Physics 661
Problem Set 3

Due: Tuesday November 28, 2017

1. (a.) A resonance $X^0(1620)$ decays via the strong interaction to the final states $n\pi^0$ and $p\pi^-$ with branching ratios of approximately 16 and 8 %, respectively. What is its isospin? Explain your reasoning.

   (b.) The $\Sigma^0(1385)$ decays by the strong interaction to $\Sigma\pi$ final states with a branching ratio of 12%. What are the branching ratios for the charged modes $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Sigma^-\pi^+$ if the $\Sigma^0(1385)$ is an isotriplet state?

2. Show that in a reaction of the form $a+b \to c+d$ the Mandelstam variables defined as

$$s = -(k_a + k_b)^2$$
$$t = (k_a - k_c)^2$$
$$u = (k_b - k_c)^2$$

   where $k_i = (\vec{p_i}, iE_i)$, are connected by the relation $s - t - u = m_a^2 + m_b^2 + m_c^2 + m_d^2$.

3. Make a rough estimate of the relative magnitude of the static potential between two quarks separated by distances of 0.1 fm and 0.001 fm.

4. Estimate the relative magnitudes of the static potential between (a) a quark-antiquark pair and (b) two protons, both separated by a distance of 3 fm, given that the value of the dimensionless Yukawa coupling,

$$\alpha_X = \frac{g^2}{4\pi\hbar c}$$

   for the $pp\pi^0$ vertex is about 14. Comment briefly on your answer in the light of QCD.

5. Would you expect the widths of three chi states listed in Table 6.14 of Martin and Shaw, 4th edition, to be bigger, smaller or about the same as the widths of the $J\psi(3097)$ and $\psi(3686)$? Check your answer with the experimental widths given in the PDG tables.

6. Show mesons with quantum numbers

$$B = Q = S = C = \tilde{B} = 0$$

and

$$J^{PC} = 0^{--} \text{and} \mathrm{J^{PC}} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, ...$$

are forbidden in the simple quark model.

7. Draw two quark diagrams corresponding to the decay sequence

$$\Lambda_b^0 \to \Lambda^* + J/\psi;$$
$$\Lambda^* \to K^- + p,$$

given that the b quark couples to lighter quarks almost exclusively via the vertex bcW analogous to the udW vertex of Figure 3.1 of Martin and Shaw, 4th edition.

8. Draw quark diagrams for the decay sequence

$$\Lambda_b^0 \to K^- + P_c^+;$$
$$P_c^+ \to J/\psi + p,$$

assuming P_c^+ is the charmonium-pentaquark state $P_c^+ = uudc\bar{c}$.

9. (a.) Estimate, theoretically, the cross-section ratio R, defined by

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

at total center-of-mass energies $E_{CM} = 2.8$, 5 and 15 GeV, assuming that there are no resonance peaks at these energies and that

$$R = R_0(1 + \alpha_s/\pi)$$

defined by Martin and Shaw remains approximately valid even at energies as low as 2.8 GeV.

(b.) How does your estimate compare to experiment?

(c.) How would you expect R to change when the energy becomes large enough to produce $t\bar{t}$ pairs, where t is the top quark?