QCD, Jets, & Gluons

- Quantum Chromodynamics (QCD)
- $e^+ e^- \rightarrow \mu^+ \mu^-$
- $e^+ e^- \text{ annihilation to hadrons } (e^+ e^- \rightarrow Q\bar{Q})$
Quantum Chromodynamics

• Another gauge theory, as QED

• Interactions described by exchange of massless, spin-1 bosons – so-called “gauge bosons”

• Force is long range, due to massless gluons – but long range force is cancelled by combinations of color in mesons and baryons
The color quantum number

- Color was invented to explain:
  - $\Delta^{++} = uuu$
  - $e^+e^- \rightarrow \text{hadrons}$
  - Also explains $\pi^0 \rightarrow \gamma \gamma$
- Color of a quark has three possible values
  - say red, blue, green
- Antiquarks carry anticolor
  - Anti-red, anti-blue, anti-green
- Bosons mediating the quark-quark interaction are called gluons
  - Gluons are to the strong force what the photon is to the EM force
  - Gluons carry a color and an anticolor
    - 9 possible combinations of color and anticolor
    - $r\bar{r} + g\bar{g} + b\bar{b}$ is color neutral, leaving 8 effective color combinations
- Results in potential $V = -\frac{4}{3} \alpha_s/l + kr$
Color

- 3 different color states
  - $\chi_C = r, g, b$
- Color hypercharge
  - $Y_C$
- Color isospin
  - $I_{3C}$

Hadrons are combinations of quarks with $Y^C = 0$ and $I_{3C} = 0$
so baryons = $r \ g \ b$
& mesons = $r$ anti-$r$, etc.

<table>
<thead>
<tr>
<th>(a) Quarks</th>
<th>(b) Antiquarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{3}^C$</td>
</tr>
<tr>
<td>$r$</td>
<td>1/2</td>
</tr>
<tr>
<td>$g$</td>
<td>$-1/2$</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
</tr>
</tbody>
</table>
Color

- In this diagram, the gluon has the color quantum numbers:
  - \( I^c_3 = I^c_3(r) - I^c_3(b) = \frac{1}{2} \)
  - \( Y^c = Y^c(r) - Y^c(b) = 1 \)

- So gluon exists in color state

\begin{center}
\begin{tabular}{c|c|c}
(a) Quarks & & \\
& & \\
\hline
& \( I^c_3 \) & \( Y^c \) \\
\hline
\( r \) & \( 1/2 \) & \( 1/3 \) \\
\( g \) & \( -1/2 \) & \( 1/3 \) \\
\( b \) & \( 0 \) & \( -2/3 \) \\
\end{tabular}
\end{center}
Quantum Chromodynamics

• Important properties of strong interactions distinguishing it from QED:
  - Color confinement
    • Observed states have zero color charges.
  - Asymptotic freedom
    • Interaction gets weaker at short distances.
    • At 0.1 fm the lowest order diagrams dominate and one gluon exchange approximates quark-quark scattering.

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Quantum Chromodynamics

- **QCD potential:**
  
  \[ V = -(4/3) \alpha_s / r \ (r \leq 0.1 \text{ fm}) \]
  
  \[ V = kr \ (r \geq 1 \text{ fm}) \]
The color quantum number

• A possible basis for the gluons:
  - $r\bar{b}$, $rg$, $bg$, $br$, $gr$, $gb$, $\frac{1}{\sqrt{2}}(r\bar{r}-b\bar{b})$, $\frac{1}{\sqrt{6}}(r\bar{r}+b\bar{b}-2g\bar{g})$
  - (color singlet - no color - excluded: $\frac{1}{\sqrt{3}}(r\bar{r}+b\bar{b}+g\bar{g})$

• Gluon exchanges are analogous to photon exchanges, but different

• Hadrons are always color neutral (or color-singlets)
  - eg. $Q_r\bar{Q}_r$, $Q_rQ_bQ_g$
  - Note - quark combinations such as $QQ$ or $QQ\bar{Q}\bar{Q}$ are not (normally) bound
The QCD Potential at Short Distance

- Jets produced in high energy collisions are direct support for the short range behavior.

UA1 at CERN

azimuth

rapidity
Running Couplings in QED

- Shielding effect is created around a charge, effectively changing the strength of coupling.

