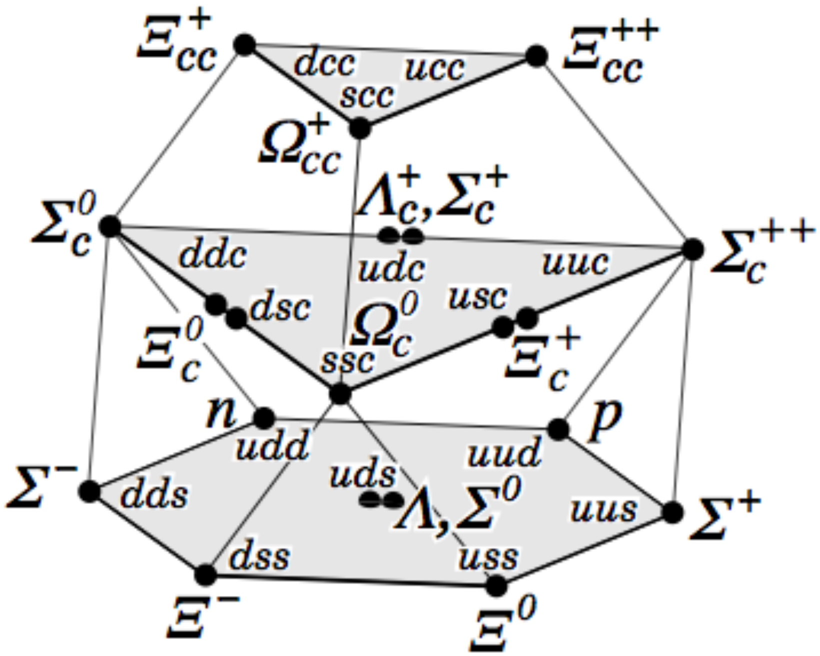
Heavy quark mass	Triplet–singlet difference	Product
$M_c \simeq 1.86$ GeV	$M_{D^*} - M_D = 0.14$ GeV	$M_c(M_{D^*} - M_D) = 0.26$ GeV
$M_b \simeq 5.28$ GeV	$M_{B^*} - M_B = 0.046$ GeV	$M_b(M_{B^*} - M_B) = 0.24$ GeV

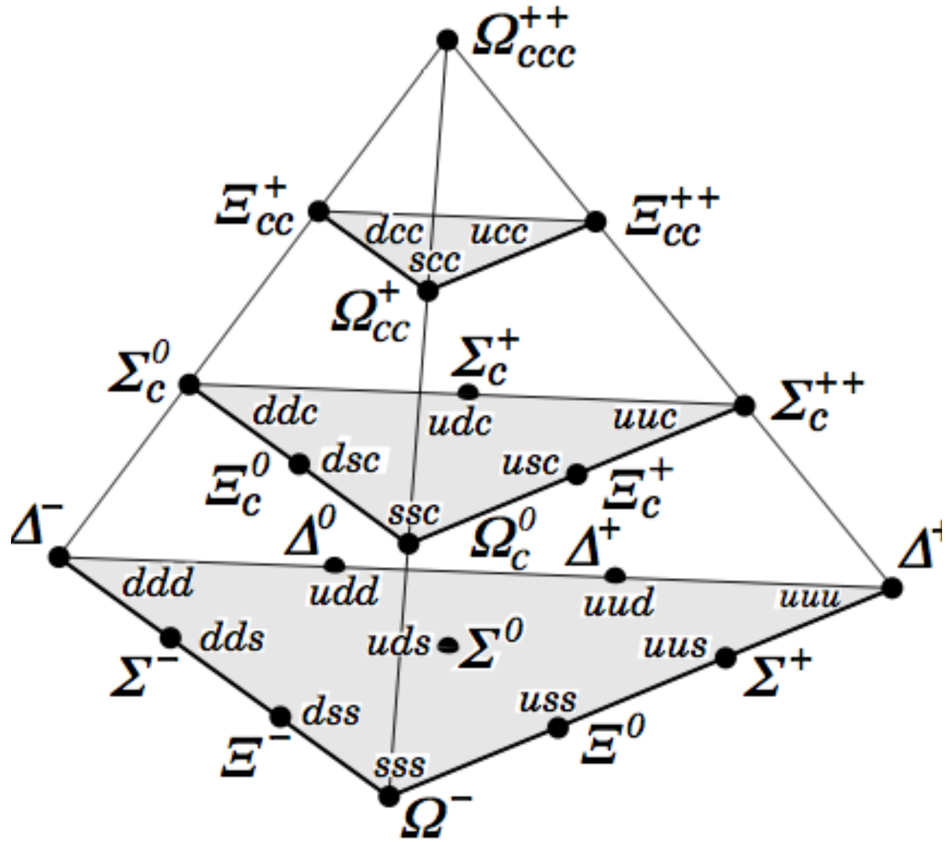
~ 1/M (color magnetic int.)   independent of M

