Electroweak Interactions and the Standard Model

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Introduction

In the late 1960s, it was realized that the electromagnetic and weak interactions were different aspects of the same force, the Electroweak Interaction

Weinberg, Salam, and Glashow developed a model of this interaction and predicted that the symmetry between the two apparently different interactions would be clear at very large momentum transfers ($q \gg 10^4$ GeV^2)

At low energy, the mass difference between the photon and the W^+, W^-, and Z^0 break the symmetry

The theory introduces one new arbitrary constant, commonly expressed as $\sin^2\theta_W$

this constant must be measured, it is not given by the theory

Divergences in the Weak Interactions

Early attempts to formulate an understanding of the weak interaction were plagued with divergences in amplitudes at high energy

Well-behaved theories must be renormalizable.

An example of the problem is in the Fermi theory of neutrino scattering

$$\nu_e + e^- \to e^- + \nu_e$$

$$\frac{d\sigma}{dq^2} = \frac{G^2}{\pi}$$

$$\sigma_{\mathrm{tot}}(\nu e) = \frac{G^2}{\pi} q^2_{\mathrm{max}} = \frac{2G^2 m E}{\pi} = \frac{G^2 s}{\pi}$$

cross section increases with energy

Divergences in the Weak Interactions

Another way to look at this:

Wave theory yields a cross section for pointlike scattering (l=0):

$$\sigma_{\max} = \pi \lambda\!\!\!^{-2}(2l+1)/(2s+1) = \pi \lambda\!\!\!^{-2}/2$$

Since $\lambda = h/p^*$, $\sigma_{\rm tot}(\nu e) = \frac{G^2 s}{\pi}$ $= 4G^2 p^{*2}/\pi > \sigma_{max}$ for

$$p^* > (\pi/G\sqrt{8})^{1/2} \simeq 300\ \mathrm{GeV}/c$$

(λ and p^* are cms values)

Therefore the Fermi Theory predicts a cross section that exceeds the Unitarity Limit at high energy

This "crisis" is resolved since the propagator replaces G^2 with $G/(1 + q^2/M_W^2)$ when $q^2 \gg M_W^2$ $\sigma_{\rm tot}(\nu e) \Rightarrow \frac{G^2 M_W}{\pi}$

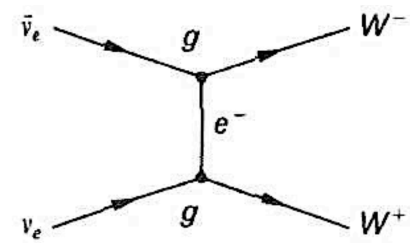
Introduction of Neutral Currents

The divergences are fixed by the finite mass of the W for

$$\nu_e + e^- \rightarrow e^- + \nu_e$$

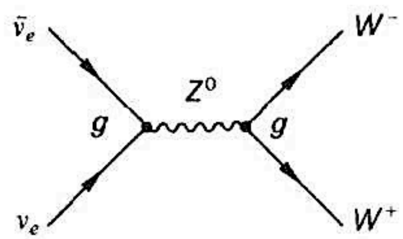
but they persist for other processes. Consider $\nu\bar{\nu} \rightarrow W^+W^-$

We still need additional physics to control this cross section. This diagram alone will diverge

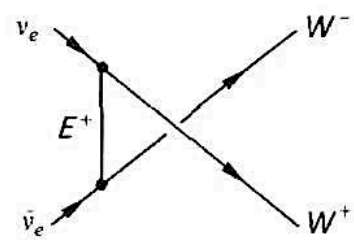


There are two possibilities:

1.) a neutral boson



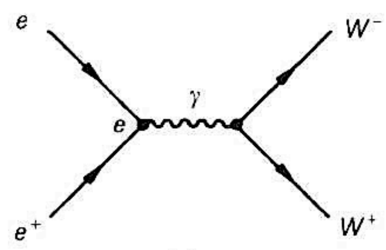
2.) a new heavy lepton



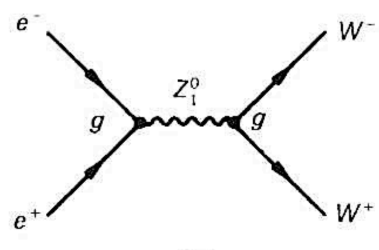
there is no evidence for this solution

Introduction of Neutral Currents

Another divergent process is



Again this can be "saved" by heavy neutral currents



Which is exactly what Nature has chosen to do

The couplings g and e are similar if $M_W \simeq \frac{g}{\sqrt{G}} \simeq \frac{e}{\sqrt{G}} \sim 100\ \text{GeV}$

The Weinberg-Salam Model

The Electroweak Theory was proposed in the late 1960s by Weinberg, Salam, and Glashow.