- Pairs of charge will be created and reabsorbed, producing shielding effect ("vacuum polarization").
Bhabha measurement at large momentum-transfer (L3)
Running Couplings in QED

![Graph showing running couplings in QED]

S. Mele

QED predictions from Burkhardt & Pietrzyk
PLB 513 (2001) 46
Running Couplings

- Dependence of coupling on momentum transfer (and therefore mass scale) follows the renormalization group equation:

\[
\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha(q^2)} + \beta_0 \ln \left( \frac{q^2}{\mu^2} \right) + \cdots
\]

- \(\beta_0\) depends on the number of degrees of freedom

\[
\beta_0 = \frac{1}{12\pi} (4n_f - 11n_b)
\]

- where \(n_b\) are bosons degrees of freedom and \(n_f\) are fermions degrees of freedom in the vacuum loops

- In QED there are no bosons in the loops, so at high energy

\(n_b = 0, \quad n_f = 3\) (where all three families are active)

consequently, \(\alpha(M_Z = 91 \text{ GeV}) = 1/129\)
Running Couplings

\[ \frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha(q^2)} + \beta_0 \ln \left( \frac{q^2}{\mu^2} \right) + \cdots \]

\[ \beta_0 = \frac{1}{12\pi} (4n_f - 11n_b) = 1/12\pi(4\times3 - 0) = 1/\pi \]

- \( q^2 = m_e^2 = (5.1 \times 10^{-4} \text{ GeV})^2 \)

- \( \frac{1}{\alpha(m_Z^2)} = 137 + \frac{1}{\pi} \ln(2.5 \times 10^{-7}/91^2) \)

\[ \alpha(m_Z^2) = 1/129 \]
Running Couplings in QCD

- In QCD there are three degrees of freedom (3 colors)
  \( n_b = 3 \)
  \( n_f = 3 \) (where all three families are active)

\[
\beta_0 = \frac{1}{12\pi} (4n_f - 11n_b) = (12-33)/12\pi = -7/4\pi
\]

\[
\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{7}{4\pi} \alpha_s(\mu^2) \ln \left( \frac{q^2}{\mu^2} \right)}
\]

or
\[
\alpha_s(q^2) = \frac{1}{B \ln(q^2/\Lambda^2)}
\]

where \( B = -\beta \quad \Lambda^2 = \mu^2 \exp\{-1/B \ \alpha_s(\mu^2)\} \)
Running Couplings in QCD

\[ \alpha_s(q^2) = \frac{1}{B \ln(q^2/\Lambda^2)} \]

• \( \alpha_s \) decreases with increasing \( q^2 \)
  – this is the opposite dependence from QED
  – it is typical of non-Abelian field, where the field particles carry charge and are self-coupling
  – the longitudinal gluons have an antishielding effect, spread out the color charge, weakening the interaction

• Large \( q^2 \) \( \alpha_s \rightarrow 0 \)
  – asymptotic freedom

• Small \( q^2 \) \( \alpha_s \rightarrow \infty \)
  – confinement

\{ Perturbation theory applies for \( q^2 >> L \) \}
Running Couplings in QCD
Running of Strong Coupling

M. Davier, Tau2010

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Running Couplings in QCD

Many observables are sensitive to the value of $\alpha_s$

- Widths of bounds state of $cc$ and $bb$

- $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(\text{point})$ requires factor $(1 + \alpha_s/\pi)$

- Event shapes depend on fraction of 3 jets

- Scaling deviations in deep inelastic lepton-nucleon scattering

- Hadronic width of the $Z$
The “discovery” of quarks

- deep inelastic lepton-nucleon scattering revealed dynamical understanding of quark substructure

\[ e \rightarrow e + (1-x)P \]

- leptoproduction of hadrons could be interpreted as elastic scattering of the lepton by a pointlike constituent of the nucleon, the quark

- theory of scattering of two spin-1/2, pointlike particles required
The electromagnetic process is dominated by single-photon exchange

\[ M_{if} = \frac{e^2}{q^2} = 4\pi\alpha/q^2 \]

\[ \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} f(\theta) \]

(neglecting the muon mass)

What about spins?

- The conservation of helicity at high energy for the EM interaction means only LR and RL states will interact
$e^+ e^- \rightarrow \mu^+ \mu^-$

- Conservation of helicity
  - consider conservation of helicity at high energy in scattering of electron

\[ e^-_R \rightarrow e^-_R \text{ and } e^-_L \rightarrow e^-_L \]
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

- **Conservation of helicity**
  - consider crossed diagrams
  - \( e^-_L \rightarrow e^-_L \) t-channel, and \( e^-_L \) only couples with \( e^+_R \) in s-channel (\( e^-_R \) only couples with \( e^+_L \))
$e^+ e^- \rightarrow \mu^+ \mu^-$