- Mass difference for pseudoscalar states only weakly depends on heavy quark mass
  - $M(D_s^+)-M(D^+)$ = 1968-1870 = 98 MeV
  - $M(B_s^0) - M(B^0)$ = 5366 – 5280 = 86 MeV

Charmed Baryons

SU(4)





There is very weak evidence for a baryon with two c quarks, a Ξ^{+}_{cc} at 3519 MeV.

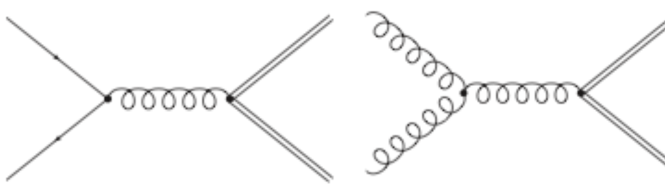
Top Quark

- First observed in 1995 at Fermilab
- M = 173 GeV
- $p\bar{p} \rightarrow t\,\bar{t} + X$ (1/10^{10} collisions)

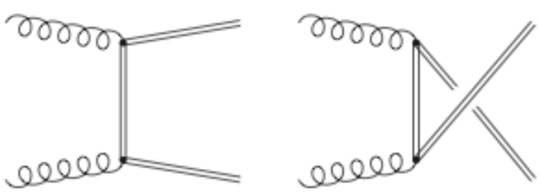
  $\rightarrow W^{+}b\,W^{-}\bar{b}$

Top quark production

