Weak Interactions

- History of Weak Interaction Theory
- Neutral Weak Currents
- Classification
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- W-lepton Interactions
- Weak decays of quarks and the GIM model
- CKM matrix
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- Top Quark
- Helicity of the Neutrino

Weak Interaction Theory

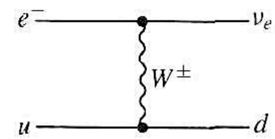
- 1890's – Beta decay discovered.
- 1930's – Pauli proposed neutrino & Fermi developed theory based on four-fermion contact interaction.
- 1938 - Heavy charged particle exchange proposed for beta decay by Oscar Klein.
- 1960's - Unified theory of weak and EM interactions with heavy neutral particle exchange proposed by Glashow, Salaam & Weinberg.
- 1972 - Neutral current interactions observed at CERN.

Classification

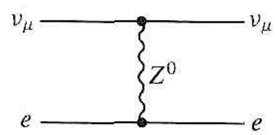
- Weak interactions are mediated by the "intermediate bosons" $W^{\pm}$ and Z^0 (M ~ 80-90 GeV ➔ R ~ 10^{-16} cm)
- Just as the EM force between two current carrying wires depends on the EM current, the weak interaction is between two weak currents, describing the flow of conserved weak charge, g

  $j \propto \psi^* \psi$

- Two types of interactions:
  - CC (charged current)

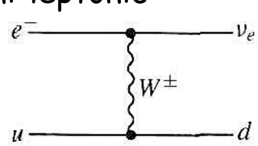


  - NC (neutral current)

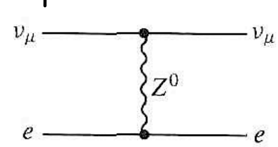


Classification

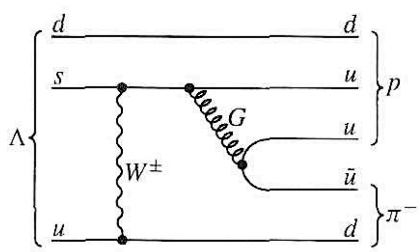
- Weak interactions occur between all types of leptons and quarks, but are often hidden by the stronger EM and strong interactions.
- Semi-leptonic



Leptonic

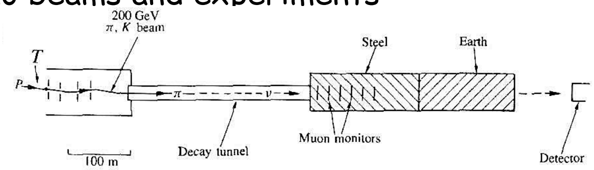


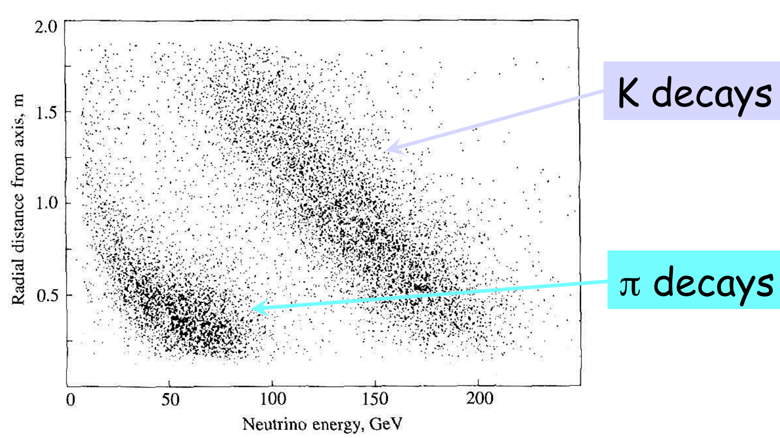
- Non-leptonic or Hadronic



Neutral weak currents

- Neutrino beams and experiments





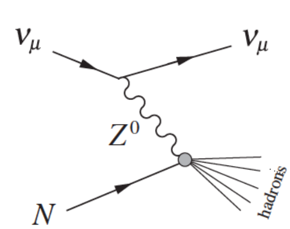
Neutral weak currents

- Accelerator neutrino beams gave evidence in 1962 for lepton flavors

$$\nu_\mu + N \rightarrow \mu^- + X$$
$$\nu_e + N \rightarrow e^- + X$$

- In 1973, neutrino interactions without a charge lepton were discovered: Neutral-current events

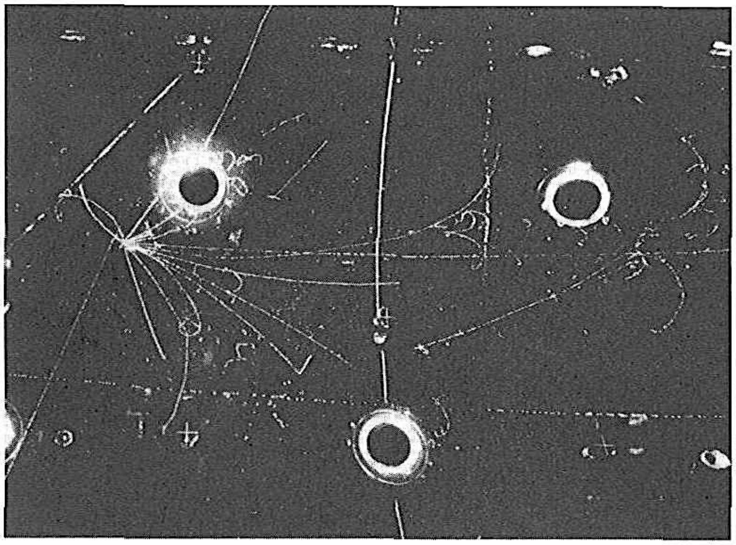
$$\nu_\mu + N \rightarrow \nu_\mu + X$$
$$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X$$
$$\nu_\mu + e^- \rightarrow e^- + \nu_\mu$$
$$\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu$$



