Physics 661
Problem Set 1

Due: Monday, October 16, 2017

Work the following problems:

1. (a) The values of mc^2 for the pion π^+ and muon μ^+ are 139.57 MeV and 105.66 MeV respectively. Find the kinetic energy of the muon in the decay at rest $\pi^+ \to \mu^+ + \nu_\mu$ assuming that the neutrino is massless. (b) For a neutrino of finite but very small mass m_ν show that, compared with the case of a massless neutrino, the muon momentum would be reduced by the fraction

$$\frac{\Delta p}{p} = -\frac{m_\nu^2(m_\pi^2 + m_\mu^2)}{(m_\pi^2 - m_\mu^2)^2} \simeq \frac{4m_\nu^2}{10^4}$$

   where m_ν is in MeV.

2. Deduce an expression for the energy of a γ-ray from the decay of the neutral pion, $\pi^0 \to 2\gamma$, in terms of the mass m, energy E and velocity βc of the pion and the angle of emission θ in the pion rest frame. Show that if the pion has spin zero, so that the angular distribution is isotropic, the laboratory energy spectrum of the γ-rays will be flat, extending from $E(1+\beta)/2$ to $E(1-\beta)/2$. Find an expression for the disparity D (the ratio of energies) of the γ-rays and show that $D > 3$ in half the decays and $D > 7$ in one quarter of them for β=1.

3. The ρ meson is a particle of spin J = 1 and mass 770 MeV/c^2 occurring in three charge states ρ^+, ρ^0, ρ^-. It decays to a pair of spinless pions. Show that while $\rho^\pm \to \pi^0\pi^\pm$ and $\rho^0 \to \pi^+\pi^-$ are allowed, $\rho^0 \to \pi^0\pi^0$ is forbidden.

4. State which of the following reactions are allowed by the conservation laws and which are forbidden, giving the reasons:

$$\pi^0 \to e^+e^-$$

$$e^- + p \to n + \nu_e$$

$$\mu^+ \to e^+e^-e^+$$

$$K^0 + n \to \Lambda + \pi^0$$

$$\Xi^0 \to \Lambda + \pi^0$$

5. (a) Show that a negative muon captured in an S-state by a nucleus of charge Ze and mass A will spend a fraction $f \simeq 0.25A(Z/137)^3$ of its time inside the nuclear matter, and that in time t it will travel a total distance $fct(Z/137)$ in the nuclear matter. (b) The law of radioactive decay of free muons is $dN/dt = -\lambda_d N$, where $\lambda_d = 1/\tau$ is the decay constant and the lifetime is $\tau = 2.16$ μs. For a negative muon captured in an atom Z the decay constant is $\lambda = \lambda_d + \lambda_c$, where λ_c is the probability per unit time of nuclear capture. For aluminium (Z = 13, A = 27) the mean lifetime of negative muons is 0.88 μs. Calculate λ_c and, using the expression for f in (a), compute the interaction mean free path Λ for a muon in nuclear matter. (c) From the magnitude of Λ estimate the magnitude of the weak coupling constant in the reaction $\mu^- + p \to n + \nu$, assuming that a coupling constant of unity corresponds to a mean free path equal to the range (1 fm) of the nuclear forces.

6. The cross-section for the reaction $\pi^- + p \to \Lambda + K^0$ at 1 GeV/c incident momentum is about 1 mb ($10^{-27} cm^2$). Both the Λ and K^0 particles decay with a mean lifetime of order 10^{-10} s. From this information, estimate the relative magnitude of the couplings responsible for the production and decay, respectively, of these strange particles.

7. The quark-antiquark potential is often taken to be of the form

$$V_s(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Assume k = 0.85 GeV fm^{-1}, and show that the QCD potential leads to a value for the confining force between a pair of quarks equal to 14 tonnes weight.

8. Draw Feynman diagrams (in terms of transitions at the quark level if hadrons are involved) for the following weak decays:

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\Lambda \to p + e^- + \overline{\nu_e}$$
$$K^0 \to \pi^+\pi^-$$
$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

Draw Feynman diagrams for the following strong decays:

$$\omega^0 \to \pi^+ + \pi^- + \pi^0$$
$$\rho^0 \to \pi^+ + \pi^-$$
$$\Delta^{++} \to p + \pi^+$$

9. Show that, in the process of pair conversion $\gamma \to e^+e^-$, it is impossible to conserve energy and momentum without the participation of another particle (a nucleus). Calculate the minimum momentum transfer from this extra particle, for a 1 GeV γ-ray.