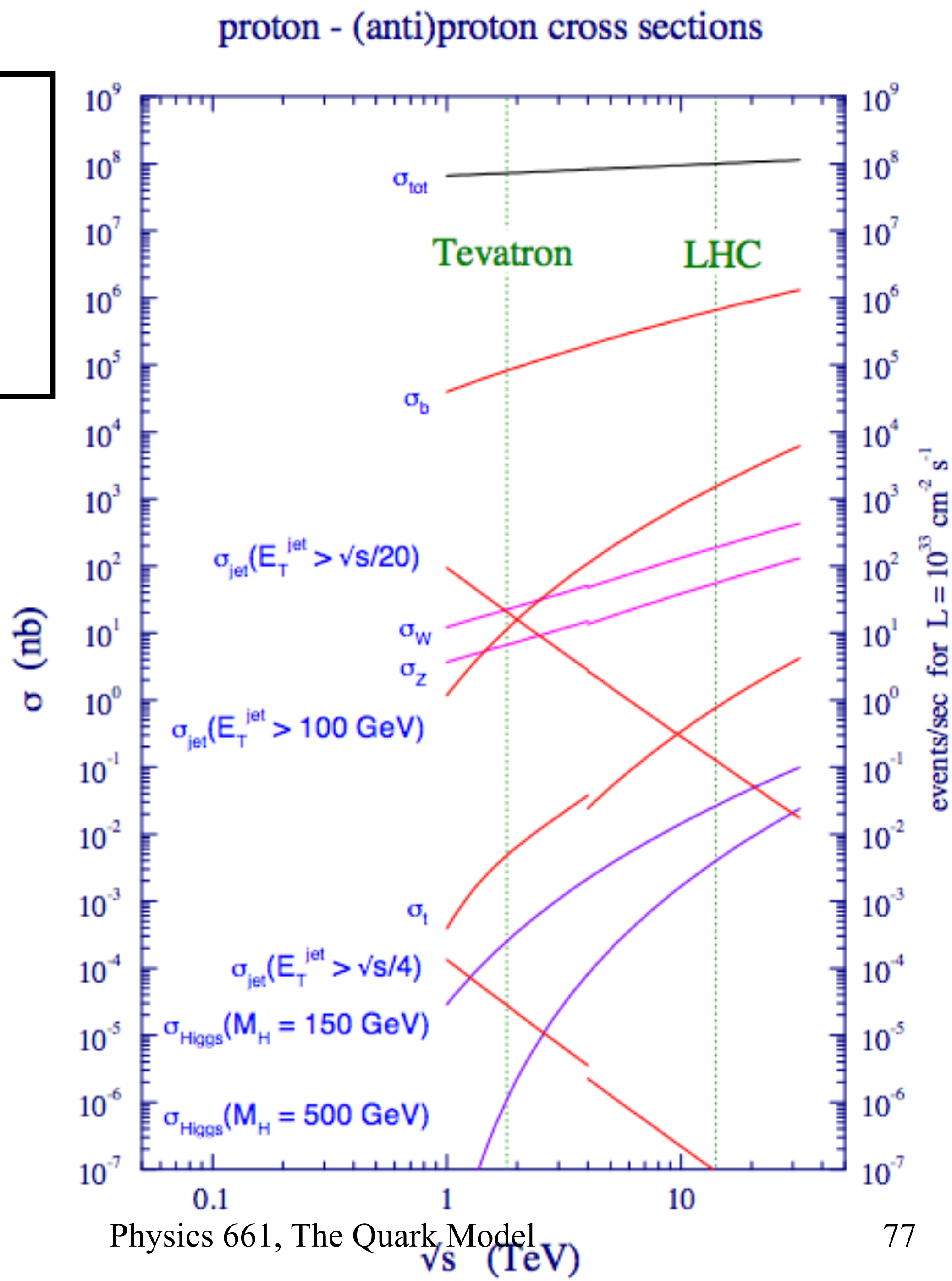
Leading order Feynman diagrams



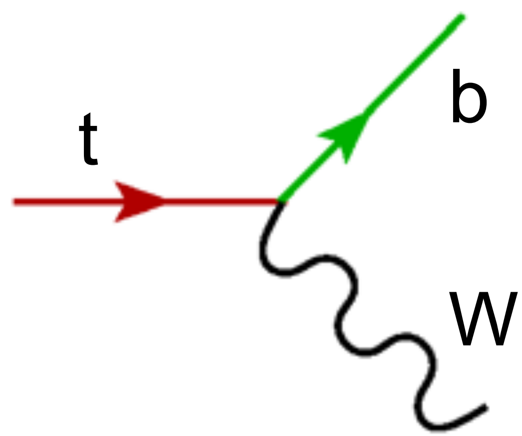
$$q + \bar{q} \longrightarrow Q + \bar{Q}$$
$$g + g \longrightarrow Q + \bar{Q}$$



Also $q + g \rightarrow q + Q + \bar{Q}$



Top Quark Decay



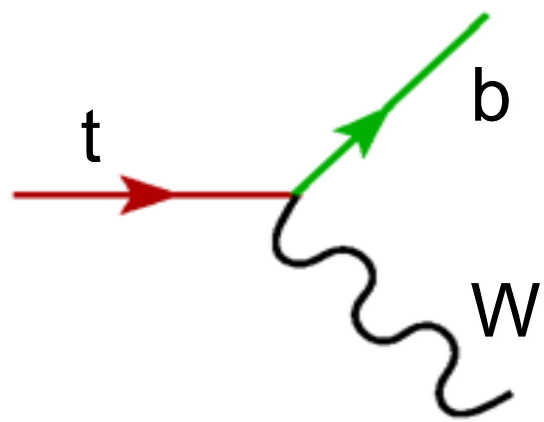
Top Quark Decay

- Top quark is very short lived, not long enough for bound states to form
- Particle Data Group:

Γ= 1.41 (+0.19 -0.15) GeV

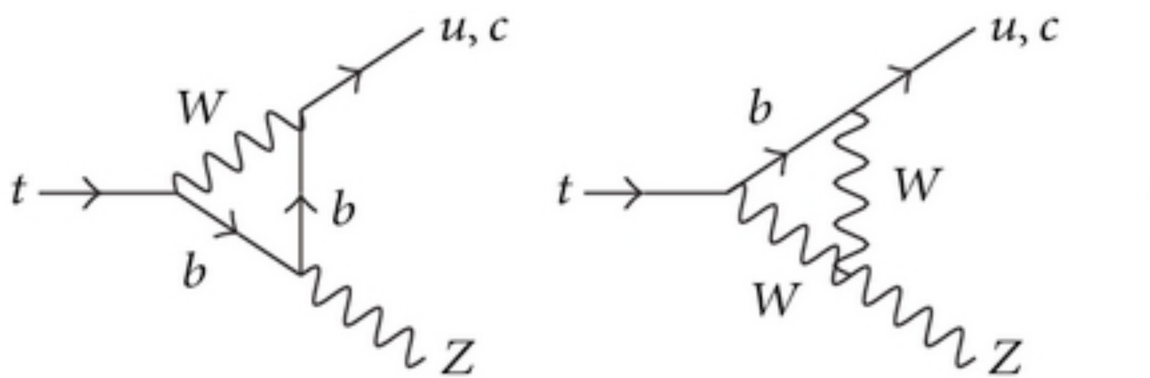
$\tau = \hbar/\Gamma$= 197 MeV-fm/3 x 10^{23} fm/sec/1410MeV