Neutral weak currents

- Charge current interaction (Gargamelle bubble chamber - CERN)
- 15 tons Freon (CF_3Br)

neutrinos

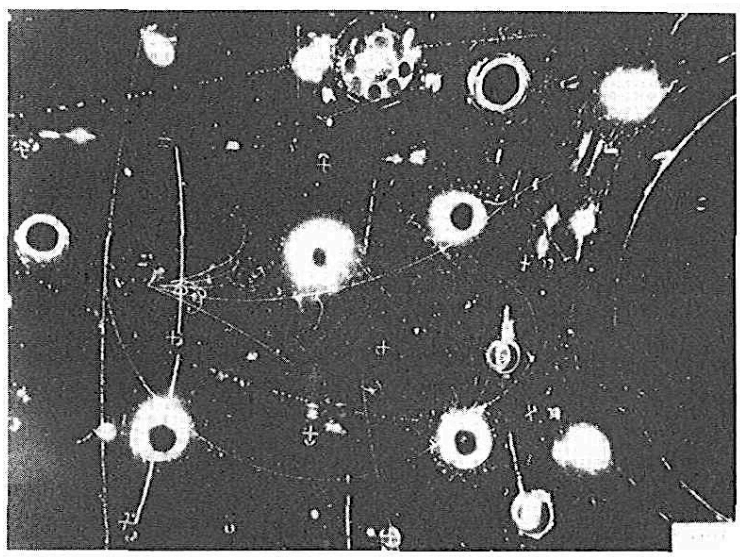


muon

Neutral weak currents

- Neutral current interaction

neutrinos



Neutral weak currents

- The discovery of neutral current interactions established the unified theory of weak and electromagnetic interactions
- Estimates of the masses of the $W^{\pm}$ and Z^0 were possible
- Once the masses were known, experiments could be planned to search for the intermediate bosons

Lepton universality

- Unit of weak charge
  - all the leptons carry the same weak charge and therefore couple to the $W^{\pm}$ with the same strength
  - The quarks DO NOT carry the same unit of weak charge
- Muon decay

$$\Gamma(\mu \to e\nu_e\bar{\nu}_\mu) = \frac{1}{\tau} \propto G^2 m_\mu^5 = \frac{G^2 m_\mu^5}{192\pi^3}$$

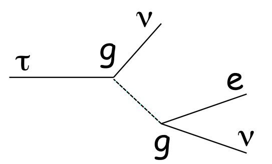
- experimental: $\tau_\mu = 2.197 \times 10^{-6}$ sec

Lepton universality

- Tau decay

$$\Gamma(\tau \rightarrow e\nu_e\bar{\nu}_\tau) = B(\tau \rightarrow e\,\nu\,\nu)\,\frac{1}{\tau} \propto G^2 m_\tau^5$$

$$= \frac{G^2 m_\tau^5}{192\pi^3}$$



- B(τ → e ν ν) = 17.80 ± 0.06%
- Test universality: since $\Gamma \sim G^2 \sim g^4$

$$g_\tau^4 \propto B(\tau \rightarrow e\,\nu\,\nu) / (m_\tau^5\,\tau_\tau)$$

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \rightarrow e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

Lepton universality

- Test universality:

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \rightarrow e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

With $\tau_\mu = 2.197 \times 10^{-6}$ s, $\tau_\tau = (291.0 \pm 1.5) \times 10^{-15}$ s, $m_\mu = 105.658$ MeV
$m_\tau = 1777.0$ MeV and B(τ → e ν ν) = 17.80 ± 0.06%

$$\frac{g_\tau}{g_\mu} = 0.999 \pm 0.003$$

$$\frac{g_\mu}{g_e} = 1.001 \pm 0.004$$

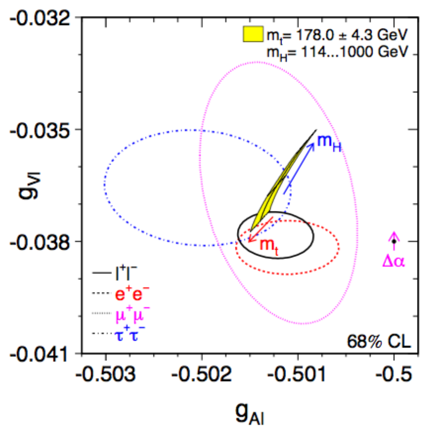
Lepton universality

- Lepton universality also holds for the Z couplings:

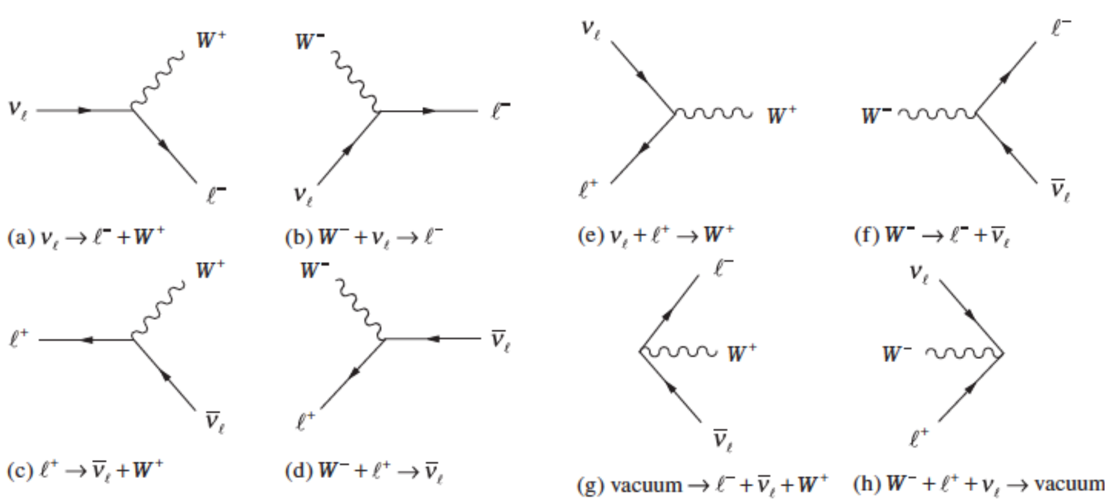
$$Z^0 \to e^+e^- : \mu^+\mu^- : \tau^+\tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$