Four massless mediating bosons are postulated, arranged as a triplet and a singlet as members of multiplets of "weak isospin" I and "weak hypercharge" Y

$W_\mu = W_\mu^{(1)}, W_\mu^{(2)}, W_\mu^{(3)}$ I =1 triplet of SU(2)

B_μ I=0 (isoscalar) of U(1) group of hypercharge

theory is referred to as SU(2) $\times$ U(1)

Higgs mechanism generates mass for three bosons leaving one (the photon) massless

The Weinberg-Salam Model

The Lagrangian energy density is

$$L = g\mathbf{J}_\mu \cdot \mathbf{W}_\mu + g' J_\mu^Y B_\mu$$

J_μ is the weak isospin current of fermions
J_μ^Y is the weak hypercharge current

$$J_\mu^Y = J_\mu^{\text{em}} - J_\mu^{(3)}$$

Note – Many authors prefer the definition $J_\mu^Y = 2\,(J_\mu^{em} - J_\mu^{((3)})$

W and B are the boson fields, which transform into the physical boson fields, the $W_\mu^\pm$, Z_μ, and A_μ (the photon)

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left[W_\mu^{(1)} \pm i W_\mu^{(2)}\right]$$

Note – this phase choice of Perkins is not that of many other authors who prefer $W^\pm = 1/\sqrt{2}\ [W_\mu^{(1)} \mp \iota W_\mu^{(2)}]$

$$W_\mu^{(3)} = \frac{gZ_\mu + g' A_\mu}{\sqrt{g^2 + g'^2}} \qquad B_\mu = \frac{-g' Z_\mu + g A_\mu}{\sqrt{g^2 + g'^2}}$$

The Weinberg-Salam Model

These definitions:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left[W_\mu^{(1)} \pm i\,W_\mu^{(2)}\right]$$

Note – this phase choice of Perkins is not that of many other authors who prefer $W^\pm = 1/\sqrt{2}\ [\,W_\mu^{(1)}\ \mp \iota\, W_\mu^{(2)}\ ]$

$$W_\mu^{(3)} = \frac{g Z_\mu + g' A_\mu}{\sqrt{g^2 + g'^2}} \qquad B_\mu = \frac{-g' Z_\mu + g A_\mu}{\sqrt{g^2 + g'^2}}$$

produce the expected Lagrangian

$$L = g\mathbf{J}_\mu \cdot \mathbf{W}_\mu + g' J_\mu^Y B_\mu$$

$$\begin{aligned} L &= g\left(J_\mu^{(1)} W_\mu^{(1)} + J_\mu^{(2)} W_\mu^{(2)}\right) + g\left(J_\mu^{(3)} W_\mu^{(3)}\right) + g'\left(J_\mu^{\mathrm{em}} - J_\mu^{(3)}\right) B_\mu \\ &= (g/\sqrt{2})(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + J_\mu^{(3)}\left(g W_\mu^{(3)} - g' B_\mu\right) + J_\mu^{\mathrm{em}} g' B_\mu \end{aligned}$$

The Weinberg-Salam Model

$$L = (g/\sqrt{2})(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + J_\mu^{(3)}\underbrace{\left(g W_\mu^{(3)} - g' B_\mu\right)}_{\frac{g}{\cos\theta_W} Z_\mu} + J_\mu^{em} g' B_\mu$$

define θ_W:

$$g'/g = \tan\theta_W$$

then (since $W_\mu^{(3)} = \dfrac{g Z_\mu + g' A_\mu}{\sqrt{g^2 + g'^2}}$ and $B_\mu = \dfrac{-g' Z_\mu + g A_\mu}{\sqrt{g^2 + g'^2}}$)

the Lagrangian becomes

$$L = \frac{g}{\sqrt{2}}(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + \frac{g}{\cos\theta_W}(J_\mu^{(3)} - \sin^2\theta_W J_\mu^{em}) Z_\mu + \boxed{g \sin\theta_W} J_\mu^{em} A_\mu$$

