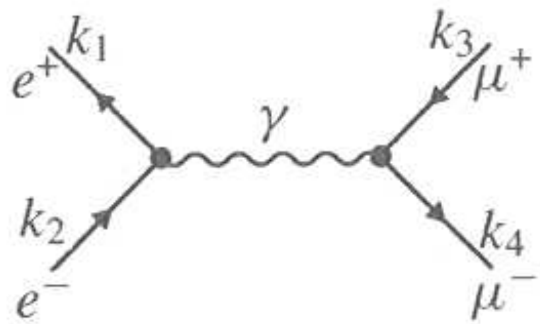
Quarks and Partons

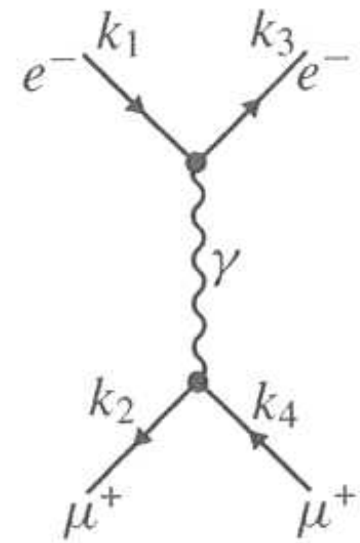
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon

Electron-muon scattering, $e^- \mu^+ \to e^- \mu^+$

s channel



t channel



- Mandelstam variables:

$s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

$t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

$u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4$

for $m = 0$

$k = (p_x, p_y, p_z, iE)$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Mandelstam variables:

  $s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

  suppose $k_1 = (p, iE)$ $k_2 = (-p, iE)$

  $s = -(p-p)^2 - (2iE)^2 = 4E^2$ for m = 0

  $t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

  suppose $k_3 = (p\cos\theta, p\sin\theta, 0, iE)$

  $t = (p - p\cos\theta)^2 + (-p\sin\theta)^2 + (E - E)^2 = 2p^2(1-\cos\theta)$

  $= 4p^2\sin^2\theta/2$

  $u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4 = 4p^2\cos^2\theta/2$

Electron-muon scattering, $e^- \mu^+ \to e^- \mu^+$

- Annihilation cross-section ($e^+e^- \to \mu^+\mu^-$)
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{t^2 + u^2}{s^2}\right) = \frac{\alpha^2}{8p^2}\left[\sin^4(\theta/2) + \cos^4(\theta/2)\right]$$

$$= \frac{\alpha^2}{4s}\left[1 + \cos^2\theta\right]$$

$2\sin^2(\theta/2) = 1 - \cos\theta$

$2\cos^2(\theta/2) = 1 + \cos\theta$

$s = 4p^2$

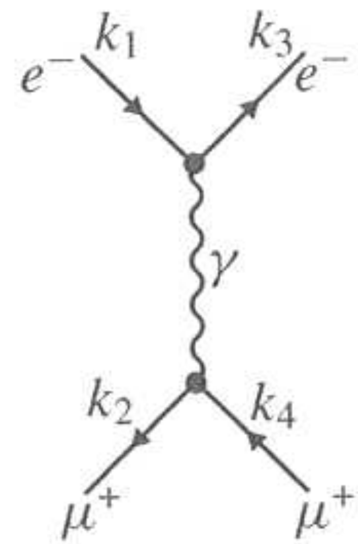
- Now consider the crossed channel, $e^- \mu^+ \to e^- \mu^+$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

<u>crossed channel</u> $(s \leftrightarrow -t)$

- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{s^2 + u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{8p^2 \sin^4(\theta/2)}\left[1 + \cos^4(\theta/2)\right]$$



So far, all of these expressions are for the center-of-momentum system

What about the laboratory frame?

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Suppose the muon is a target at rest in the laboratory
  (Note: this is not practical since the lifetime of the muon is 2.2 microseconds)
- γ is the boost from cms to lab
- p and θ are the projectile parameters in the cms

- In the lab (neglecting lepton masses):

$$E_\mu = \gamma (p - \beta p \cos\theta) \quad \text{(scattered muon)}$$
$$= \gamma p (1 - \cos\theta)$$
$$E_e = \gamma (p + \beta p) = 2\gamma p \quad \text{(incident electron)}$$
$$y \equiv E_\mu / E_e = (1 - \cos\theta)/2$$

$$\text{so } 1-y = 1 - (1 - \cos\theta)/2 = (1+\cos\theta)/2 = \cos^2\theta/2$$
$$d\Omega = 2\pi\, d(\cos\theta) = 4\pi\, dy$$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} [1 + \cos^4(\theta/2)]$$

$d\,\Omega = 4\pi\, dy$
$\cos^2 \theta/2 = 1-y$
$q^2 = t = 4p^2 \sin^2 \theta/2$

$$\frac{d\sigma}{4\pi dy} = \frac{\alpha^2}{q^4 / 2p^2} [1 + (1-y)^2]$$

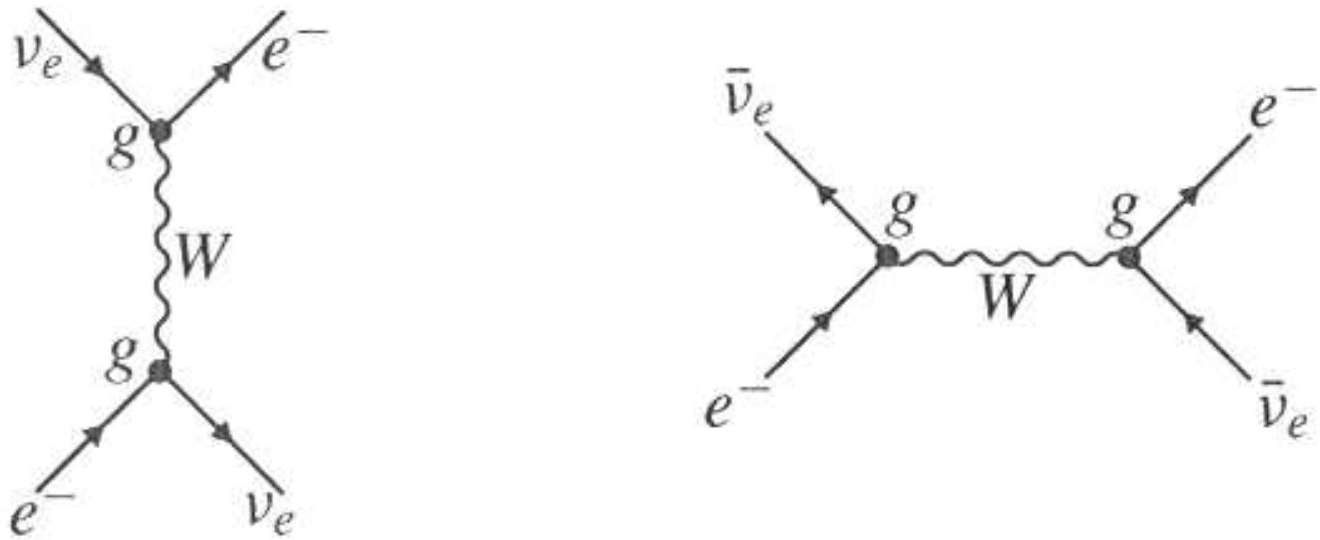
same helicity scattering

$$\frac{d\sigma}{dy} = \frac{2\pi\, \alpha^2 s}{q^4} [1 + (1-y)^2]$$

opposite helicity scattering

As y -> 0, we recover the Rutherford formula

Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$



Consider only the charged-current reaction

$$M(\nu_e\, e^- \to \nu_e\, e^-) = (g/\sqrt{2})^2 / (q^2 + M_W^2)$$

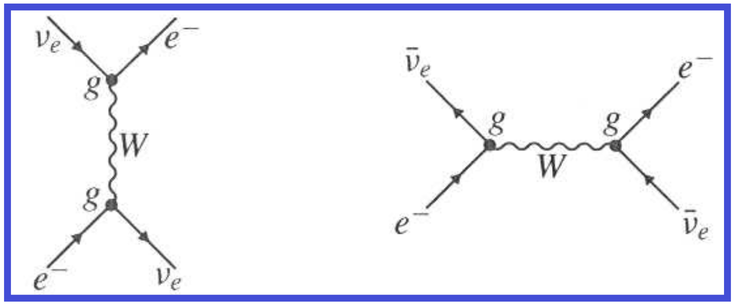
$$= g^2 / 2M_W^2 \quad \text{for } q^2 << M_W^2$$

Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$

$M(\nu_e\, e^- \to \nu_e\, e^-) = g^2/2M_W^2$

$G \equiv \sqrt{2}\, g^2/8M_W^2$

$M(\nu_e\, e^- \to \nu_e\, e^-) = 2\sqrt{2}\, G$



$$\frac{d\sigma}{d\Omega} = \frac{W}{\phi_i} = \frac{W}{v_i} = \frac{2\pi}{\hbar}\frac{|M_{if}|^2}{v_i}\frac{1}{(2\pi\hbar)^3}p_f^2\frac{dp_f}{dE_0}$$

$d\sigma/dq^2 = G^2/\pi$

$\sigma = G^2 s/\pi$ since $q^2_{max} = s$

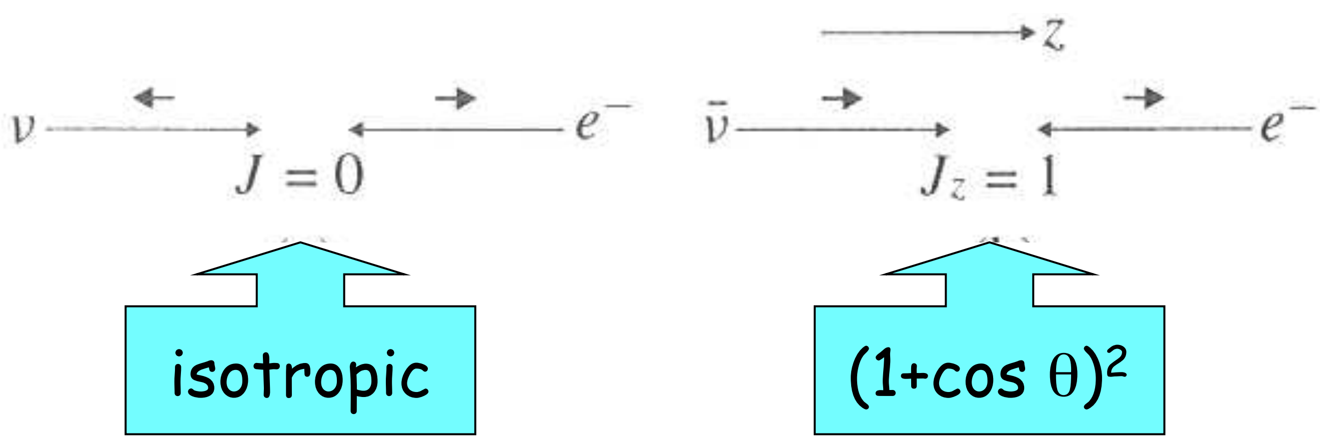
Neutrino-electron scattering, $\nu_e\, e^- \rightarrow \nu_e\, e^-$