- From the muon lifetime we can compute the Fermi constant, G:

$$G/(\hbar c)^3 = 1.1664 \times 10^{-5}\ \mathrm{GeV}^{-2}$$



W-lepton Interactions

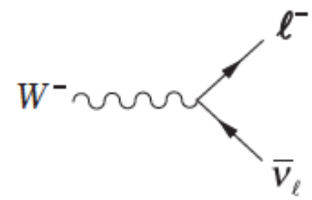


Plus all particles replaced by anti-particles

Strength of W-lepton Interaction

- $\Gamma(W \rightarrow e\nu) \approx 0.230 \pm 0.008$ GeV

  Since only mass scale is W
  - (lepton masses are negligible):



- $\Gamma(W \rightarrow e\nu) \approx \alpha_W M_W$

- So α_W = 0.230 GeV/80.4 GeV = .003 = 1/350
  - This is on the same scale as α = 1/137

  - Note: precise theoretical calculation yields

    $\Gamma(W \rightarrow e\nu) \approx 2\alpha_W M_W/3$

    and α_W = .0043 ± .0002 = 1/233 = 0.6 α

Weak decays of quarks

ΔS = 1 weak decays are suppressed relative to ΔS = 0 weak decays by a factor of about 20

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \qquad (\Delta S = 1)$$

$$n \rightarrow p + e^- + \bar{\nu}_e \qquad (\Delta S = 0)$$

Fermi constant (G) deduced from neutron β-decay is slightly smaller than G deduced from muon decay ($\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$)

Can these facts be explained?

Cabibbo theory: d and s quarks interacting in weak force are not flavor eigenstates, but rotated by a mixing angle (θ_c)

Weak decays of quarks

lepton doublets $\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$

quark doublet $\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} = \begin{pmatrix} u \\ d' \end{pmatrix}$

quark mixing

These three quarks were the only known quarks at the time Cabibbo wrote down his theory

With this assumption the "effective" couplings will be:

$G \cos\theta_c$ for $\Delta S = 0$ ($d \to u$)

$G \sin\theta_c$ for $\Delta S = 1$ ($s \to u$)

experiment yields $\theta_c \approx 13^0$ ($\sin\theta_c \approx 0.22$)

Weak decays of quarks

$\Delta S = 1$ weak decays are suppressed relative to $\Delta S = 0$ weak decays by a factor of about 20

$$\Sigma^- \to n + e^- + \bar{\nu}_e \qquad (\Delta S = 1)$$

$$n \to p + e^- + \bar{\nu}_e \qquad (\Delta S = 0)$$

this results from $d' = d\cos\theta_c + s\sin\theta_c$

$\sin^2\theta_c = (0.22)^2 = 1/20$

Fermi constant (G) deduced from neutron β-decay is slightly smaller than G deduced from muon decay ($\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$)

this results from $d' = d\cos\theta_c + \ldots$

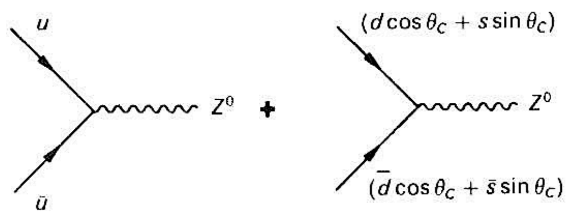
Weak decays of quarks.
The GIM model.

Flavor-changing neutral currents

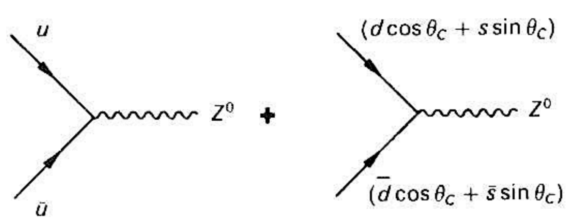
$$\frac{K^+ \rightarrow \pi^+ + \nu + \bar{\nu}}{K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu} \leq 10^{-5}$$

$$\frac{1.7 \pm 1.1 \times 10^{-10}}{3.353 \pm 0.034\ \%} = 4.8 \times 10^{-9}$$

2011



Weak decays of quarks.
The GIM model.



Neutral current interaction should be:

$$\underbrace{u\bar{u} + (d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c)}_{\Delta S = 0} + \underbrace{(s\bar{d} + d\bar{s})\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

so flavor-changing neutral interactions should be allowed, with a slight suppression.

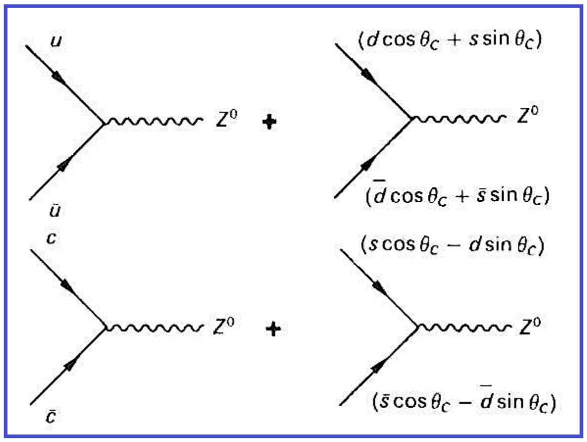
Why were they so rare?

GIM mechanism (1970)

Weak decays of quarks. The GIM model.