↑ weak CC ↑ weak NC ↑ em NC

we recognize the electron charge $e = \boxed{g \sin\theta_W}$

Intermediate Boson Masses

The Fermi constant G_F is related to g and M_W by:

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

It then follows:

$$M_{W^\pm} = \left(\frac{g^2\sqrt{2}}{8G}\right)^{1/2} = \left(\frac{e^2\sqrt{2}}{8G\sin^2\theta_W}\right)^{1/2} = \frac{37.4}{\sin\theta_W}\,\text{GeV}$$

Note: this is tree level expression

Since we expected $M_W \sim 100$ GeV,
this implies

$\sin\theta_W \sim 0.4$
or $\sin^2\theta_W \sim 0.2$

at this point, this is just a guess on $\sin^2\theta_W$

Intermediate Boson Masses

What about the Z mass?

We can invert the equations

$$W_\mu^{(3)} = \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2 + g'^2}} \qquad B_\mu = \frac{-g'Z_\mu + gA_\mu}{\sqrt{g^2 + g'^2}}$$

and find:

$$Z_\mu = W_\mu^{(3)} \cos\theta_W - B_\mu \sin\theta_W$$
$$A_\mu = W_\mu^{(3)} \sin\theta_W + B_\mu \cos\theta_W$$

from these we obtain:

$$M_Z^2 = M_W^2 \cos^2\theta_W + M_B^2 \sin^2\theta_W - 2M_{BW}^2 \cos\theta_W \sin\theta_W$$
$$M_\gamma^2 = 0 = M_W^2 \sin^2\theta_W + M_B^2 \cos^2\theta_W + 2M_{BW}^2 \cos\theta_W \sin\theta_W$$
$$M_{Z\gamma}^2 = 0 = (M_W^2 - M_B^2) \sin\theta_W \cos_W + M_{BW}^2(\cos^2\theta_W - \sin^2\theta_W)$$

Intermediate Boson Masses

So we have:

$$M_Z^2 = M_W^2 \cos^2\theta_W + M_B^2 \sin^2\theta_W - 2M_{BW}^2 \cos\theta_W \sin\theta_W$$

$$M_\gamma^2 = 0 = M_W^2 \sin^2\theta_W + M_B^2 \cos^2\theta_W + 2M_{BW}^2 \cos\theta_W \sin\theta_W$$

$$M_{Z\gamma}^2 = 0 = (M_W^2 - M_B^2) \sin\theta_W \cos_W + M_{BW}^2(\cos^2\theta_W - \sin^2\theta_W)$$

eliminating M_{BW} we obtain:

$$M_{Z^0} = \frac{M_{W^\pm}}{\cos\theta_W} = \frac{75}{\sin 2\theta_W}\ \text{GeV}$$

Note: this is tree level expression

For $\sin^2\theta_W \sim 0.2$, the Z mass should be about 10% larger than the W mass

Intermediate Boson Masses

These predictions we have “derived” represent those of the simplest model of EW symmetry breaking:

that is one involving one isospin doublet of scalar Higgs particles

Many other models are possible. Some arrive at these same predictions, some do not.

We can limit the possible such alternative models since a factor ρ can be different from 1 in other models, and

$$M_{Z^0}^2 = \frac{M_W^2}{\rho \cos^2 \theta_W}$$

Data to date indicates ρ = 1, consistent with the simplest model, as well as some others (eg. SUSY with two doublets)

Electroweak Couplings of Leptons and Quarks

The Standard Model assigns leptons to a left-handed doublet and a right-handed singlet in weak isospin

$$\psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \left.\begin{matrix} I = \tfrac{1}{2}, & I_3 = +\tfrac{1}{2}, & Q = 0 \\ I = \tfrac{1}{2}, & I_3 = -\tfrac{1}{2}, & Q = -1 \end{matrix}\right\} \qquad Y = -\tfrac{1}{2}$$

$$\psi_R = (e^-)_R \qquad I = 0, \qquad Q = -1, \qquad Y = -1$$

$Y = Q - I_3$

Many authors prefer $Y = 2(Q-I_3)$

the coupling to the Z is $\left(J_\mu^{(3)} - \sin^2\theta_W J_\mu^{\text{em}}\right)$

so $g_L = I_3 - Q\sin^2\theta_W, \qquad g_R = -Q\sin^2\theta_W$

this means the vector and axial vector couplings to the Z are:

$$c_V = g_L + g_R = I_3 - 2Q\sin^2\theta_W, \qquad c_A = g_L - g_R = I_3$$

Electroweak Couplings of Leptons and Quarks

Quarks are also assigned to left-handed doublets and right-handed singlets in weak isospin