CMS FRAME

- Amplitude is $d_{\mu \mu'}^J (\theta) = d_{1,1}^1 (\theta) = \frac{1+\cos \theta}{2}$
  - if RL $\rightarrow$ RL
- Amplitude is $d_{\mu \mu'}^J (\theta) = d_{1,-1}^1 (\theta) = \frac{1-\cos \theta}{2}$
  - If LR $\rightarrow$ RL
- $M^2 = [(1+\cos \theta)/2]^2 + [(1-\cos \theta)/2]^2 = (1+\cos^2 \theta)/2$
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

CMS FRAME

- Now, for RL → RL and LR → RL we have
  \[ M^2 \sim \left[ \frac{1+\cos \theta}{2} \right]^2 + \left[ \frac{1-\cos \theta}{2} \right]^2 = \frac{1+\cos^2 \theta}{2} \]
- Sum over final states, adding RL → LR and LR → LR, to double \( M^2 \)
  \[ M^2 = (1+\cos^2 \theta) \]
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

- \[ \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \]

- \[ \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3s} \]
  \[ = 87 \text{nb} / s(\text{GeV}^2) \]
  (point cross section)
At higher energy, the $Z^0$ exchange diagram becomes important.

$e^+ e^- \rightarrow \mu^+ \mu^-$ at higher energy

\[ a_{em} \sim \frac{4\pi\alpha}{s} \]
\[ a_{wk} \sim G \text{ (Fermi const.)} \]

interference $\sim \frac{a_{wk}a_{em}}{a_{em}^2} \sim \frac{Gs}{4\pi\alpha} \sim 10^{-4}s$
$e^+ e^- \rightarrow \mu^+ \mu^-$

$f \sim a_{wk} a_{em} / a_{em}^2 \sim Gs / (4\pi \alpha)$

$\sim 10^{-4}s \quad \text{(interference)}$

Asymmetry

$(B-F)/(F+B) = f$

$\approx 10\% \text{ at } s=1000 \text{ GeV}^2$

interference of $Z^0$
$e^+ e^- \text{ annihilation to hadrons}$

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)
\]

\[
\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{87\text{nb}}{s(\text{GeV}^2)}
\]

(point cross section)
\[ \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \]

- \( \sigma(e^+ e^- \rightarrow q^+ q^-) \)
  \[ = 4\pi\alpha^2 Q_q^2 N_c / 3s \]
  \[ = 87\text{nb} Q_q^2 N_c / s(\text{GeV}^2) \]
Three-jet events

\[ e^+ \quad \bar{q} \quad q \quad e^- \]

\[ e^+ \quad \bar{q} \quad g \quad q \quad e^- \]
Three-jet events

- Three-jet event observed by JADE Collaboration at DESY
Three-jet events

- Three-jet event angular distribution observed by TASSO Collaboration at DESY

![Graph showing angular distribution](graph.png)

- Dashed line – spin 0 gluon
- Solid line – spin 1 gluon
Correction for three-jet events

- Multiply by \((1 + \alpha_s/\pi)\)

\[
\sigma(e^+ e^- \rightarrow q^+ q^-) = 4\pi\alpha^2 Q^2_q N_c / 3s \\
= 87\text{nb} \frac{Q^2_q N_c}{s(\text{GeV}^2)} (1 + \alpha_s/\pi)
\]
$e^+ e^-\text{ annihilation to hadrons}$
$e^+ e^- \text{ annihilation to hadrons}$

![Graph showing cross-sections for various decay channels](image)

- $\rho$, $\omega$, $\phi$
- $J/\psi$
- $\psi(2S)$
- $\tau$
- $Z$
\[ R = \frac{\sigma (e^+ e^- \rightarrow \text{hadrons})}{\sigma (\text{point})} \]

$\sqrt{s}$ [GeV]    

Particle Data Group

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$e^+ e^- \text{ annihilation to hadrons}$

- $R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(\text{point})}$
  - consider $e^+ e^- \rightarrow \text{hadrons}$ as $e^+ e^- \rightarrow Q\bar{Q}$, summed over all quarks

- $R = \sum e_i^2 (1 + \frac{\alpha_s}{\pi})/ e^2$
  
  $= N_c ((1/3)^2 + (2/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2 + \ldots) (1 + \frac{\alpha_s}{\pi})$

- $N_c = 3$

- Therefore, $R$ should increase by a well defined value as each flavor threshold is crossed
$e^+ e^-$ annihilation to hadrons