GIM mechanism (1970): introduce fourth quark (charm)

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$



Neutral current interaction becomes:

$$\underbrace{u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})\cos^2\theta_c + (s\bar{s} + d\bar{d})\sin^2\theta_c}_{\Delta S = 0}$$

$$\underbrace{+(s\bar{d} + \bar{s}d - \bar{s}d - s\bar{d})\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

Weak decays of quarks. The GIM model.

The quark mixing,

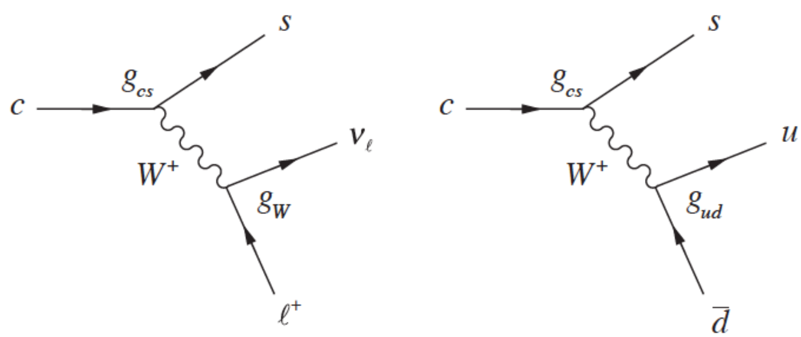
$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

can be expressed in matrix form:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The weak interactions involving the four light quarks are consistent with this model, being defined by a unique value of θ_c

Charm decays



Decays of charmed quarks are dominated by strange quarks in the final state.

W boson decays

- W → l ν, u d′, c s′
- Since d′ = d cos θ_c + s sin θ_c

  ud will appear with prob ~ $\cos^2 \theta_c$

  and us with prob ~ $\sin^2 \theta_c$
- We expect approximately:

  Γ(W→ud′) = Γ(W→cs′) = 3 Γ(W→eν)

  due to color
- So B (W→hadrons) ~ 2/3

and B (W→eν) = B (W→eν) = B (W→eν) = 1/9

W boson decays

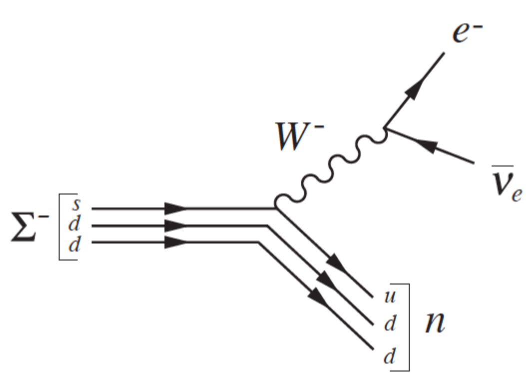
W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

Mode		Fraction (Γ_i/Γ)	Confidence level
$\ell^+\nu$	[a]	(10.80 ± 0.09) %	
$e^+\nu$		(10.75 ± 0.13) %	
$\mu^+\nu$		(10.57 ± 0.15) %	
$\tau^+\nu$		(11.25 ± 0.20) %	
hadrons		(67.60 ± 0.27) %	
$\pi^+\gamma$		$< 8 \times 10^{-5}$	95%
$D_s^+\gamma$		$< 1.3 \times 10^{-3}$	95%
cX		(33.4 ± 2.6) %	
$c\bar{s}$		$(31 {}^{+13}_{-11})$ %	
0 invisible	[b]	(1.4 ± 2.9) %	

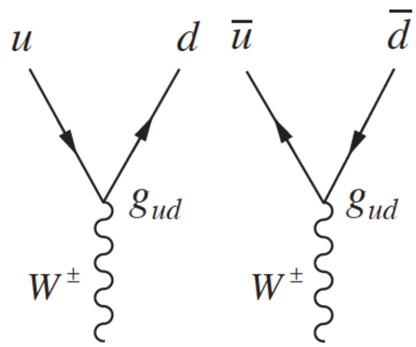
Weak Decay Selection Rules

$$\frac{\Gamma(\Sigma^+ \to n + e^+ + \nu_e)}{\Gamma(\Sigma^- \to n + e^- + \bar{\nu}_e)} < 5 \times 10^{-3}$$



Weak Decay Selection Rules

- $\Delta S = 0$, $\Delta Q = \pm 1$



- $\Delta S = \Delta Q = \pm 1$

  $s \rightarrow c$ or u

Weak decays of quarks and the CKM matrix

Six quark model, mixing matrix (CKM matrix) is 3x3:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

NxN unitary matrix has n(n-1)/2 real parameters (Euler angles) and (N-1)(N-2)/2 non-trivial phase angles

so this 3x3 matrix has 3 Euler angles and 1 phase
- this phase results in T-violation (or CP-violation)

(CKM = Cabibbo, Kobayashi, Maskawa)

Weak decays of quarks and the CKM matrix

V_{ud} (=cos θ_c) determined by comparing nuclear β-decay and μ-decay rates

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

most precise from superallowed 0+ → 0+ nuclear beta decays, pure vector transitions:

$|V_{ud}| = 0.97425 \pm 0.00022$ (avg of 20 meas.)