- Leptons participate in charged-current weak interactions as longitudinally polarized particles

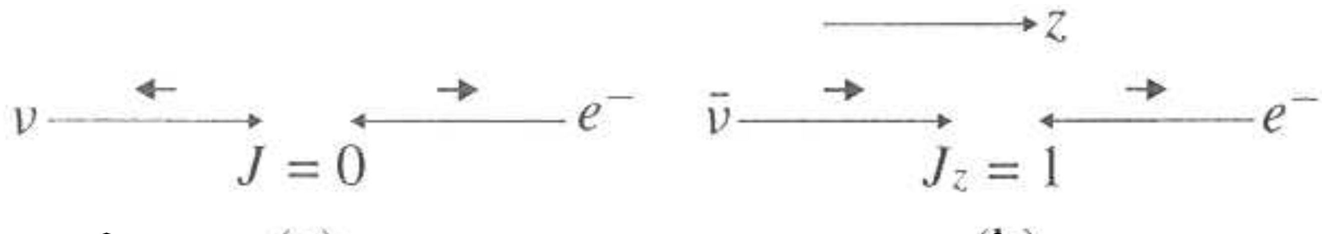
$$P = (I_+ - I_-)/(I_+ + I_-) = \alpha\, p/E = \alpha\, v/c$$

$\alpha = -1$ for leptons

$\alpha = +1$ for antileptons



Neutrino-electron scattering, $\nu_e\, e^- \rightarrow \nu_e\, e^-$



isotropic

$(1+\cos\theta)^2$

$\sigma = G^2 s/\pi$

$d\sigma/d\cos\theta = (G^2/8\pi)s\,(1+\cos\theta)^2$

$\sigma = (G^2/3\pi)s$

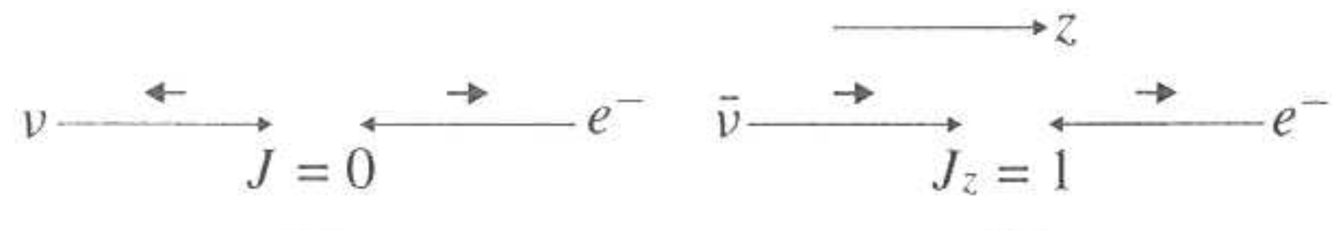
$$\sigma(\nu_e\, e^- \rightarrow \nu_e\, e^-) = 3\ \sigma(\overline{\nu}_e\, e^- \rightarrow \overline{\nu}_e\, e^-)$$

Neutrino-electron scattering, $\nu_e\, e^- \rightarrow \nu_e\, e^-$

Consider the cross section in terms of $y = E_e/E_\nu$ in the lab:

as before, $(1+\cos\theta)^2 = 4(1-y)^2$

and $d \cos\theta = 2\, dy$



$\nu_e\, e^- \rightarrow \nu_e\, e^-$

$d\sigma/dy = G^2 s/\pi$

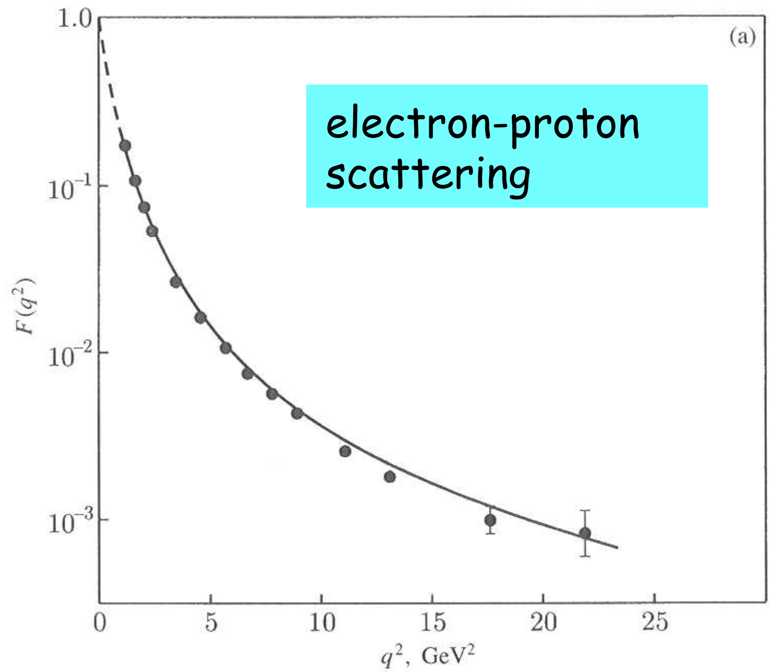
$\overline{\nu}_e\, e^- \rightarrow \overline{\nu}_e\, e^-$

$d\sigma/dy = (G^2/\pi)\, s\, (1-y)^2$

Elastic Lepton-Nucleon Scattering

If the nucleon were a point like particle the cross section would follow the $e\mu \rightarrow e\mu$ calc.

$$\frac{d\sigma}{dq^2} = \frac{2\pi\,\alpha^2}{q^4}\left[1 + (1-q^2/s)^2\right]$$



But this equation must be modified by the addition of a form factor

$$\frac{d\sigma}{dq^2} \rightarrow \frac{d\sigma}{dq^2}\, F(q^2)$$

therefore, the nucleon has structure

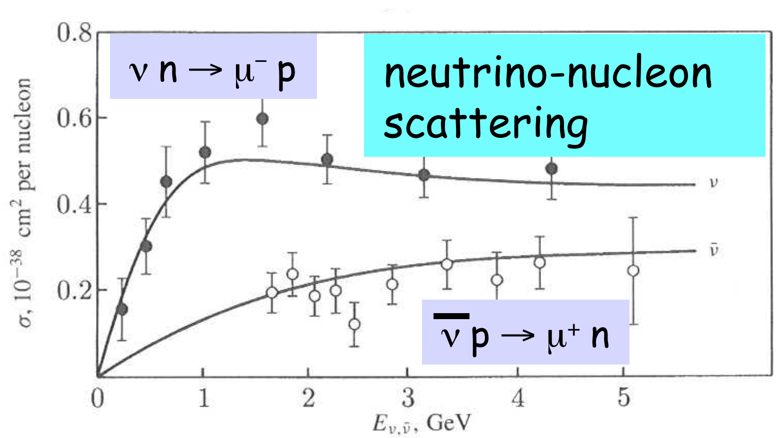
Elastic Lepton-Nucleon Scattering

Neutrino cross-section is expected to rise with s:

$$d\sigma/dy = G^2 s/\pi$$

However, elastic and quasi-elastic cross section saturate due to strong q^2 dependence of form factor

This appears to represent an exponential charge distribution



Deep Inelastic Scattering and Partons

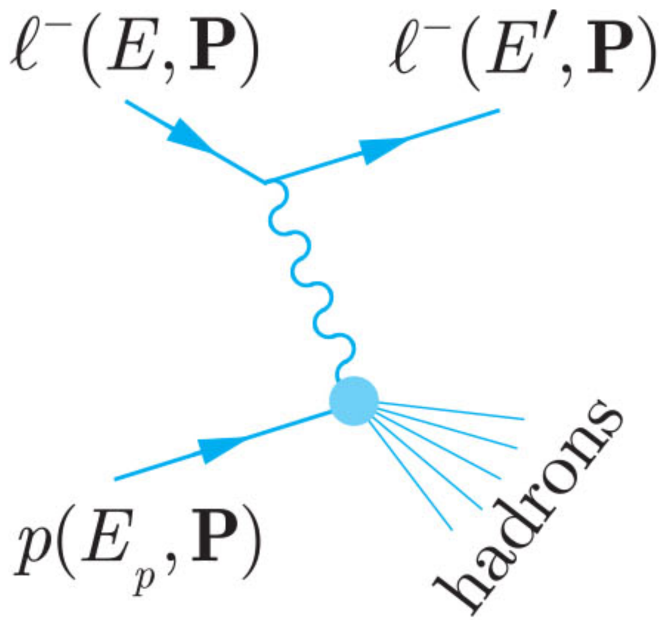


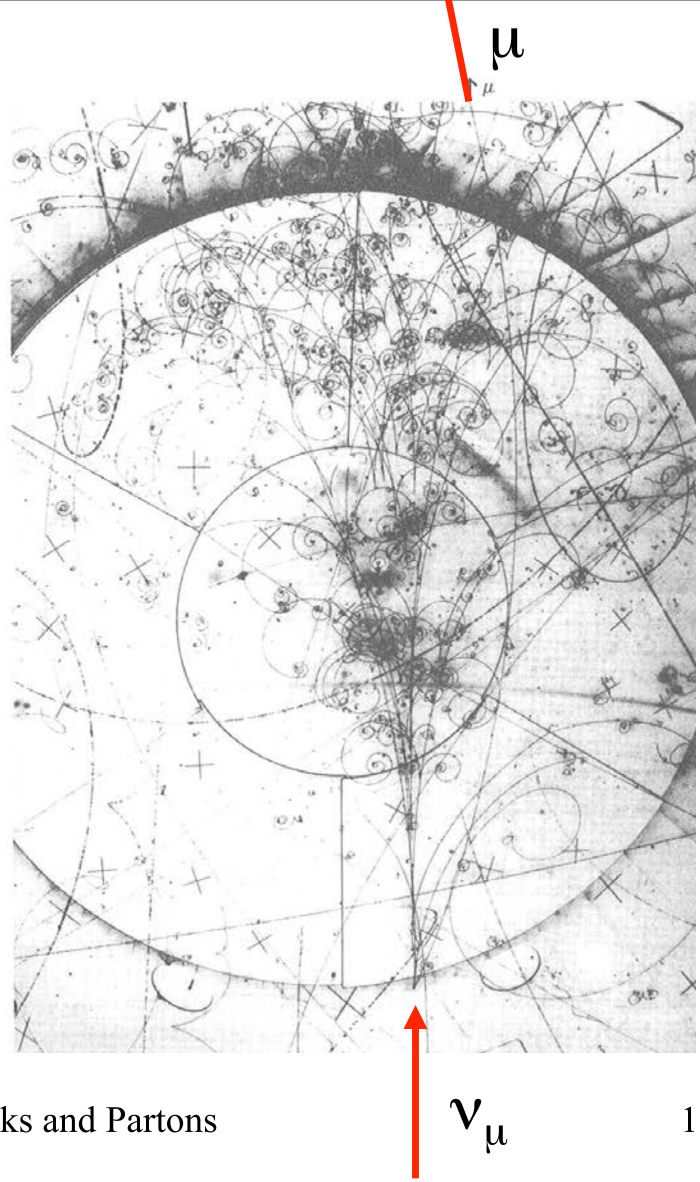
Figure 8.3 Dominant one-photon exchange mechanism for inelastic lepton–proton scattering, where $\ell = e$ or μ.