$\psi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	$I = \frac{1}{2}$,	$I_3 = +\frac{1}{2}$,	$Q = 2/3$	$Y = 1/6$
	$I = \frac{1}{2}$,	$I_3 = -\frac{1}{2}$,	$Q = -1/3$	
$\psi_R = (u)_R$	$I = 0$,		$Q = 2/3$	$Y = 2/3$
$\psi_R = (d)_R$	$I = 0$,		$Q = -1/3$	$Y = -1/3$

$Y = Q - I_3$

Many authors prefer $Y = 2(Q-I_3)$

As with the leptons, we have:

$$g_L = I_3 - Q\sin^2\theta_W, \qquad g_R = -Q\sin^2\theta_W$$

and

$$c_V = g_L + g_R = I_3 - 2Q\sin^2\theta_W, \qquad c_A = g_L - g_R = I_3$$

Electroweak Couplings of Leptons and Quarks

So, in summary, we now have:

Fermion	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1
e, μ, τ	$-1 + 4\sin^2\theta_W$	-1
u, c, t	$1 - \frac{8}{3}\sin^2\theta_W$	1
d, s, b	$-1 + \frac{4}{3}\sin^2\theta_W$	-1

Electroweak Couplings of Leptons and Quarks

Now suppose $\sin^2\theta_W \sim 0.23$ (as it is)

Fermion	$2c_V$		$2c_A$
ν_e, ν_μ, ν_τ	1	$= 1$	1
e, μ, τ	$-1 + 4\sin^2\theta_W$	$= -0.08$	-1
u, c, t	$1 - \frac{8}{3}\sin^2\theta_W$	$= 0.39$	1
d, s, b	$-1 + \frac{4}{3}\sin^2\theta_W$	$= -0.69$	-1

Electroweak Couplings of Leptons and Quarks

$\sin^2\theta_W \sim 0.23$

Fermion	$g_L = \frac{1}{2}(c_V + c_A)$	$g_R = \frac{1}{2}(c_V - c_A)$
ν_e, ν_μ, ν_τ	0.5	0
e, μ, τ	–0.27	0.23
u, c, t	0.35	–0.15
d, s, b	–0.42	0.08

$g_L = I_3 - Q\sin^2\theta_W$
$g_R = -Q\sin^2\theta_W$

Review of the Weinberg-Salam Model

$$L = g\mathbf{J}_\mu \cdot \mathbf{W}_\mu + g' J_\mu^Y B_\mu \qquad g'/g = \tan\theta_W$$

$$L = \frac{g}{\sqrt{2}}(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + \frac{g}{\cos\theta_W}\left(J_\mu^{(3)} - \sin^2\theta_W J_\mu^{\text{em}}\right)Z_\mu + \boxed{g\sin\theta_W} J_\mu^{\text{em}} A_\mu$$

↑ weak CC — ↑ weak NC — ↑ em NC

$$e = \boxed{g\sin\theta_W}$$

$$M_{W^\pm} = \left(\frac{g^2\sqrt{2}}{8G}\right)^{1/2} = \left(\frac{e^2\sqrt{2}}{8G\sin^2\theta_W}\right)^{1/2} = \frac{37.4}{\sin\theta_W}\ \text{GeV}$$

$$M_{Z^0} = \frac{M_{W^\pm}}{\cos\theta_W} = \frac{75}{\sin 2\theta_W}\ \text{GeV}$$

Electroweak Couplings of Leptons and Quarks

$\sin^2\theta_W \sim 0.23$

Fermion	$g_L = c_V + c_A$	$g_R = c_V - c_A$
ν_e, ν_μ, ν_τ	0.5	0
e, μ, τ	–0.27	0.23
u, c, t	0.35	–0.15
d, s, b	–0.42	0.08