~ 5 x 10^{-25} sec



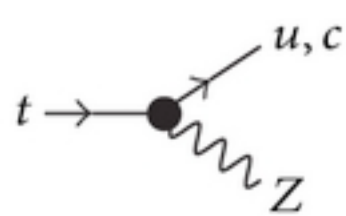
Top FCNC Decays

- Top quark flavor changing neutral current (FCNC) interactions are highly suppressed in the Standard Model.
- $B(t \to Xq) \sim 10^{-17} - 10^{-12}$,
  where X = H,γ,Z or g.



Top FCNC Decays

- Top quark flavor changing neutral current (FCNC) interactions are highly suppressed in the Standard Model.
- Any large signal of FCNCs will indicate the existence of new interactions.



Top FCNC Decays

- BSM theory predictions

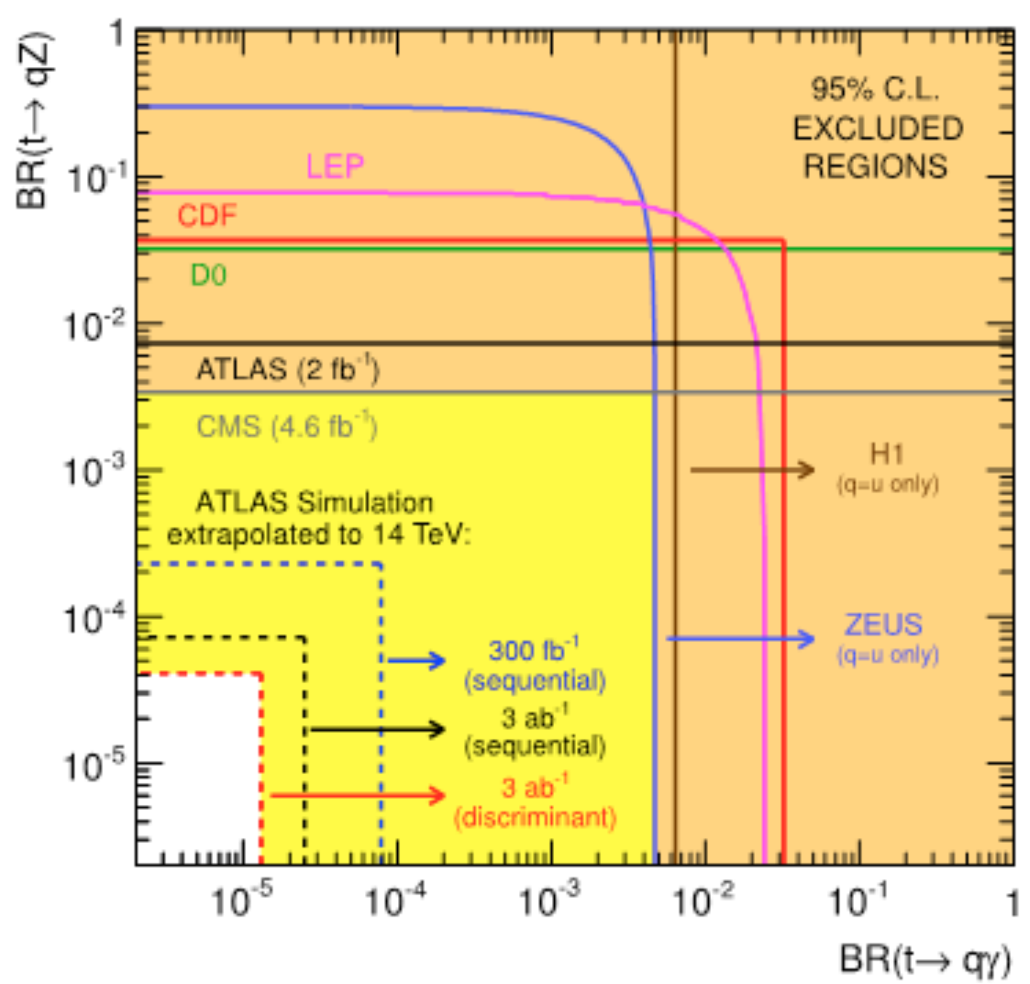
BR FCNC	$t \to q\gamma$	$t \to qZ$
SM	10^{-14}	10^{-14}
QS	10^{-9}	10^{-4}
2HDM	10^{-6}	10^{-7}
MSSM	10^{-6}	10^{-6}
RPV SUSY	10^{-6}	10^{-5}
TC2	10^{-6}	10^{-4}
RS	10^{-9}	10^{-5}