Deep Inelastic Scattering and Partons

Consider inclusive processes

lepton + nucleon
→ lepton' + anything

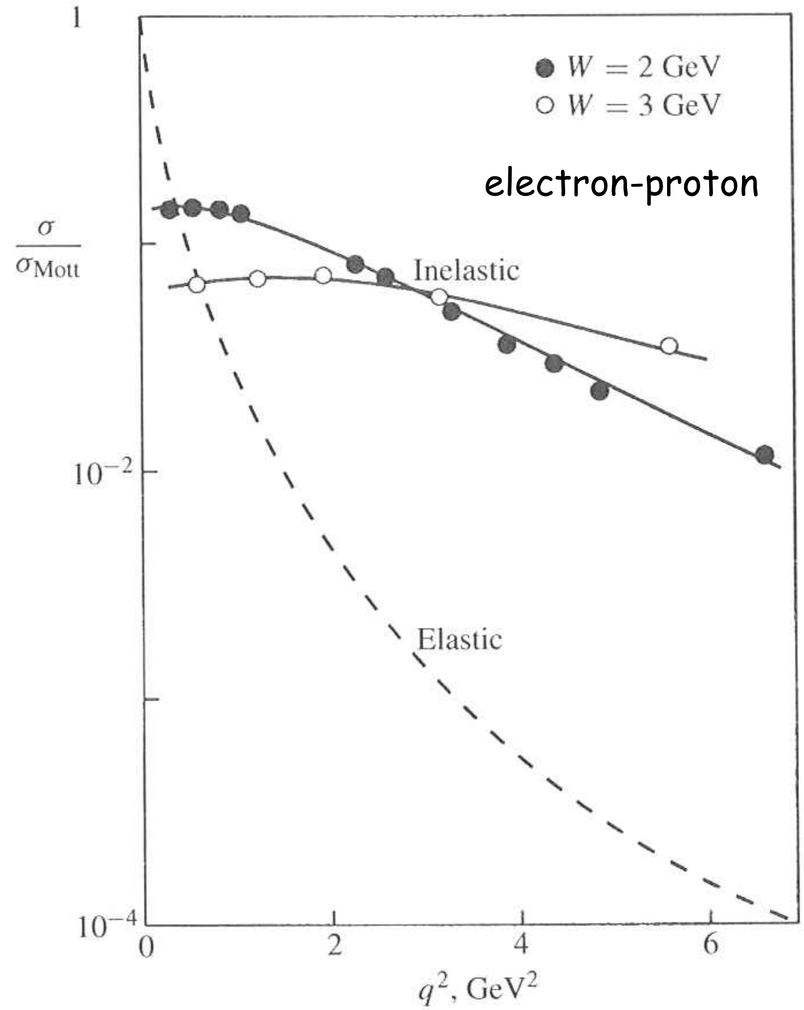
Example of such an interaction with neutrinos



Deep Inelastic Scattering and Partons

These inclusive reactions are observed to be only weakly q^2 dependent, unlike the case for exclusive reactions

This is a sign of elastic scattering from point-like constituents within the nucleon



Deep Inelastic Scattering and Partons

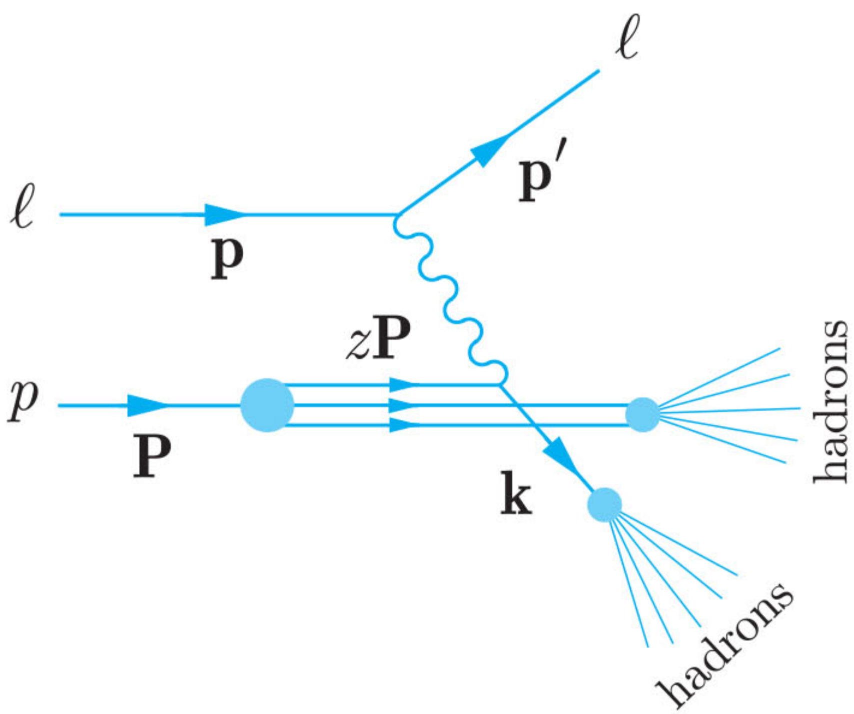
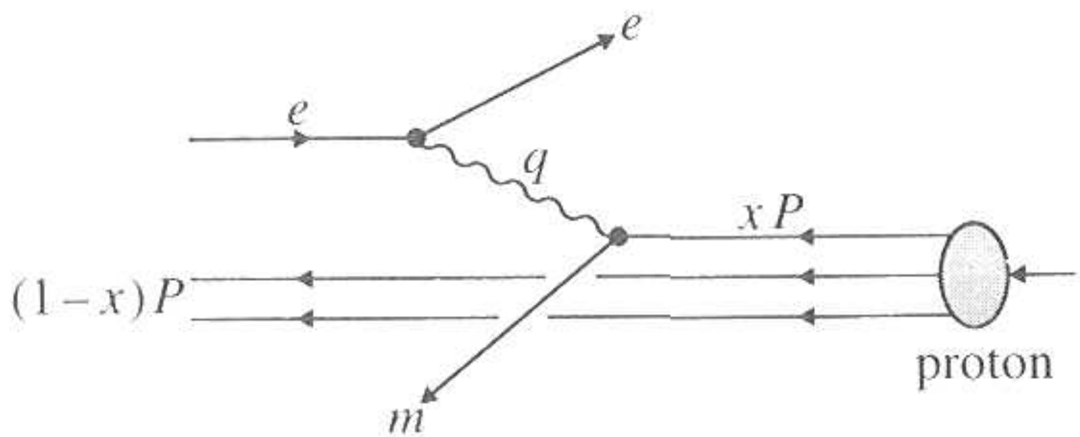


Figure 8.4 Dominant contribution to deep inelastic lepton–proton scattering in the quark model, where $\ell = e$ or μ.

Deep Inelastic Scattering and Partons

Infinite momentum frame (Feynman)

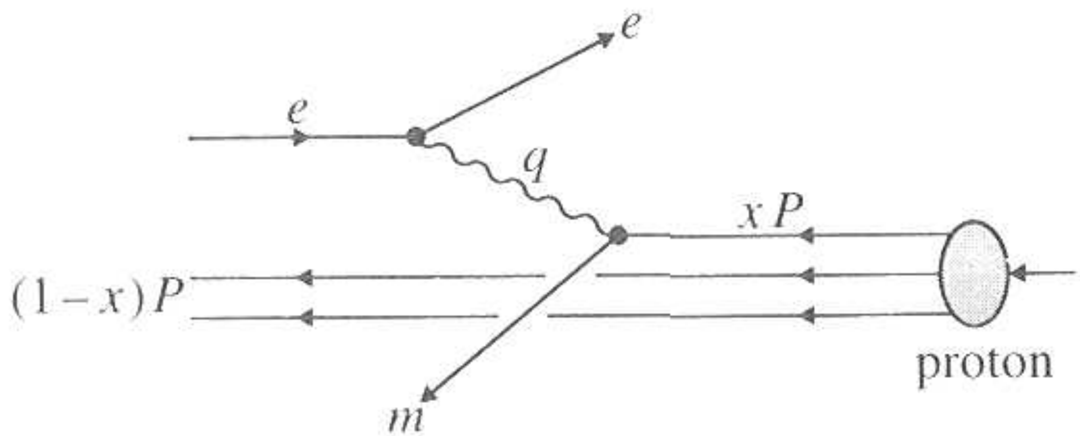


Very large proton momentum

proton $P = (p,0,0,ip)$ as mass can be neglected
each parton has 4-momentum xp
now scatter by absorbing q

$(xP + q)^2 = -m^2 \approx 0$

Deep Inelastic Scattering and Partons



$$(xP + q)^2 = -m^2 \approx 0$$

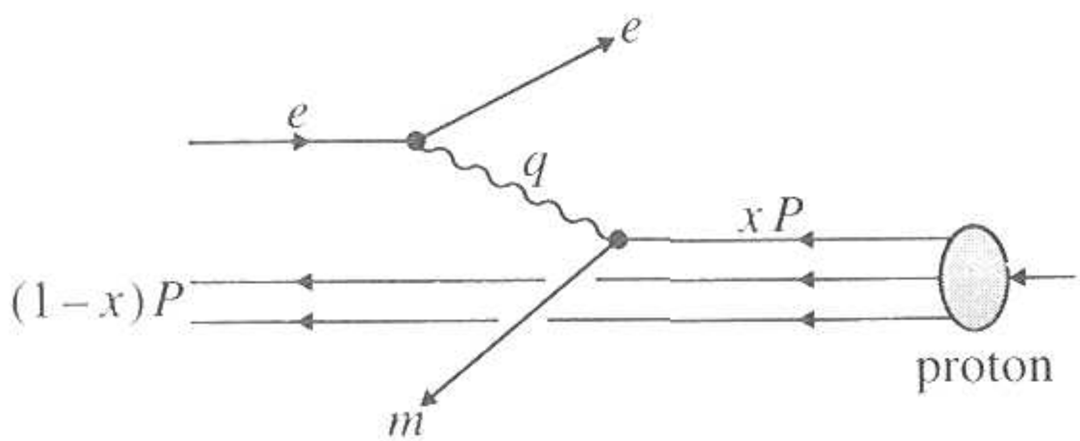
$$x^2 P^2 + q^2 + 2xPq \approx 0$$

If $|x^2 P^2| = x^2 M^2 \ll q^2$,

$$x = -q^2/2Pq = q^2/2M\nu$$

(ν is the energy transfer in the lab.)