V_{us} determined from semi-leptonic decays of strange particles,

eg. $K_L^0 \to \pi e \nu,\ K_L^0 \to \pi \mu \nu,\ K^\pm \to \pi^0 e^\pm \nu,$
$K^\pm \to \pi^0 \mu^\pm \nu$ and $K_S^0 \to \pi e \nu$

$|V_{us}| = 0.2252 \pm 0.0009$

Weak decays of quarks and the CKM matrix

V_{ub} measured by selecting B semi-leptonic decays to non-strange particles (inclusive $B \to X_u\, l\, \nu$ decays) for example, at the end-point

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

$|V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}$

V_{cb} from exclusive and inclusive semileptonic decays of B mesons to charm (eg. $B \to D l \nu$)

$$\Gamma(b \to c l \nu) = \frac{R(b \to c l \nu)}{\tau_B} = \frac{G^2 m_b^5}{192\pi^3} |V_{cb}|^2 f$$

$|V_{cb}| = (40.6 \pm 1.3) \times 10^{-3}$

Weak decays of quarks and the CKM matrix

V_{cd} from ν_μ di-μ events

$$\nu_\mu + d \to \mu^- + c$$
$$\hookrightarrow s + \mu^+ + \nu_\mu$$

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

and $D \to \pi\ l\ \nu$

$|V_{cd}| = 0.230 \pm 0.011$

V_{cs} from semileptonic D decays (eg. $D^+ \to K^0\ e^+\ \nu_e$) or leptonic D_s decays (eg. $D_s^+ \to \mu^+\nu_\mu$)

$|V_{cs}| = 1.023 \pm 0.036$

Weak decays of quarks and the CKM matrix

V_{td}, V_{ts} from B–Bbar oscillations mediated by box diagrams with top quarks, or loop-mediated rare K and B decays

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

$|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$

$|V_{ts}| = (38.7 \pm 2.1) \times 10^{-3}$

V_{tb} from top decay branching fractions
$R = B(t \to Wb)/B(t \to Wq)$
$= |V_{tb}|^2/(\Sigma|V_{tq}|^2) = |V_{tb}|^2$, where q = b,s,d.

$|V_{tb}| = 0.88 \pm 0.07$

Weak decays of quarks and the CKM matrix

$V_{CKM} =$

$V_{ud} = 0.97425 \pm 0.00022$	$V_{us} = 0.2252 \pm 0.0009$	$V_{ub} = 0.00389 \pm 0.00044$
$V_{cd} = 0.230 \pm 0.011$	$V_{cs} = 1.023 \pm 0.036$	$V_{cb} = 0.0406 \pm 0.0013$
$V_{td} = 0.0084 \pm 0.0006$	$V_{ts} = 0.0387 \pm 0.0021$	$V_{tb} = 0.88 \pm 0.07$

Diagonal terms are close to unity
Off-diagonal terms are small

Particle Data Group

Unitarity of the CKM Matrix

$V_{CKM} =$

$V_{ud} = 0.9742$	$V_{us} = 0.2252$	$V_{ub} = 0.0039$
$V_{cd} = 0.23$	$V_{cs} = 1.02$	$V_{cb} = 0.041$
$V_{td} = 0.0084$	$V_{ts} = 0.039$	$V_{tb} = 0.88$

From the independently measured CKM elements, the unitarity of the CKM matrix can be checked.

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006$ (1st row)
$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.101 \pm 0.074$ (2nd row)
$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.002 \pm 0.005$ (1st column)
$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.098 \pm 0.074$ (2nd column)

Weak decays of quarks and the CKM matrix

"Standard" parametrization of CKM matrix (PDG)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $\quad s_{ij} = \sin\theta_{ij}$
(unitarity requirement limits to 3 angles and one phase)

Can we approximate?

$$V_{CKM} = \begin{vmatrix} V_{ud} = 0.9742 & V_{us} = 0.2252 & V_{ub} = 0.0039 \\ V_{cd} = 0.23 & V_{cs} = 1.02 & V_{cb} = 0.041 \\ V_{td} = 0.0084 & V_{ts} = 0.039 & V_{tb} = 0.88 \end{vmatrix}$$

Diagonal terms are close to unity
Off-diagonal terms are small: $s_{13} \ll s_{23} \ll s_{12}$

Weak decays of quarks and the CKM matrix

Diagonal terms are close to unity

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Wolfenstein Parametrization
$\lambda = s_{12}$, introduce A, ρ, η

$$V_{\text{CKM}} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$

Global fit in the Standard Model

- The CKM matrix elements can be most precisely determined by a global fit that uses all available measurements and imposes the SM constraints (i.e., three generation unitarity).

$$\lambda = 0.2253 \pm 0.0007\,, \qquad A = 0.808^{+0.022}_{-0.015}\,,$$

$$\bar{\rho} = 0.132^{+0.022}_{-0.014}\,, \qquad \bar{\eta} = 0.341 \pm 0.013\,.$$

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \ldots)$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

Particle Data Group

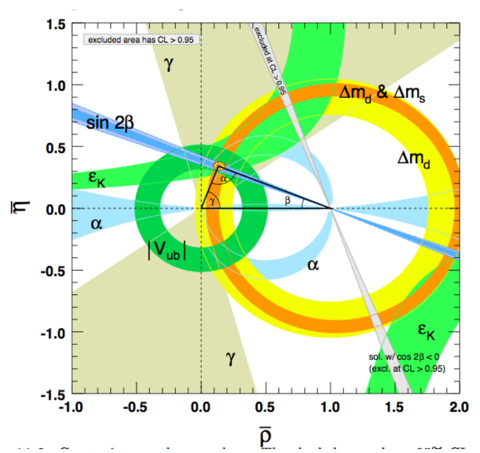
Weak decays of quarks and the CKM matrix

Wolfenstein parametrization
plot on η–ρ plane

$$V_{\mathrm{CKM}} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$

$\lambda = 0.2253 \pm 0.0007$
$A = 0.808 + 0.022 - 0.015$

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$



$$
\begin{pmatrix}
1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\
-\lambda-A^2\lambda^5(\rho+i\eta-\frac{1}{2}) & 1-\frac{\lambda^2}{2}-(\frac{1}{8}+\frac{A}{2})\lambda^4 & A\lambda^2 \\
A\lambda^3[1-(\rho+i\eta)(1-\frac{\lambda^2}{2})] & -A\lambda^2-A\lambda^4(\rho+i\eta-\frac{1}{2}) & 1-\frac{1}{2}A^2\lambda^4
\end{pmatrix} + \mathcal{O}(\lambda^6)
$$