Top FCNC Decays

- Top quark flavor changing neutral current (FCNC) interactions are highly suppressed in the Standard Model.
- Any large signal of FCNCs will indicate the existence of new interactions.
- LHC experiments are searching for FCNC interactions in top quark production and decay
- $pp \rightarrow t+j$, $pp \rightarrow t+Z$
- $t \rightarrow qZ$, $t \rightarrow qh$

Top FCNC Decays

BR FCNC	$t \to q\gamma$	$t \to qZ$
SM	10^{-14}	10^{-14}
QS	10^{-9}	10^{-4}
2HDM	10^{-6}	10^{-7}
MSSM	10^{-6}	10^{-6}
RPV SUSY	10^{-6}	10^{-5}
TC2	10^{-6}	10^{-4}
RS	10^{-9}	10^{-5}



EXTRAS

Group Theory

- Quarks are fundamental representations of the group SU(3)

$$3 \otimes 3 = 6 \oplus 3 \qquad\qquad 3 \otimes \overline{3} = 8 \oplus 1$$

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

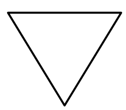
1 is anti-symmetric under interchange of two quarks
10 is symmetric under interchange of two quarks
8's are mixed under interchange of two quarks

Group Theory - Combining Multiplets

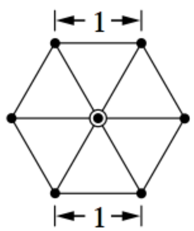
- Multiplet Labels
  - SU(n) -> (n-1) non negative integers

    $(\alpha, \beta, \gamma, \ldots)$

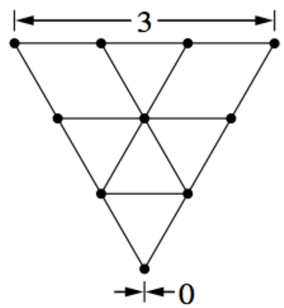
    so for SU(3) (α, β) (lengths of top, and bottom)



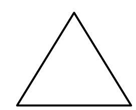
(1,0)



(1,1)



(3,0)



(0,1)

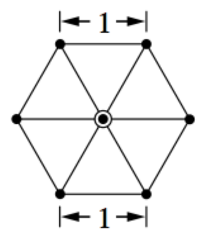
Group Theory - Combining Multiplets

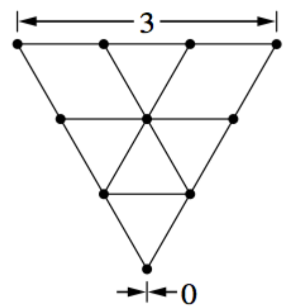
- Number of Particles in the Multiplet
  - For SU(3)

$$N = (\alpha+1)(\beta+1)(\alpha+\beta+2)/2$$

(1,1) (3,0)