Threshold $R$
- below $c$: 2
- charm: $3 \frac{1}{3}$
- bottom: $3 \frac{2}{3}$

$\times (1 + \frac{\alpha_s}{\pi})$
$e^+ e^- \text{ annihilation to hadrons}$

Threshold $R$
- below $c$: 2
- charm: 3 1/3
- bottom: 3 2/3

$\times (1 + \frac{\alpha_s}{\pi})$

Particle Data Group
$e^+ e^- \rightarrow Q \bar{Q}$

If $Q$ is spin $\frac{1}{2}$, this distribution should be

$$(1 + \cos^2 \theta)$$

Angular distribution of two jets

34 GeV
$e^+ e^- \text{ annihilation to hadrons}$

- Constancy of $R$ indicates pointlike constituents

- Angular distributions of hadron jets prove spin 1/2 partons

- Values of $R$ consistent with partons of quarks with expected charge and color quantum number
Exotic Hadrons

- Exotic hadrons are not within the simple quark model as either $qq\overline{q}$, or $qqq$

- They are theoretically possible, not being inconsistent with QCD

- The known low mass states are dominated by hadrons composed of the simple quark model configurations, $qq\overline{q}$-bar and $qqq$
Exotic Hadrons: Glueballs

- Composed of gluons alone
- Strongly interaction with $B = Q = S = C = \widetilde{B} = 0$
- Approximate lattice gauge theory calculations yield quark-less states with mass around 1.5-1.7 GeV/$c^2$
Exotic Hadrons: Glueballs

- Example predicted glueball mass spectrum
Exotic Hadrons: Glueballs

• Quark anti-quark pairs will mix with the pure gluon state
• To find an exotic particle that we can be sure is not explained by quark anti-quark we look for quantum numbers that are not possible from quark anti-quark
• These are $P=(-1)^J$ $C=(-1)^{J+1}$
• Which means $0^{+-}$, $1^{-+}$, $2^{+-}$, $3^{-+}$ ....
• The lightest glueball on the spectrum is $2^{+-}$ at about 4 GeV, too heavy to be very stable
Exotic Hadrons: Hybrids

- There is evidence for hybrids:
  - $\pi_1(1400)$ and $\pi_1(1600)$
  - Both $J^{PC} = 1^{-+}$
  - These may be hybrids, or 4 quark states
  - Hybrid is most likely

- There are also many states in 1-2 GeV range with
  \[ I = S = C = \bar{B} = 0 \quad \text{and} \quad J^{pc} = 0^{++} \]
  Quark model does not predict so many
  Are these hybrids?
Exotic Hadrons: Heavy Quarkonia

\[ e^+ + e^- \rightarrow \gamma + \pi^+ + \pi^- + J/\psi \]

and

\[ e^+ + e^- \rightarrow X(4260) \rightarrow \pi^+ + \pi^- + J/\psi, \]

\[ M = 4251 \pm 9 \text{ MeV}/c^2, \quad \Gamma = 120 \pm 12 \text{ MeV}. \]

Why doesn’t it decay to charm pairs?
Is this a four quark state?
Figure 7.10 Invariant mass distribution of the heaviest $J/\psi\pi^-$ and $J/\psi\pi^-$ pairs observed in the reaction $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ by the BESIII Collaboration (adapted from Ablikin et al., 2013) and the Belle Collaboration (adapted from Liu et al., 2013). (Copyright, American Physical Society, reproduced with permission.)

Likely $c\, c\bar{b}ar\, u\, d\bar{b}ar$ and $c\, c\bar{b}ar\, d\, u\bar{b}ar$
Pentaquarks
Pentaquarks

\[ \Lambda_b^0 \rightarrow K^- + p + J/\psi. \]

**Figure 7.11** Invariant mass of (a) \( K^-p \) and (b) \( J/\psi p \) combinations from \( \Lambda_b^0 \rightarrow J/\psi K^-p \) decays. The black line is the expectation from phase space; the blue line is the data. (Adapted from Aaij et al. 2015, with permission from the American Physical Society.)

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Pentaquarks

\[ \Lambda_b^0 \rightarrow K^- + P_c^+; \quad P_c^+ \rightarrow J/\psi + p \]

\[ P_c^+ = u u d c \overline{c} \overline{u} \]
The quark-gluon plasma

(a) Heavy nucleus

(b) Gluon field

(c) \( \gamma, e^-, e^+ \)

(d) Hadrons
Figure 7.13 Dimuon invariant-mass distributions in PbPb (left) and $pp$ (right) data at a total centre-of-mass energy of 2.76 TeV. (Adapted from S. Chatrchyan et al. 2012, with permission from the American Physical Society.) The same reconstruction and analysis selection criteria are applied to both datasets. The solid (signal + background) and dashed (background) curves show the results of the simultaneous fit to the two datasets.