Deep Inelastic Scattering and Partons



This model assumes the process occurs in two stages:

1. One parton is scattered

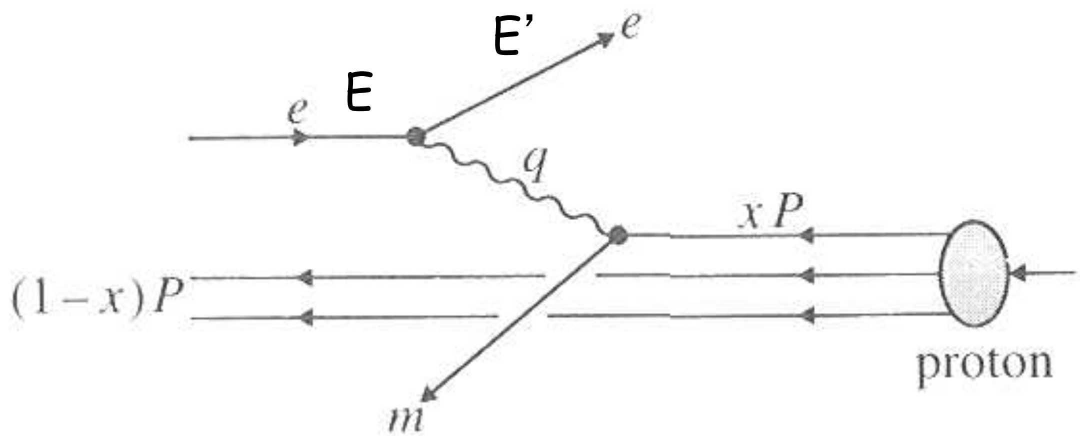
    collision time $t_1 \sim \hbar/\nu$

2. Partons recombine to form final state (M=W)

    time $t_2 \sim \hbar/W$

    $t_2 \gg t_1$

Deep Inelastic Scattering and Partons



$x = q^2/2M\nu$

(ν is the energy transfer in the lab)

$\nu = E\text{-}E'$

$q^2 = (\vec{p}-\vec{p}')^2 - (E\text{-}E')^2$

Deep Inelastic Electron-Nucleon Scattering and Quarks

Evidence that partons are quarks:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} [1 + \cos^4(\theta/2)]$$

$e^- \mu^+ \rightarrow e^- \mu^+$

1. ang. dist. characterized by two terms:
   a. $\cos^2(\theta/2)$ - electric (Mott)
   b. $\sin^4(\theta/2)$ - magnetic (spin-flip)

$$[1 + \cos^4(\theta/2)] = 2\cos^2(\theta/2) + \sin^4(\theta/2)$$

Data reveals just this dependence

Deep Inelastic Electron-Nucleon Scattering and Quarks

Consider a model of the partons

$u(x)\,dx$ = number of u quarks carrying momentum fraction $x \rightarrow x+dx$

$d(x)\,dx$ = number of d quarks carrying momentum fraction $x \rightarrow x+dx$

Cross section depends on charge squared:

$$\frac{d^2\sigma(ep)}{dydx} = \frac{4\pi\,\alpha^2 xs}{q^4}\,\frac{F_2^{ep}(x)}{x}\,\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\,\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

What about electron-neutron scattering?

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2 xs}{q^4}\,\frac{F_2^{en}(x)}{x}\left[\frac{1 + (1-y)^2}{2}\right]$$

$$F_2^{en}(x) = x\,\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\,\{5/18[d(x)+\bar{d}(x)+ u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{en}(x)}{x}\,\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\,\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

$q(x)\,dx$ = number of q quarks carrying momentum fraction $x \rightarrow x+dx$

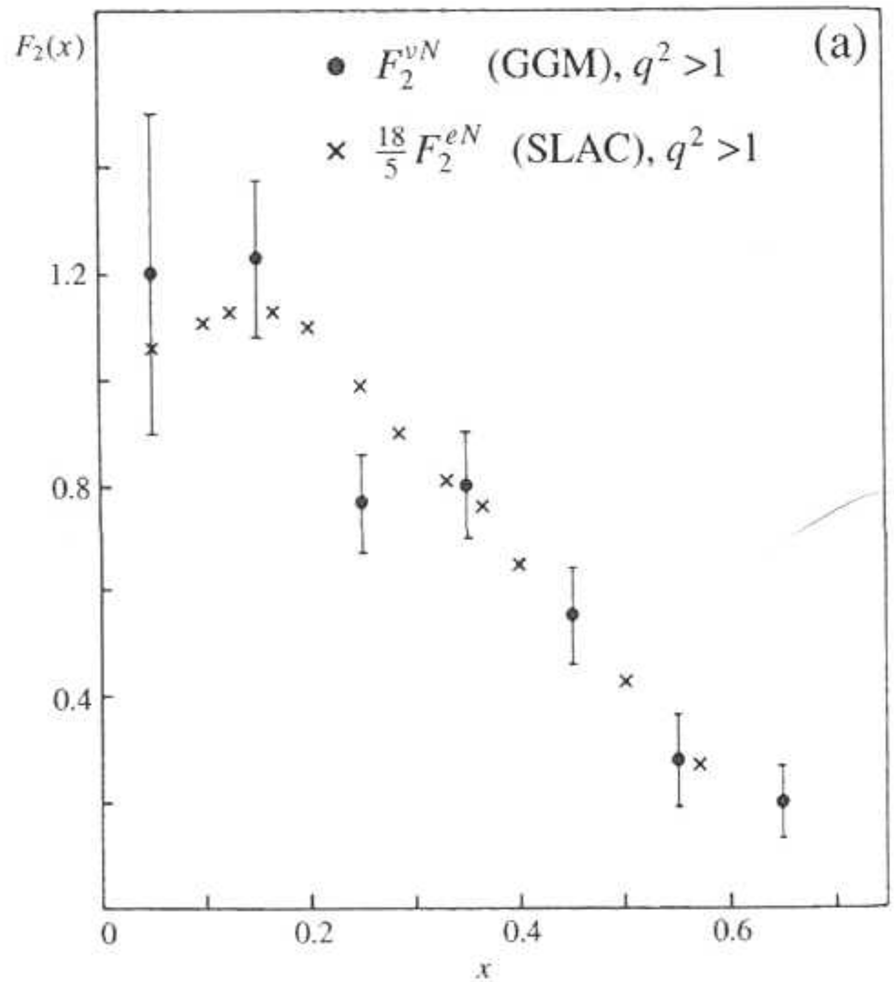
$$F_2^{en}(x) = x\,\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\,\{5/18[d(x)+\bar{d}(x)+u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$

Deep Inelastic Scattering and Quarks



Deep Inelastic Scattering and Quarks

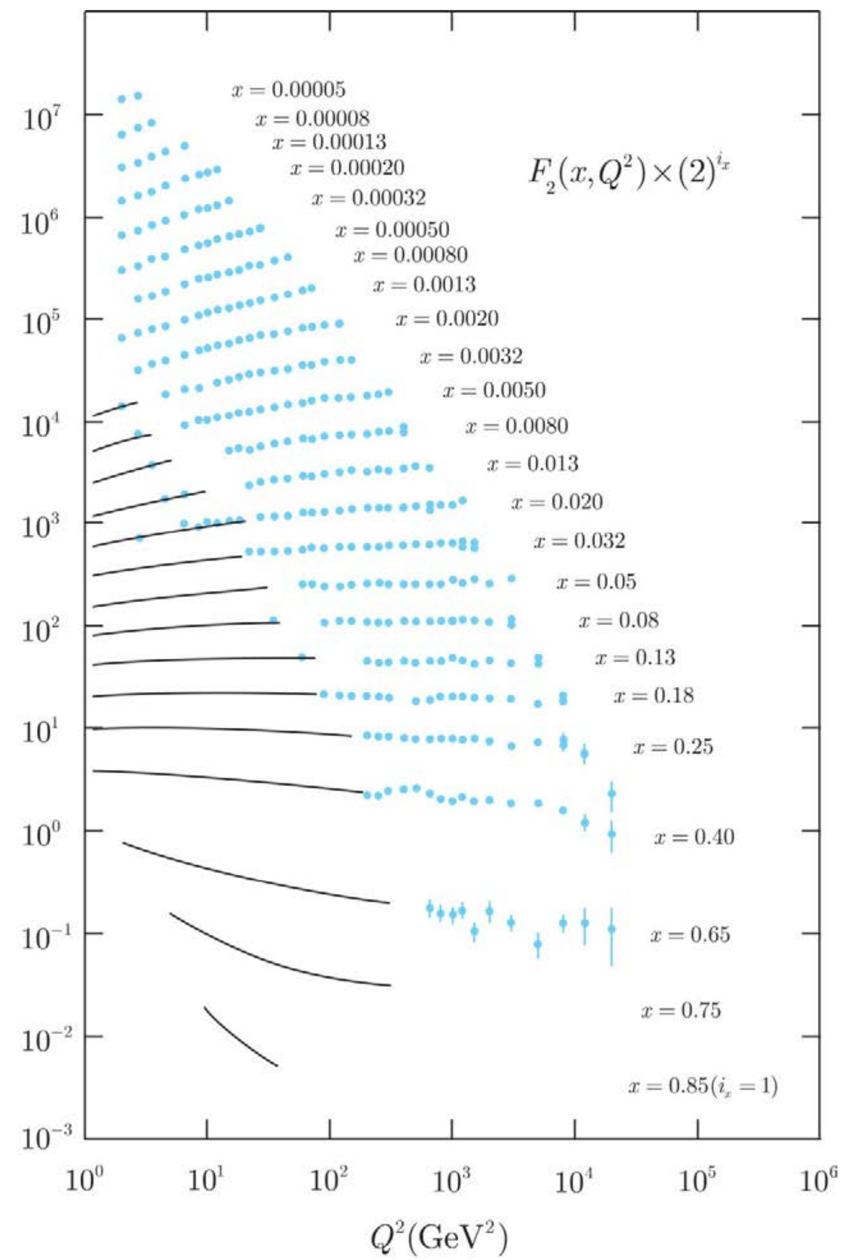


Figure 8.5 Measured values $2^{i_x} \times F_2(x, Q^2)$ for $Q^2 > 1\ \mathrm{GeV}^2$, where $i_x = 1, 2, \ldots, 24$ is an integer labelling the various x-vales in turn starting from $x = 0.85$. The blue points are data obtained using the HERA electron–proton collider at DESY, Hamburg, while the curves indicate the behaviour obtained in several earlier experiments using lepton scattering from a fixed proton target. (Adapted from the review of Foster *et al.*, 2013).