$g_L = I_3 - Q \sin^2\theta_W$
$g_R = - Q \sin^2\theta_W$

Couplings of Leptons and Quarks

$$L = \frac{g}{\sqrt{2}}(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + \frac{g}{\cos\theta_W}(J_\mu^{(3)} - \sin^2\theta_W J_\mu^{\text{em}})Z_\mu + g\sin\theta_W J_\mu^{\text{em}} A_\mu$$

	photon		W^+, W^-		Z^0	
Fermion	g^{em}_L	g^{em}_R	L	R	g_L	g_R
ν_e, ν_μ, ν_τ	0	0	1	0	0.5	0
e, μ, τ	−1	−1	1	0	−0.27	0.23
u, c, t	0.67	0.67	1	0	0.35	−0.15
d, s, b	−0.33	−0.33	1	0	−0.42	0.08
	$\times e = g \sin\theta_W$		$\times g/\sqrt{2}$ $= e/\sqrt{2}\sin\theta_W$		$\times g / \cos\theta_W$ $= e / \cos\theta_W \sin\theta_W$	

$\sin^2\theta_W \sim 0.23$ $\sin\theta_W \sim 0.48$ $\cos\theta_W \sim 0.88$

Neutrino Scattering via Z Exchange

The value of $\sin^2\theta_W$ can be measured in neutrino scattering

Recall the charged current cross sections:

$$\frac{d\sigma}{dy}(\nu_e e \to \nu_e e)|_{CC} = \frac{G^2 s}{\pi} \qquad (LL \to LL)$$

$$\frac{d\sigma}{dy}(\bar{\nu}_e e \to \bar{\nu}_e e)|_{CC} = \frac{G^2 s}{\pi}(1-y)^2 \qquad (RL \to RL)$$

$Y = E_e/E_\nu$

The neutral current cross-section will be:

$$\frac{d\sigma}{dy}(\nu_e e \to \nu_e e)|_{NC} = \frac{G^2 s}{\pi}\left[g_L^2 + g_R^2(1-y)^2\right]$$

LL → LL LR → LR

$$\frac{d\sigma}{dy}(\bar{\nu}_e e \to \bar{\nu}_e e)|_{NC} = \frac{G^2 s}{\pi}\left[g_R^2 + g_L^2(1-y)^2\right]$$

RR → RR RL → RL

$$g_L = I_3 - Q\sin^2\theta_W$$
$$g_R = -Q\sin^2\theta_W$$

Neutrino Scattering via Z Exchange

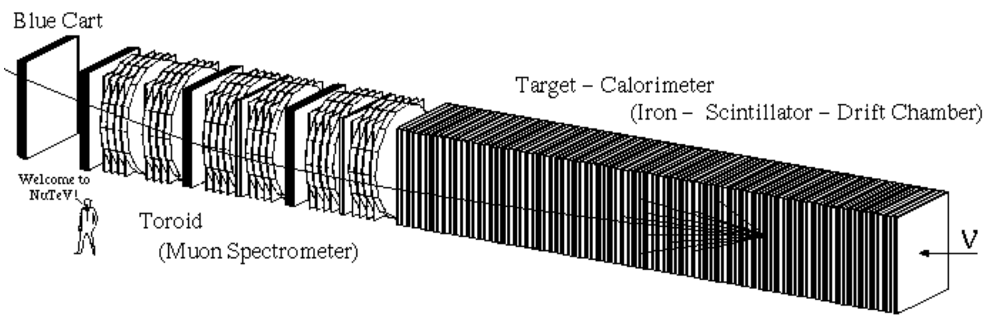
Scattering from quarks

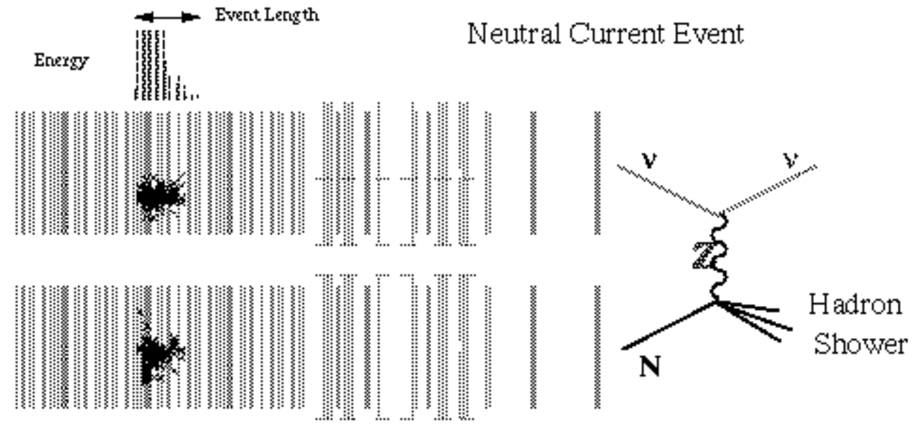
	u	d
$g_L = I_3 - Q\sin^2\theta_W$	$^1/_2 - {}^2/_3 \sin^2\theta_W$	$-{}^1/_2 + {}^1/_3 \sin^2\theta_W$
$g_R = -Q\sin^2\theta_W$	$- {}^2/_3 \sin^2\theta_W$	$+ {}^1/_3 \sin^2\theta_W$