Unitarity requires:

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$$
$$|V_{cd}|^2+|V_{cs}|^2+|V_{cb}|^2=1$$
$$|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2=1$$
$$|V_{ud}|^2+|V_{cd}|^2+|V_{td}|^2=1$$
$$|V_{us}|^2+|V_{cs}|^2+|V_{ts}|^2=1$$
$$|V_{ub}|^2+|V_{cb}|^2+|V_{tb}|^2=1$$

$$V^*_{ud}V_{cd}+V^*_{us}V_{cs}+V^*_{ub}V_{cb}=0$$
$$V^*_{ud}V_{td}+V^*_{us}V_{ts}+V^*_{ub}V_{tb}=0$$
$$V^*_{cd}V_{td}+V^*_{cs}V_{ts}+V^*_{cb}V_{tb}=0$$
$$V_{ud}V^*_{us}+V_{cd}V^*_{cs}+V_{td}V^*_{ts}=0$$
$$V_{ud}V^*_{ub}+V_{cd}V^*_{cb}+V_{td}V^*_{tb}=0 \quad \Leftarrow$$
$$V_{us}V^*_{ub}+V_{cs}V^*_{cb}+V_{ts}V^*_{tb}=0$$

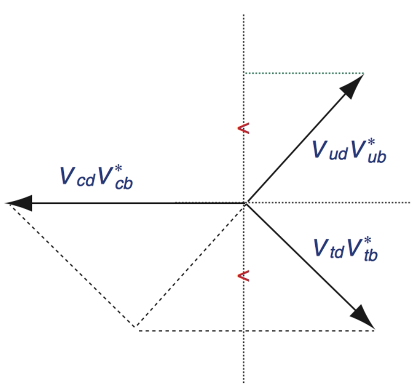
$$V_{ud}V^*_{ub}+V_{cd}V^*_{cb}+V_{td}V^*_{tb}=0$$

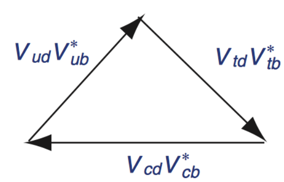
vectors in complex plane:

$V_{cd} = |V_{cd}|\, e^{i\vartheta}$ $\quad V_{cb}^* = |V_{cb}|\, e^{-i\varphi}$

Phase factor common to row or column can be eliminated!

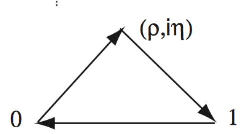
The three vectors define a triangle:





Use Wolfenstein third order:

$V_{ud}V^*_{ub} = A\lambda^3(1-\lambda^2/2)(\rho+i\eta)$
$V_{cd}V^*_{cb} = -A\lambda^3$
$V_{td}V^*_{tb} = A\lambda^3(1-\rho-i\eta)$



$V^*_{ud}V_{cd} + V^*_{us}V_{cs} + V^*_{ub}V_{cb} = 0$ $\lambda, \lambda, \lambda^5$

$V^*_{ud}V_{td} + V^*_{us}V_{ts} + V^*_{ub}V_{tb} = 0$ $\lambda^3, \lambda^3, \lambda^3$

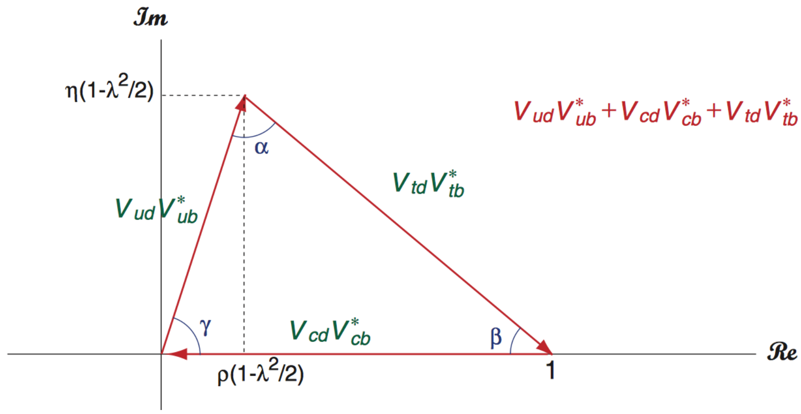
$V^*_{cd}V_{td} + V^*_{cs}V_{ts} + V^*_{cb}V_{tb} = 0$ $\lambda^4, \lambda^2, \lambda^2$

$V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0$ $\lambda, \lambda, \lambda^5$

$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$ $\lambda^3, \lambda^3, \lambda^3$

$V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0$ $\lambda^4, \lambda^2, \lambda^2$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



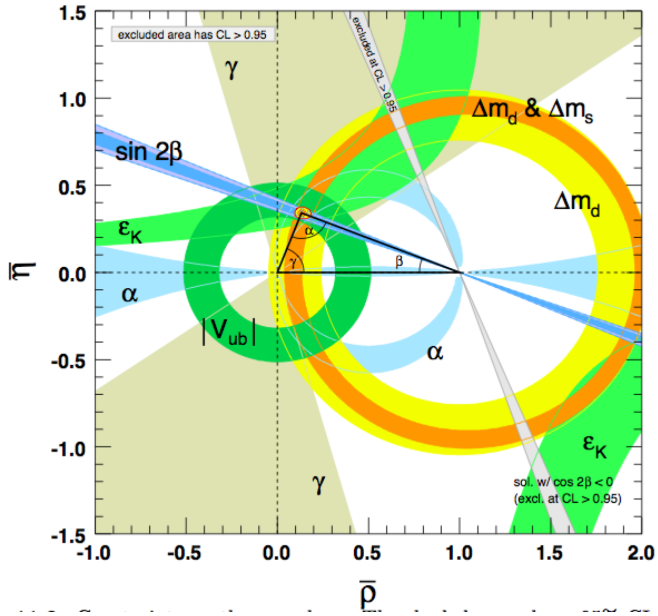
Physics with the Unitary Triangles:

Sides:

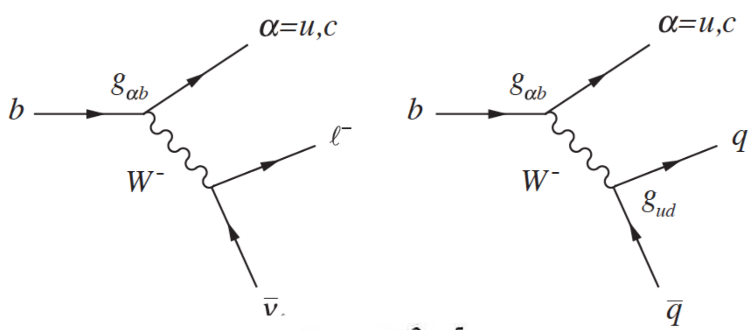
V_{ud}	β-decay	$(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e$	$\cos\vartheta_C$
V_{us}	K-decay	$K^+ \rightarrow \pi^0 + \ell^+ + \nu_\ell$	$\sin\vartheta_C$
		$K^0 \rightarrow \pi^- + \ell^+ + \nu_\ell$	
V_{cd}	ν-production of c's	$\nu_\ell + d \rightarrow \ell^- + c$	$\cos\vartheta_C$
V_{cs}		$D^\pm \rightarrow K^0 + \ell^\pm + \nu_\ell$	$\sin\vartheta_C$
V_{ub}	B-decay	$b \rightarrow u + \ell^- + \bar{\nu}_\ell$	
V_{cb}		$b \rightarrow c + \ell^- + \bar{\nu}_\ell$	
V_{td}	Δm in B^0-$\bar{B}^0$		

Is the triangle a triangle? Check on Standard Model/ New Physics!

Weak decays of quarks and the CKM matrix



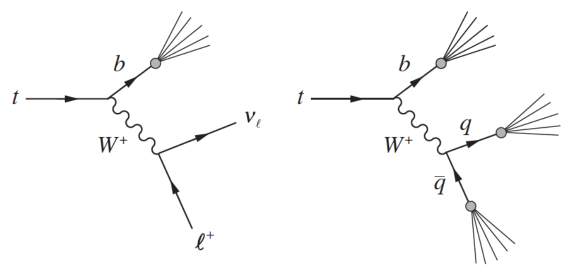
b lifetime



Recall $\Gamma(\mu \to e \nu_e \bar{\nu}_\mu) = \frac{1}{\tau} = \frac{G^2 m_\mu^5}{192\pi^3}$

Top quark

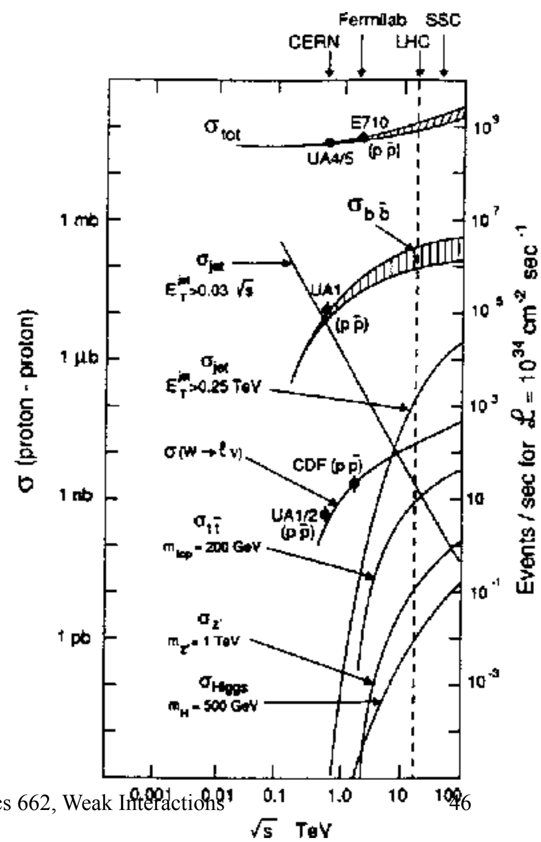
- Decays dominated by $t \rightarrow b\, W^+$
- $\Gamma(t \rightarrow b\, W^+) = 1.7$ GeV = 4×10^{-25} sec
  - $\Gamma \sim \alpha_W m_t$
    - α_W = .0043 ± .0002 = 1/233 = 0.6 α
- Hadron cannot form in less than time than
  $t \sim 1$ fm/3×10^{23} fm/s ~ 3×10^{-24} sec



Discovery of the top quark

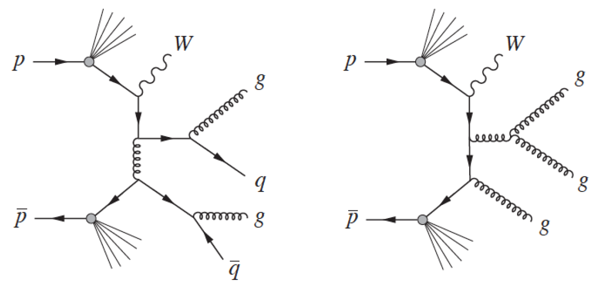
- Tevatron at Fermilab
- $t \rightarrow b\, W^+ \rightarrow l\, \nu$ 1/3
  $\rightarrow qq$ 2/3
- Produced in pairs t tbar
- decay to
  - b b l ν l ν 1/9 (2/3 are eμ)
  - b b l ν q q 4/9
  - b b q q q q 4/9
- Tevatron
  - Signal cross section
    < 10^{-9} backgrounds