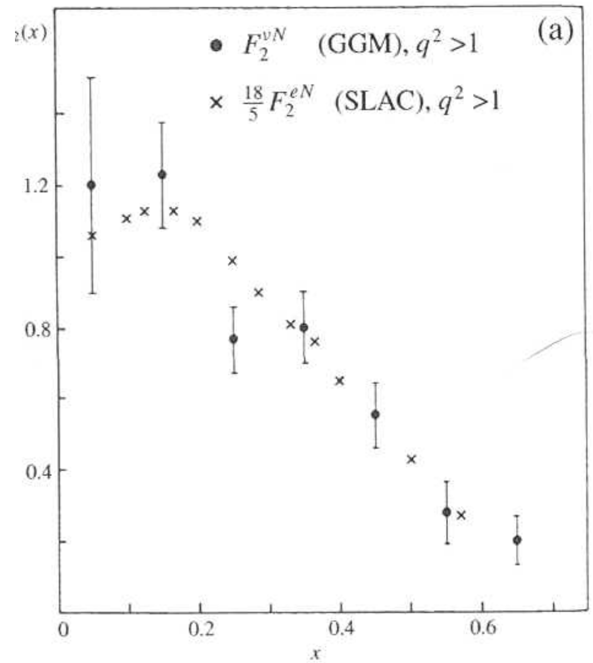
Deep Inelastic Neutrino-Nucleon Scattering and Quarks

Electron-nucleon scattering is sensitive to the parton charge, showing we have +2/3 and -1/3 quarks.

Neutrino nucleon scattering is not sensitive to charge.

Theory predicts:

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



Deep Inelastic Neutrino-Nucleon Scattering and Quarks

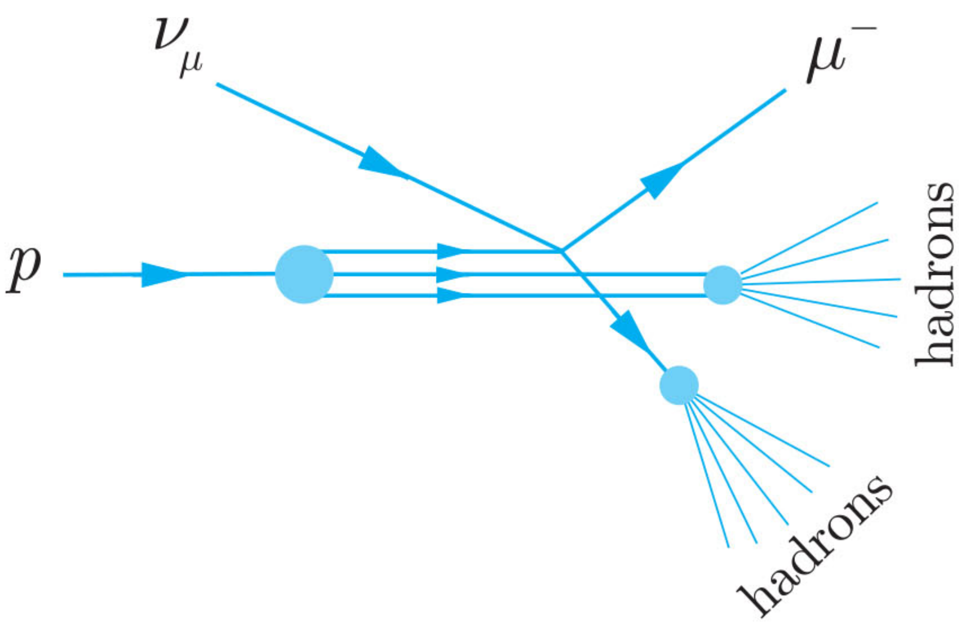
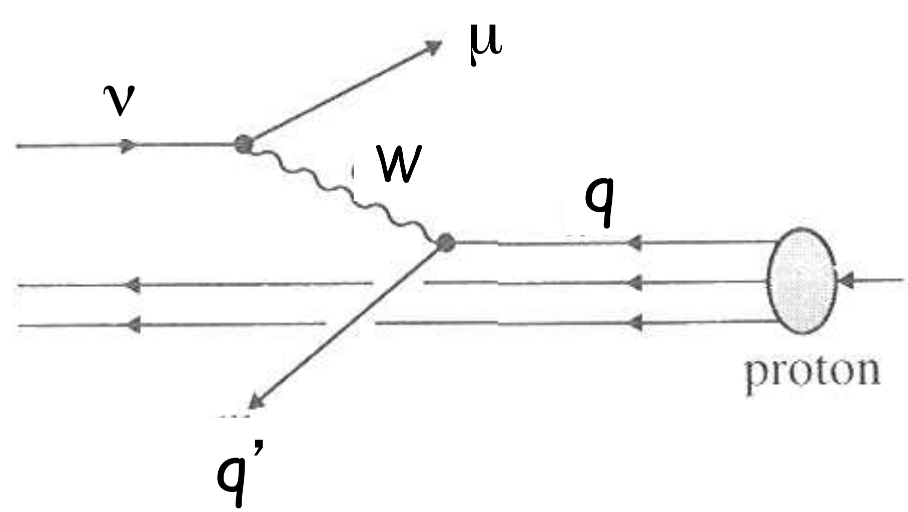


Figure 8.8 Dominant contribution to deep inelastic neutrino scattering in the parton model.

Deep Inelastic Neutrino-Nucleon Scattering and Quarks



Dominant charged current interactions

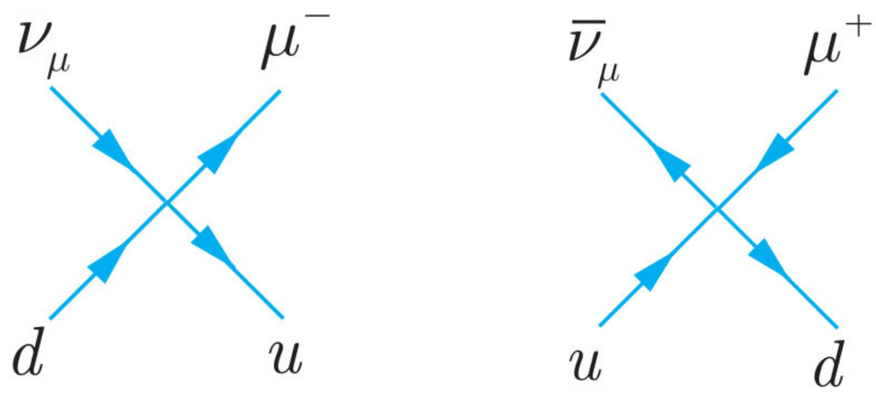
$\nu_\mu + d \rightarrow \mu^- + u$ (W^+)

$\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{d}$ (W^+)

$\bar{\nu}_\mu + u \rightarrow \mu^+ + d$ (W^-)

$\bar{\nu}_\mu + \bar{d} \rightarrow \mu^+ + \bar{u}$ (W^-)

Deep Inelastic Neutrino-Nucleon Scattering and Quarks



(a) (b)

Dominant charged current interactions

$\nu_\mu + d \rightarrow \mu^- + u$ (W+)

$\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{d}$ (W+)

$\bar{\nu}_\mu + u \rightarrow \mu^+ + d$ (W−)

$\bar{\nu}_\mu + \bar{d} \rightarrow \mu^+ + \bar{u}$ (W−)

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

left-handed right-handed

$$d\sigma(\nu p)/dydx = G^2xs/\pi\ [d(x) + \overline{u}(x)(1-y)^2]$$

- through isospin invariance:

$$d\sigma(\nu n)/dydx = G^2xs/\pi\ [u(x) + \overline{d}(x)(1-y)^2]$$

left-handed right-handed

- for isoscalar targets (# p = # n)

$$d\sigma(\nu N)/dydx = G^2xs/2\pi \{[u(x) + d(x)] + [\overline{u}(x) + \overline{d}(x)](1-y)^2]\}$$

- and

$$d\sigma(\bar{\nu} N)/dydx = G^2xs/2\pi \{[u(x) + d(x)](1-y)^2 + [\overline{u}(x) + \overline{d}(x)]]\}$$

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

- Define the neutrino structure functions:

  $F_2^{\nu N}(x) = x\,[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]$

  $F_3^{\nu N}(x) = x\,[u(x)-\bar{u}(x)+d(x)-\bar{d}(x)]$

  Then

$$d\sigma^{\nu N,\,\bar{\nu}N}/dydx = G^2ME/\pi\,\{[F_2 \pm F_3]/2 + [F_2 \mp F_3](1-y)^2/2]\}$$

Defining:

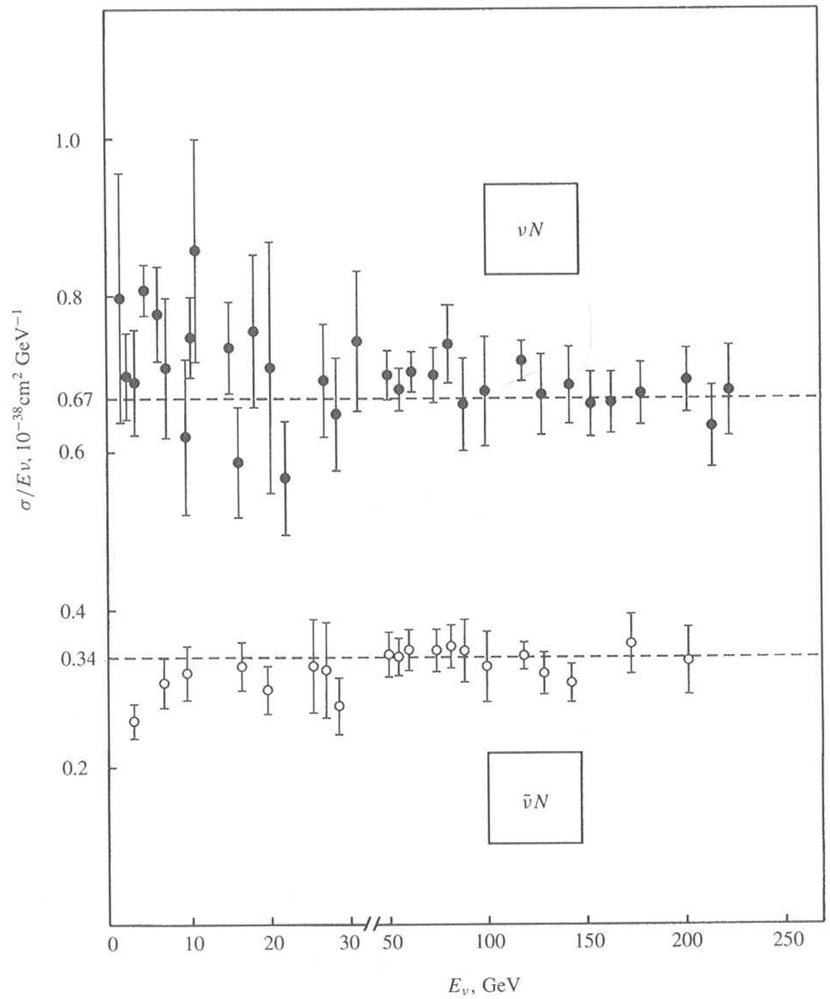
$Q = \int x[u(x)+d(x)]dx$ $\qquad \bar{Q} = \int x[\bar{u}(x)+\bar{d}(x)]dx$