$$\frac{d\sigma}{dy}(\nu_\mu\, Q \rightarrow \nu_\mu\, Q)\Big|_{NC} = \frac{G^2 s}{\pi}\left[g_L^2 + g_R^2(1-y)^2\right]$$

$$\frac{d\sigma}{dy}(\overline{\nu}_\mu\, Q \rightarrow \overline{\nu}_\mu\, Q)\Big|_{NC} = \frac{G^2 s}{\pi}\left[g_R^2 + g_L^2(1-y)^2\right]$$

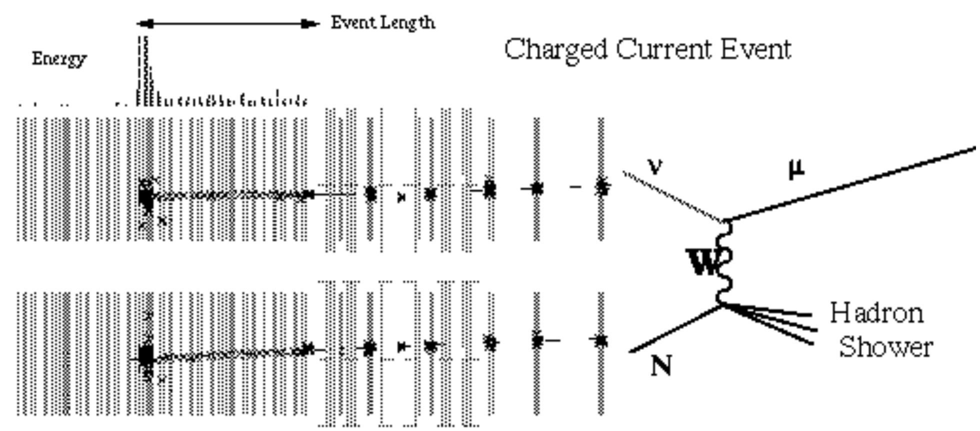
Neutrino Scattering via Z Exchange





$$R = \frac{\sigma^{\nu N}(\mathrm{NC})}{\sigma^{\nu N}(\mathrm{CC})} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

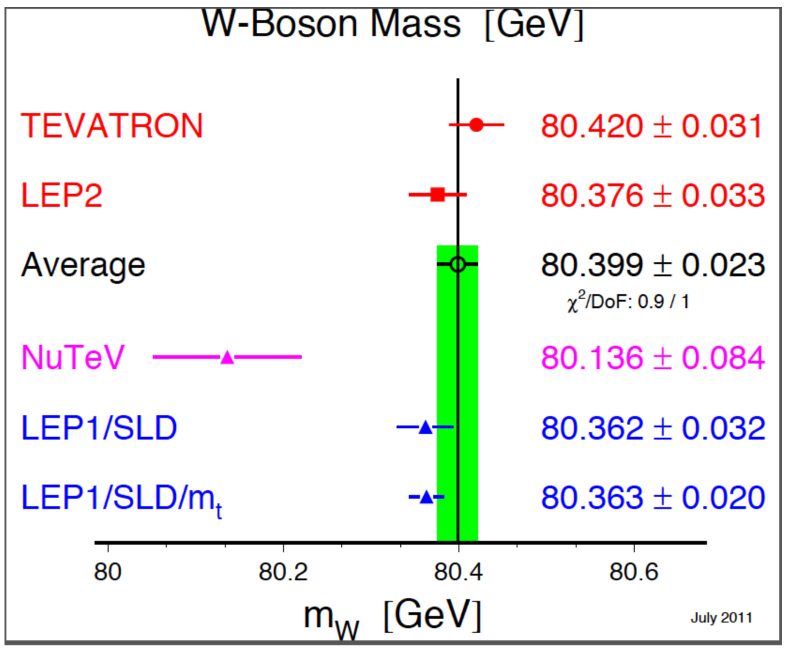
$$\bar{R} = \frac{\sigma^{\bar{\nu} N}(\mathrm{NC})}{\sigma^{\bar{\nu} N}(\mathrm{CC})} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W$$