  Need to reduce backgrounds



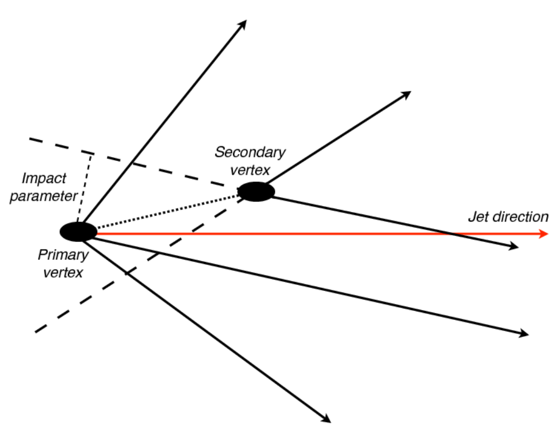
Top Quark

- Event selection
  - l ν N jets (N ≥ 3) (some jets merge)
  - Remove Z → $l^+ l^-$
- Background reduction
  - Large background from W production
  - This bkgd /signal ≈ 4/1



Top Quark

- B jet tagging
  - Find secondary vertices
    - Reduce background by x20 while selecting 40% signal



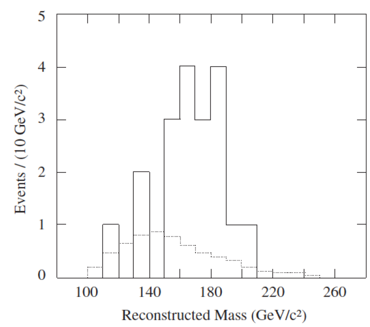
Top Quark Discovery Results

TABLE 8.1 The number of lepton + N-jet events of the type (8.56) observed with and without b-jets. (Data from Abe *et al.*, 1995.)

N	Observed events	Observed b-jet tags	Background tags expected
1	6578	40	50 ± 12
2	1026	34	5021 ± 6.5
3	164	17	5.2 ± 1.7
4	39	10	1.5 ± 0.4

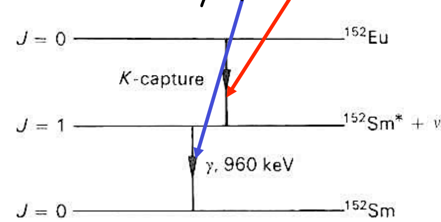
M_t = 176 ± 8 ± 10 GeV

current PDG value
M_t = 173.07 ± 0.52 ± 0.72 GeV



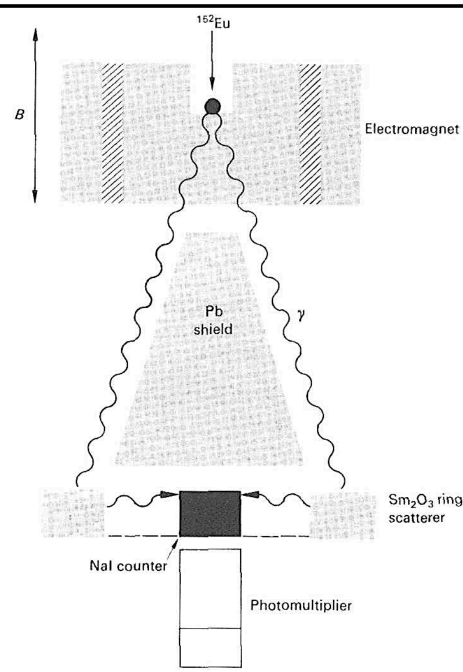
Helicity of the neutrino

- The elegant experiment of Goldhaber, Grodzins, and Sunyar(1958) established the helicity of the neutrino
- The neutrino is emitted along with a positron in β^+ decay
- The antineutrino is emitted along with an electron in β^- decay
- Reaction used is K-capture in ^{152}Eu to an excited state of ^{152}Sm
  - the excited state of ^{152}Sm decays by emitting gamma ray
  - resonance scattering of gamma-rays from ground state of Sm reveals the polarization of the event, and the helicity of the neutrino



$$\gamma + {}^{152}\mathrm{Sm} \rightarrow {}^{152}\mathrm{Sm}^* \rightarrow \gamma + {}^{152}\mathrm{Sm}$$

Helicity of the neutrino

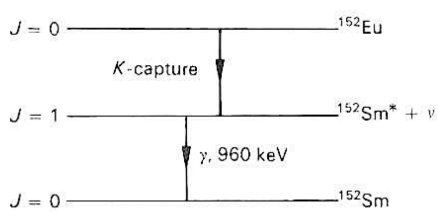


Electromagnet permits control on absorption of γ rays, determining polarization of γ rays, and consequently, neutrinos (d)

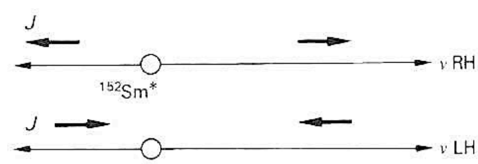
Resonance scattering in ring selects γ rays emitted in direction of recoiling $^{152}Sm^*$

(to allow for nuclear recoil)

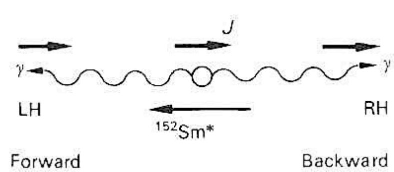
Helicity of the neutrino



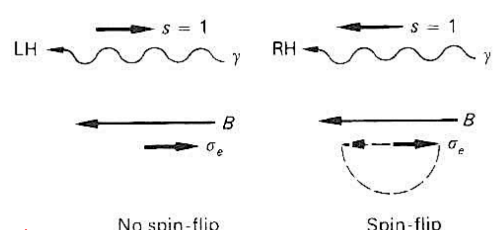
(a)



Helicity of $^{152}Sm^*$ is the same as that of the neutrino



Gammas emitted forward have same polarization as the neutrino (and the highest energy)



Gammas with spin aligned with B field in electromagnet are more absorbed (since electrons are aligned against B)