$R = \sigma(\bar{\nu}N)\,/\sigma(\nu N) = (1+3\bar{Q}/Q)/(3+\bar{Q}/Q)$

Quark distributions within the nucleon

- Several predictions of the nucleon model are validated
  - 1.) Total neutrino-nucleon cross section are proportional to energy

    $s = (E_\nu + m_p)^2 - E_\nu^2 = 2 E_\nu m_p$



Quark distributions within the nucleon

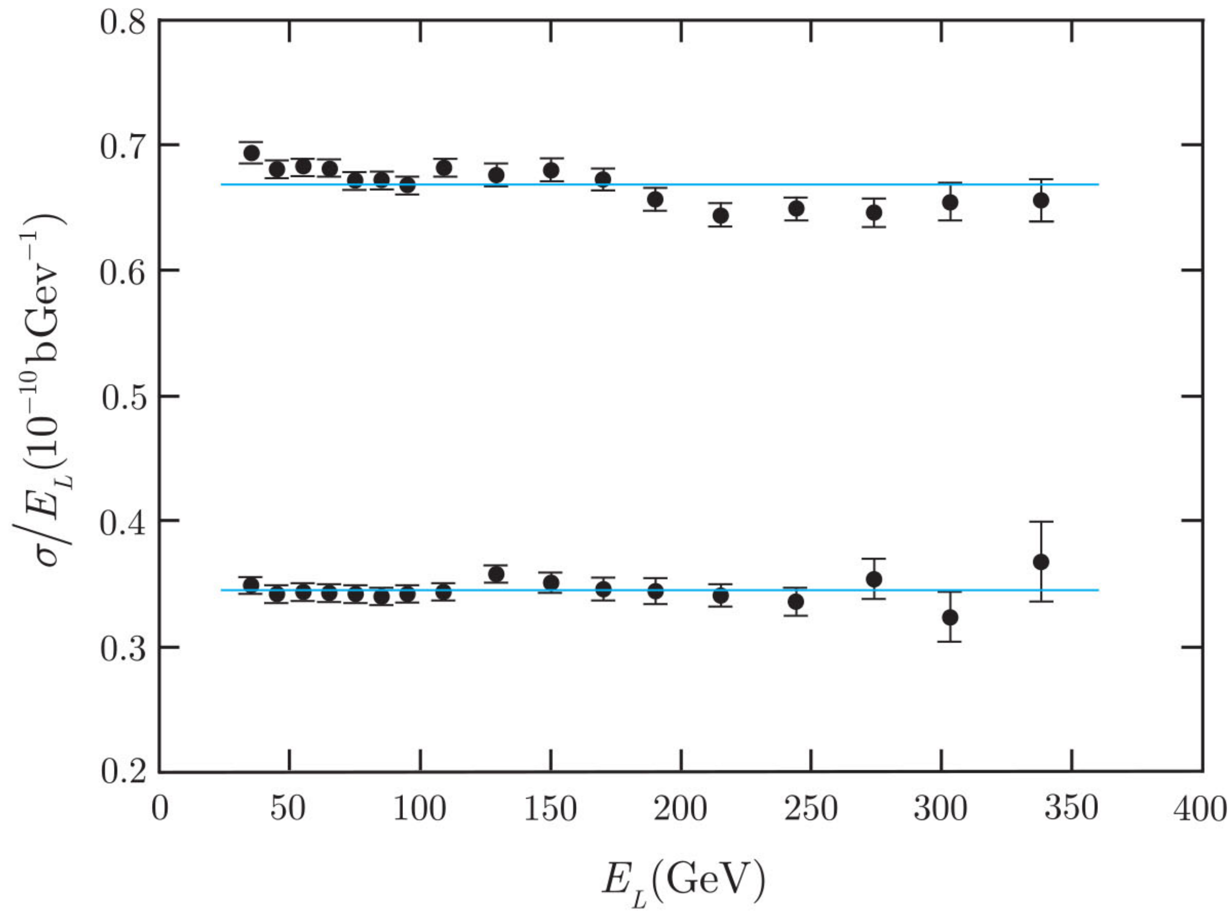
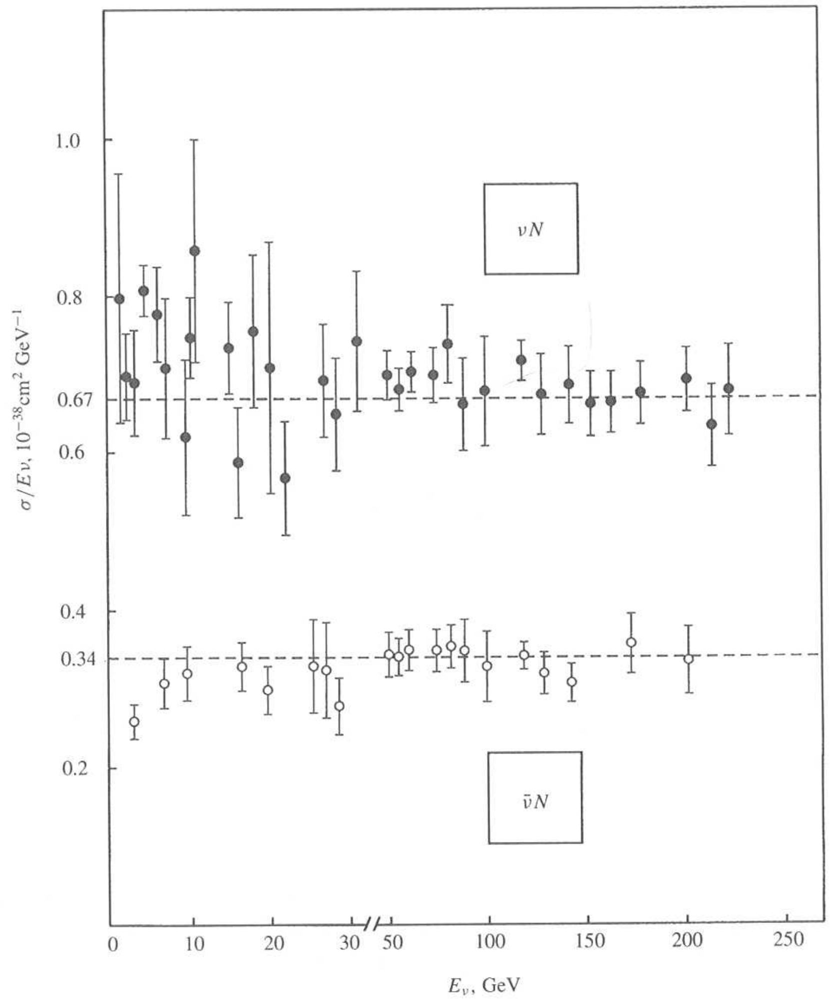


Figure 8.11 Neutrino and antineutrino total cross-sections. (Data from Seligman, 1997)

Quark distributions within the nucleon

- 2.) Ratio of antineutrino to neutrino cross sections indicates nucleon contains antiquarks with $\overline{Q}/Q \approx 0.15$

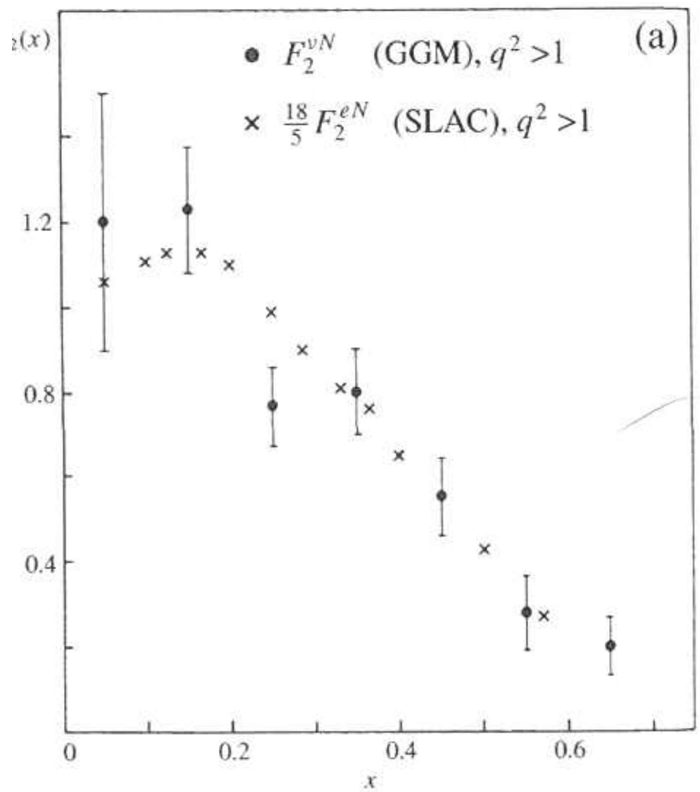
$$R = \frac{1 + 3\,\overline{Q}/Q}{3 + \overline{Q}/Q} \approx 0.45$$



Quark distributions within the nucleon

- 3.) Comparison of neutrino-nucleon cross sections with electron-nucleon cross sections establish fractionally charged quarks are real dynamical entities