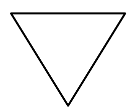
N=8

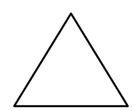




N=10

N=3



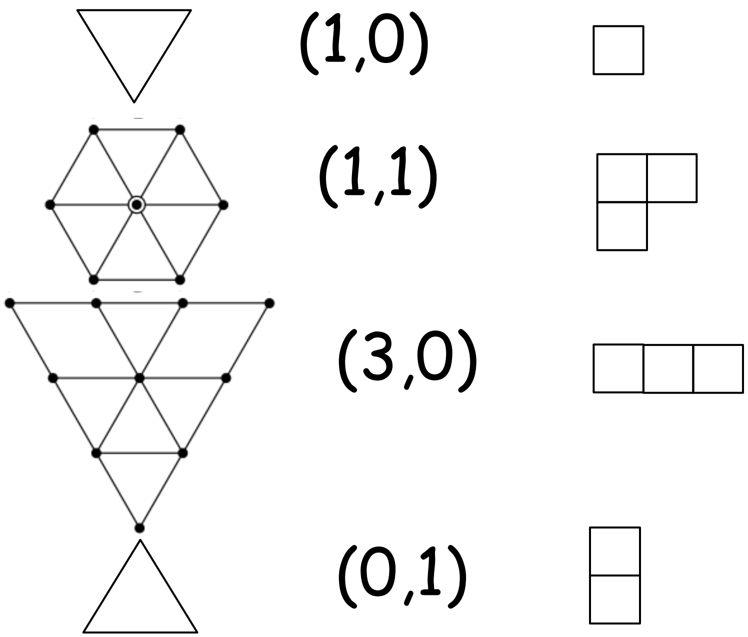


N=3

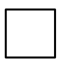
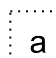
(1,0) (0,1)

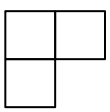
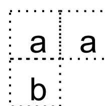
Group Theory – Combining Multiplets

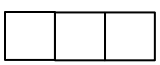
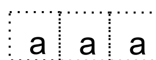
- Young's diagrams
  - Top row is α boxes past end of 2nd row
  - 2nd row is β boxes past end of 3rd row

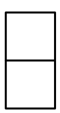
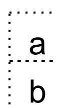


Group Theory - Combining Multiplets

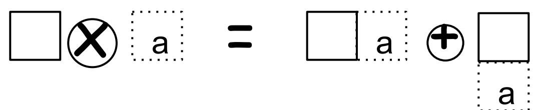
- Coupling multiplets
  - Sequence of letters okay if everywhere at least as many of an early letter (eg. a) has occurred as a later letter (eg. b)
  - In one diagram, replace boxes by a's (1st row), b's (2nd row), etc.
  - So  becomes 

 becomes 

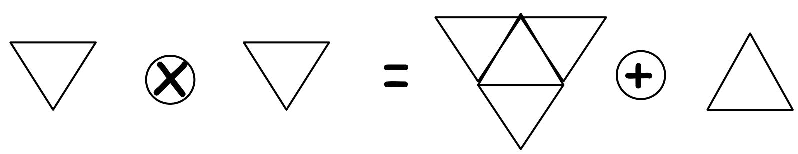
 becomes 

 becomes 

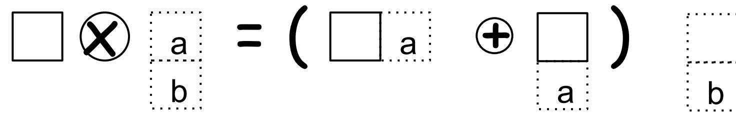
Group Theory - Combining Multiplets

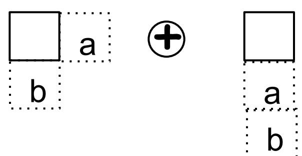
$3 \otimes 3 =$  $= (2,0) \oplus (0,1) = 6 \oplus 3$

$$3 \otimes 3 = 6 \oplus 3$$

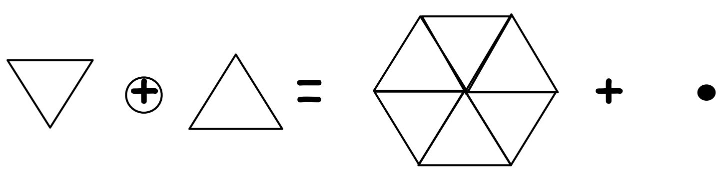


Group Theory - Combining Multiplets

$3 \otimes \bar{3} =$ 

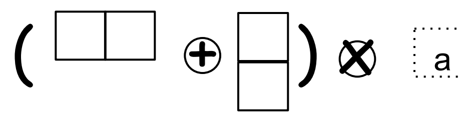
$=$ 

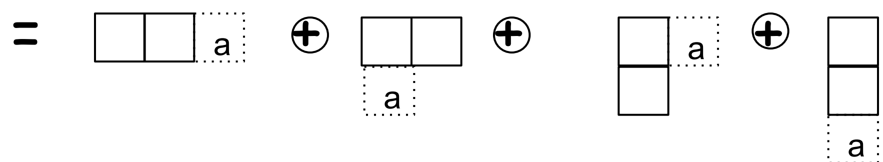
$= (1,1) \oplus (0,0) = 8 \oplus 1$



Group Theory - Combining Multiplets

$$3 \otimes 3 \otimes 3 = (6 \oplus 3) \otimes 3 =$$





$$= (3,0) \oplus (1,1) \oplus (1,1) \oplus (0,0) = 10 \oplus 8 \oplus 8 \oplus 1$$