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



Quark distributions within the nucleon

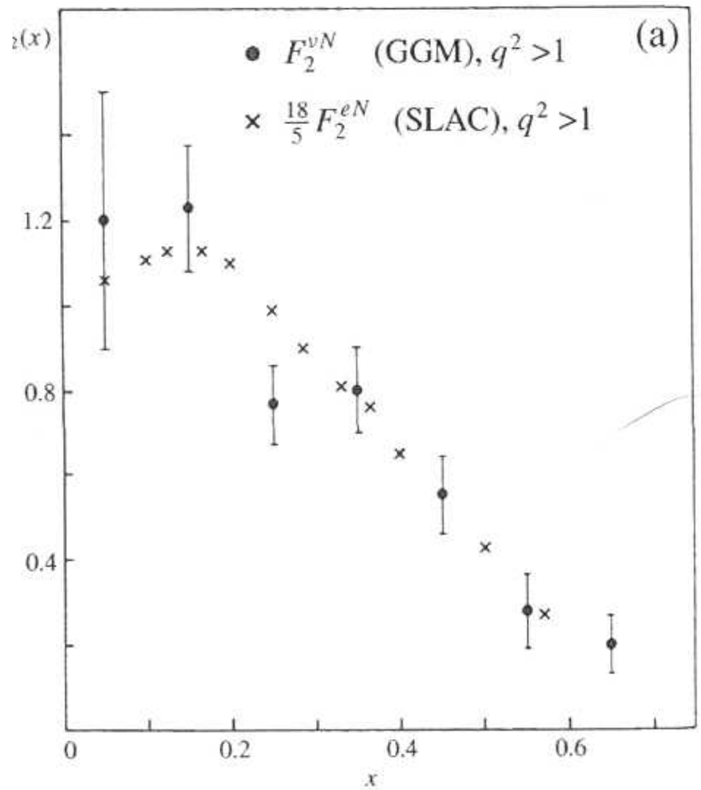
- 4.) Total momentum fraction carried by quarks and anti-quarks is ≈ 0.5

$$\int F_2^{\nu N}(x)\,dx = \tfrac{18}{5}\int F_2^{eN}(x)\,dx$$

$$\approx 0.5$$

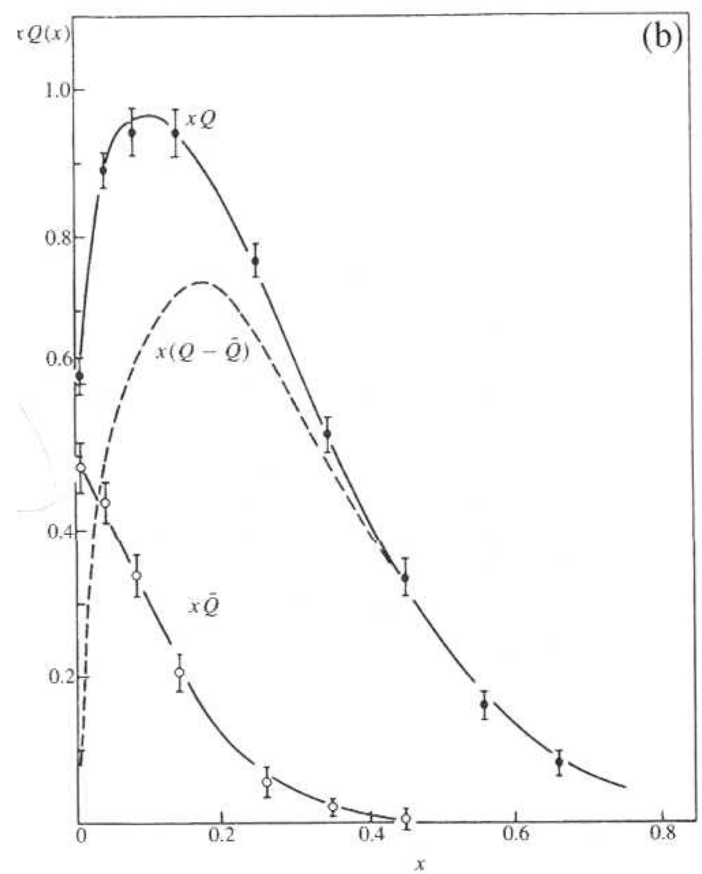
$$= \int x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]\,dx$$

Therefore 50% is carried by partons which do not couple to weak or EM forces (gluons)

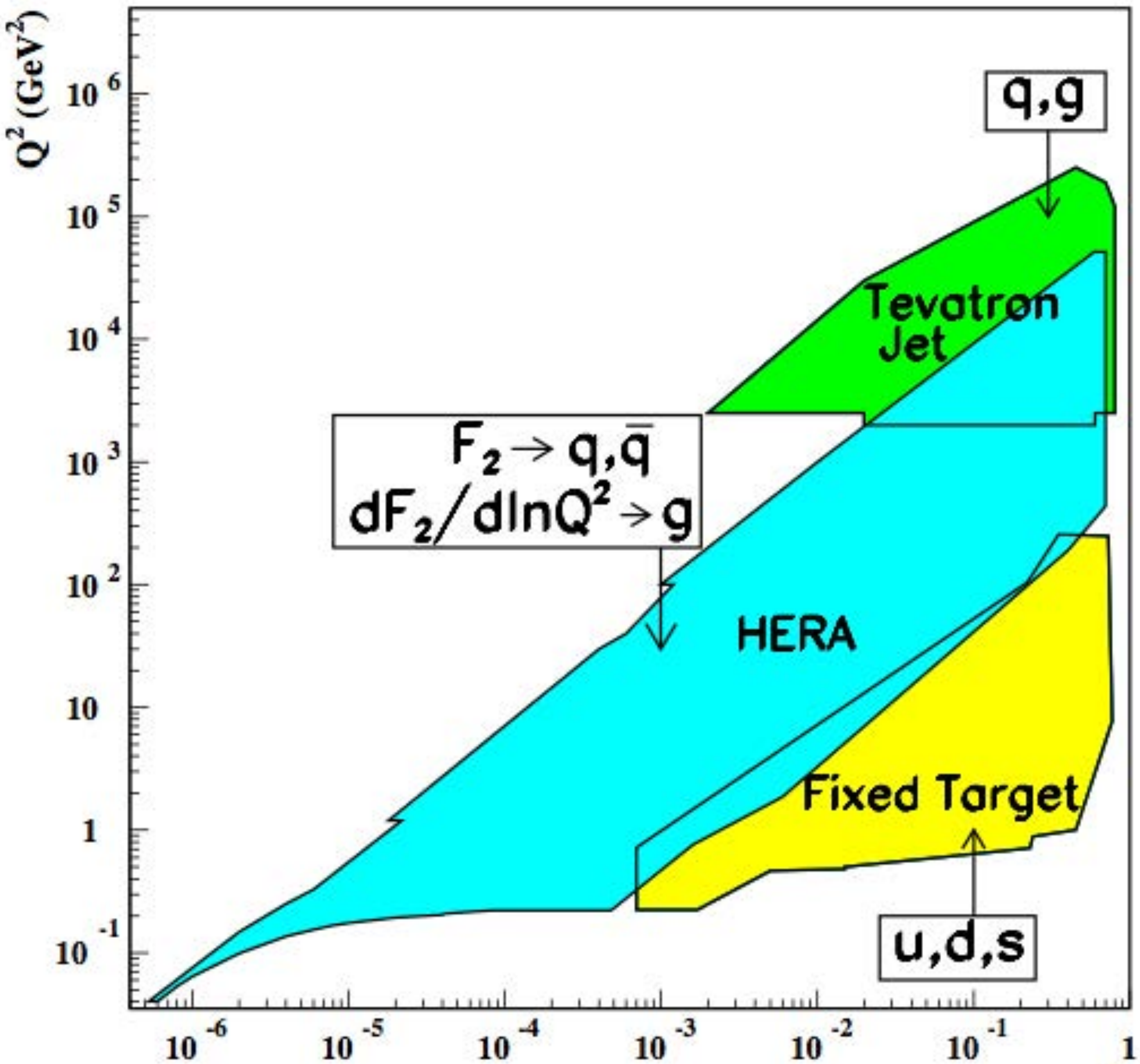


Quark distributions within the nucleon

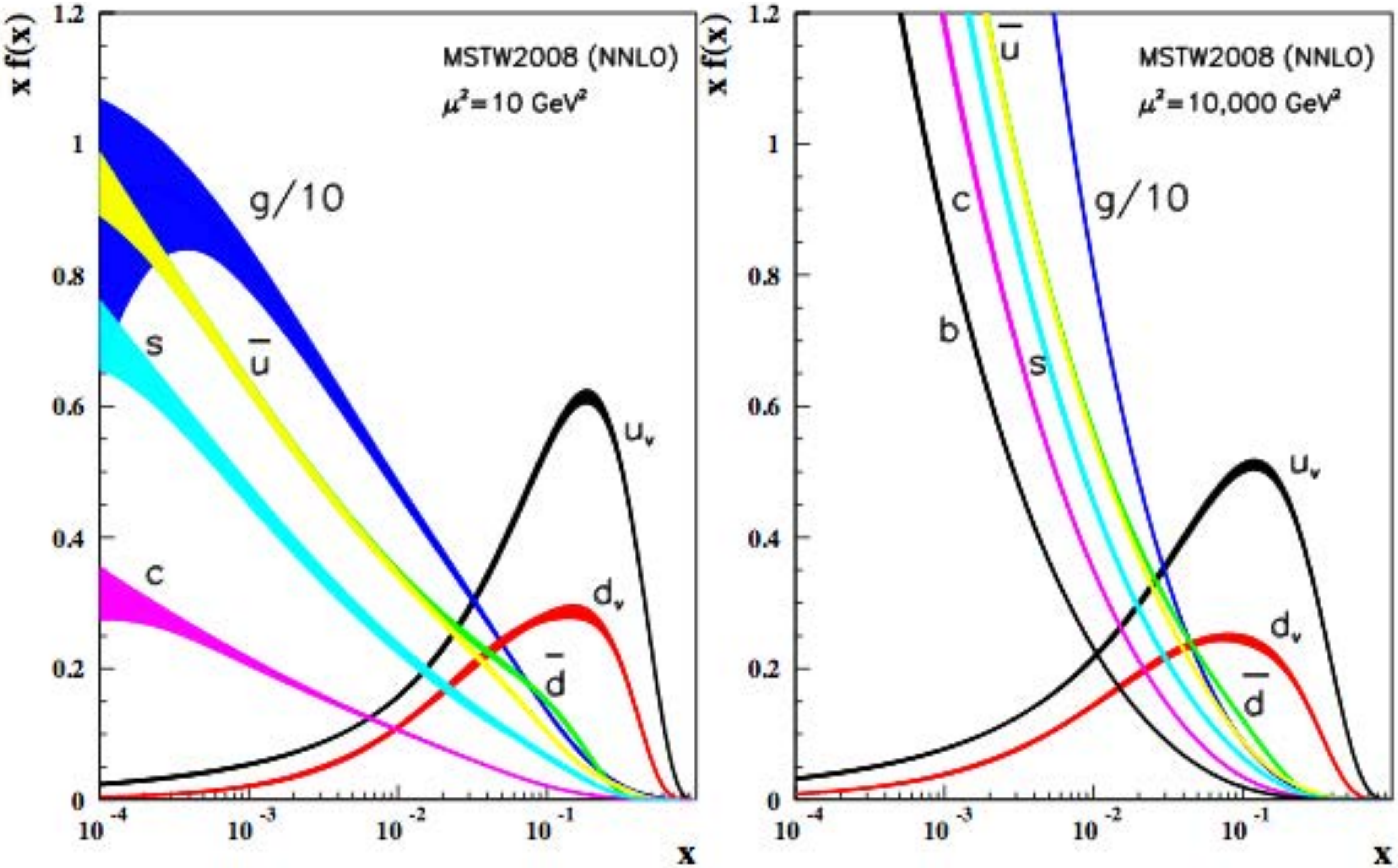
Quark and antiquark distributions can be determined from F_2 and F_3



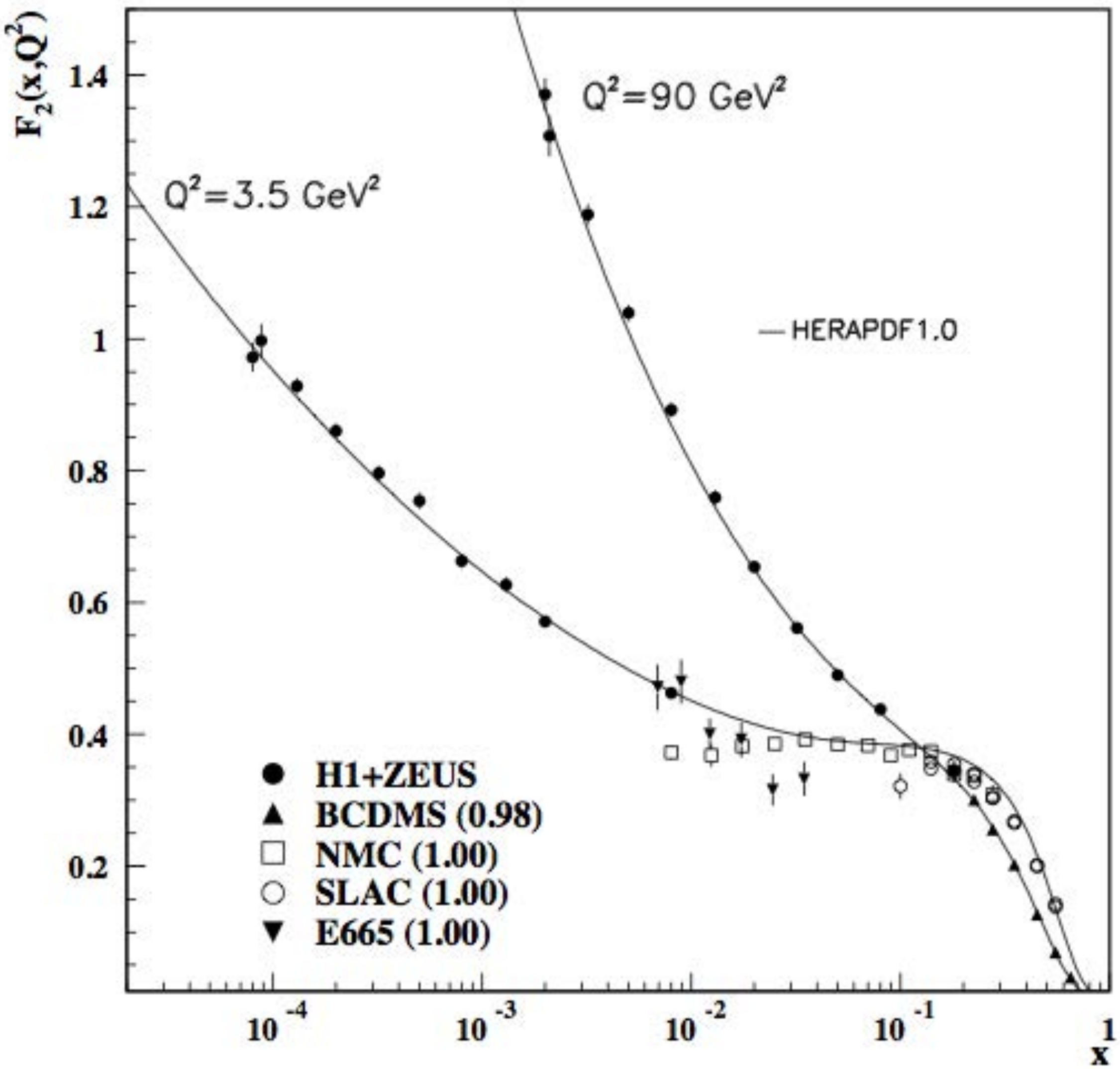
Quark distributions within the nucleon



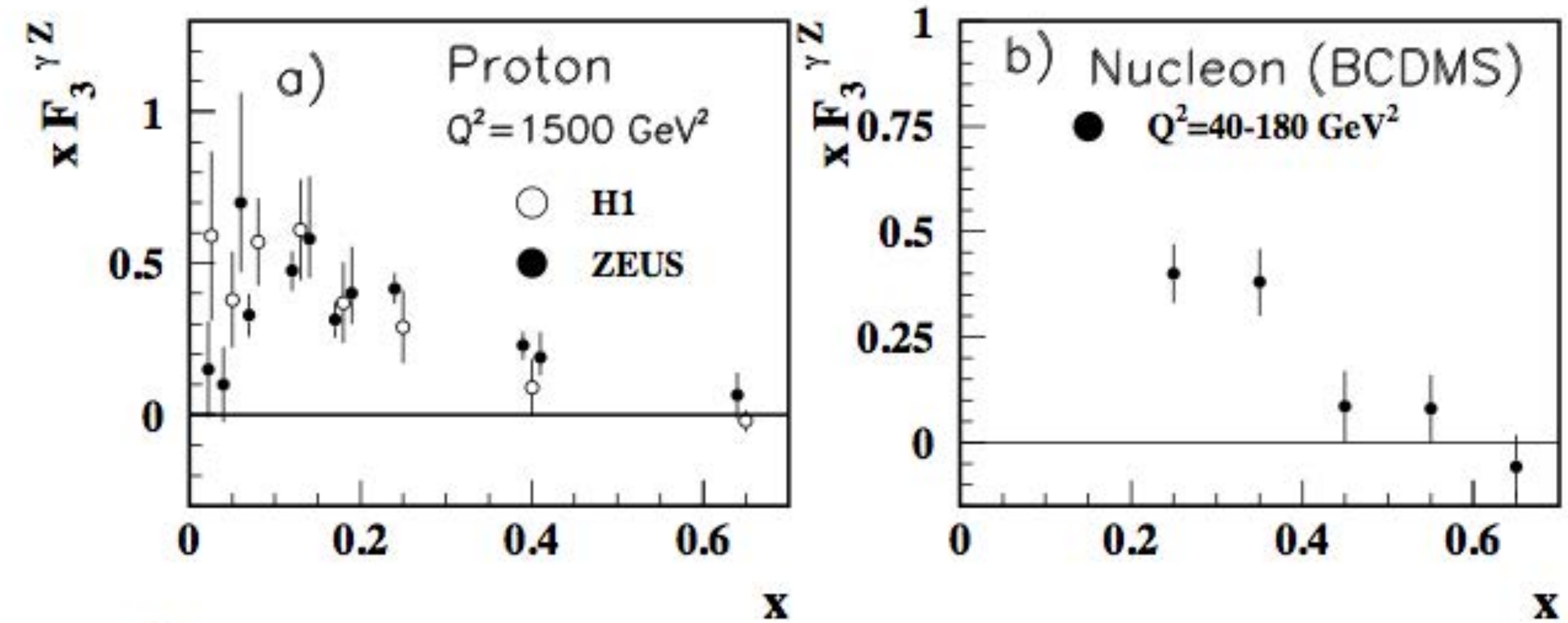
Quark distributions within the nucleon



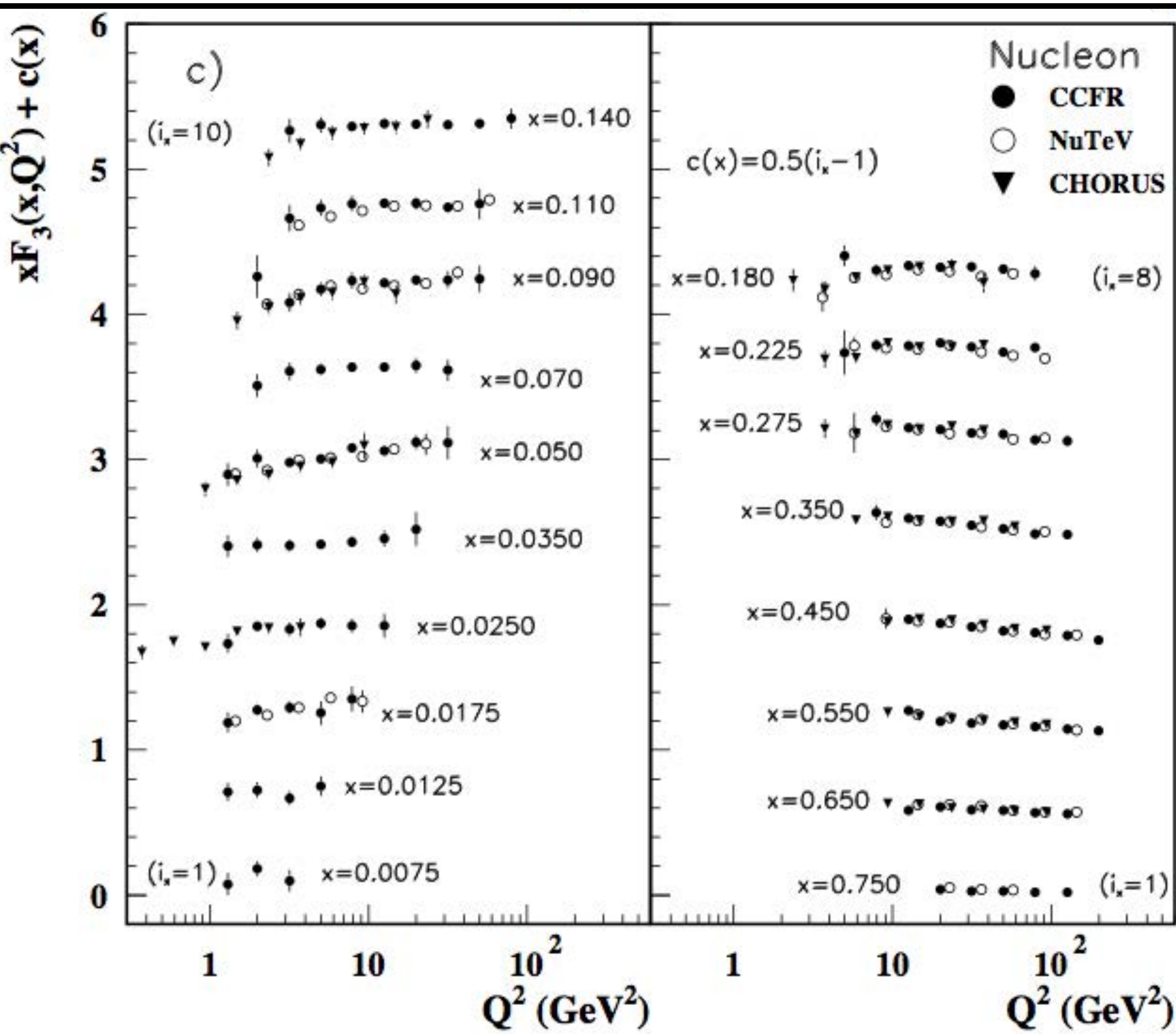
Quark distributions within the nucleon



Quark distributions within the nucleon



Quark distributions within the nucleon



Lepton and Quark Scattering

- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon

Data broadly in agreement with parton model

hadrons built of pointlike, quasi-free constituents with QNs of quarks and anti-quarks & gluons accounting for 50%

Deviations associated with interactions between quarks