Neutrino Scattering via Z Exchange

Since M_W and M_Z are related through the value of $\sin^2\theta_W$,
M_Z is very precisely known from LEP,
one can compare the $\sin^2\theta_W$ measurement of neutrino scattering to the direct M_W measurements

$$M_{Z^0} = \frac{M_{W^\pm}}{\cos\theta_W} \quad \text{or} \quad \sin^2\theta_W = 1 - M_W^2/M_Z^2$$



Asymmetries in the Scattering of Polarized Electrons by Deuterons

An experiment at SLAC in 1978 confirmed the neutral current measurements in neutrino scattering, and provided a measurement of the weak mixing angle

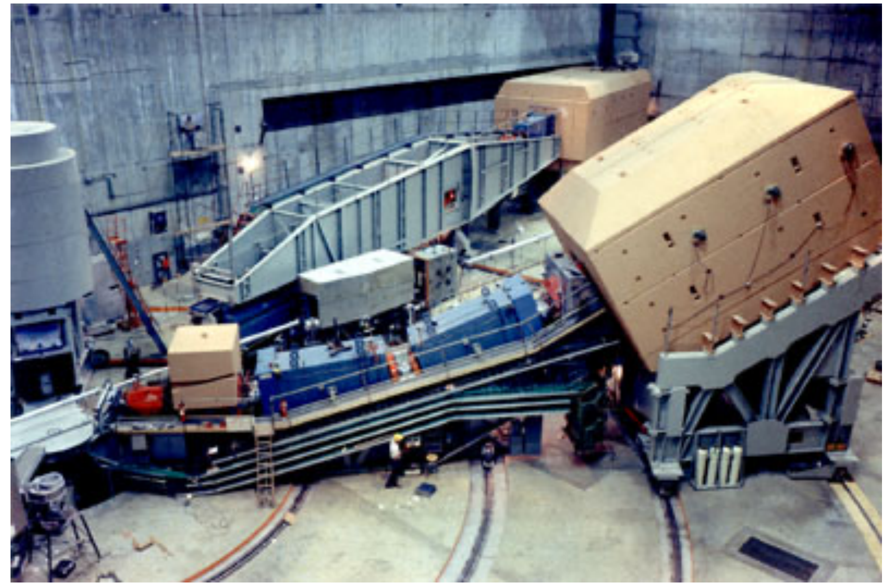
$$e^-_{R,L} + d_{unpolarized} \rightarrow e^- + X$$

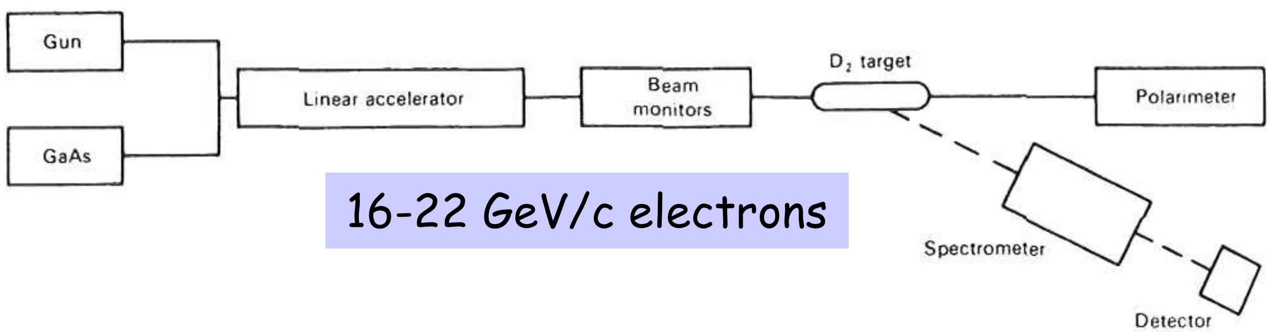
In the scattering of electrons from deuterons, γ exchange dominates, but a small contribution from Z^0 exchange results in a parity-nonconserving asymmetry between right- and left-handed electrons:

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \simeq \frac{Gq^2}{e^2} = \frac{137 \times 10^{-5}}{4\pi} \frac{q^2}{M_p^2}$$

$$\simeq 10^{-4} q^2 \qquad (q^2 \text{ in GeV}^2)$$

Asymmetries in the Scattering of Polarized Electrons by Deuterons





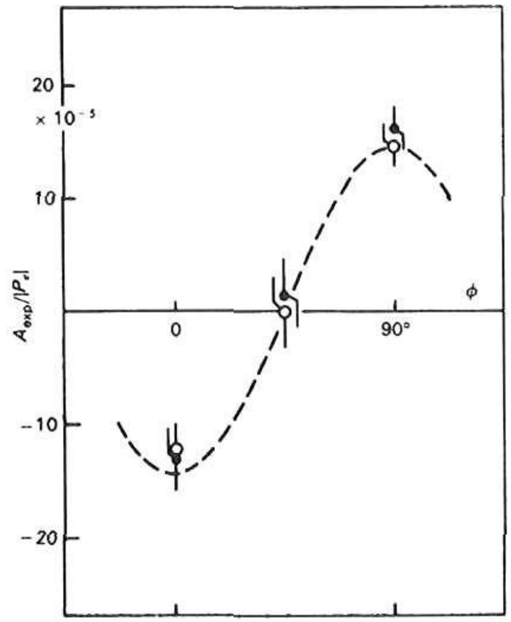
16-22 GeV/c electrons

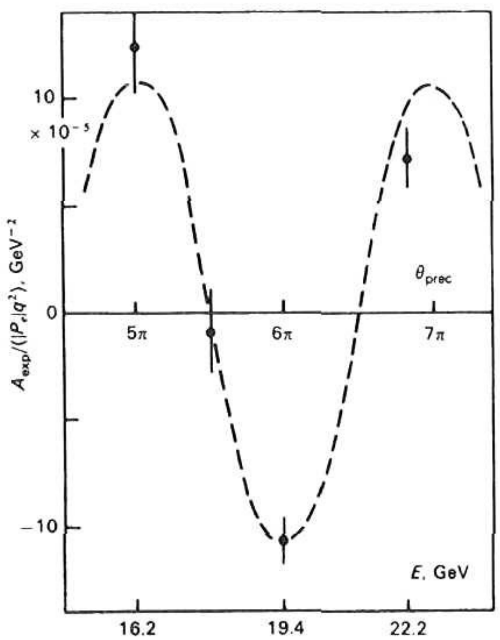
Asymmetries in the Scattering of Polarized Electrons by Deuterons

Asymmetry with unpolarized source:

$A = (-2.5 \pm 2.2) \times 10^{-5}$

Variation of the electron beam polarization





Electron beam dependence is consistent with g-2 precession

$$\theta_{\text{precession}} = \frac{E_0}{mc^2}\left(\frac{g-2}{2}\right)\theta_{\text{bend}}.$$

Asymmetries in the Scattering of Polarized Electrons by Deuterons

Final result showed clear asymmetry:

$A/q^2 = -(9.5 \pm 1.6) \times 10^{-5}\ (\mathrm{GeV}/c)^{-2}$

A was measured as a function of $y = (E_0 - E) / E_0$, the fractional electron energy loss

$$\frac{A}{q^2} = -\frac{9G}{20\sqrt{2}\pi\alpha}\left\{a_1 + a_2\frac{1-(1-y)^2}{1+(1-y)^2}\right\}$$

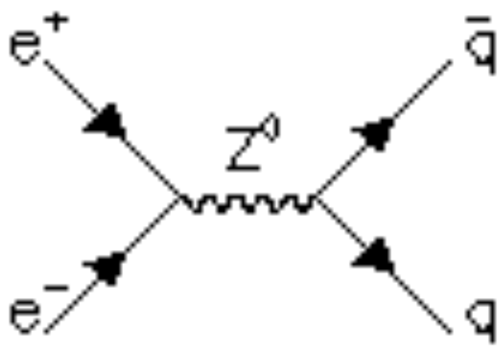
$$a_1 = 1 - \tfrac{20}{9}\sin^2\theta_W, \qquad a_2 = 1 - 4\sin^2\theta_W$$

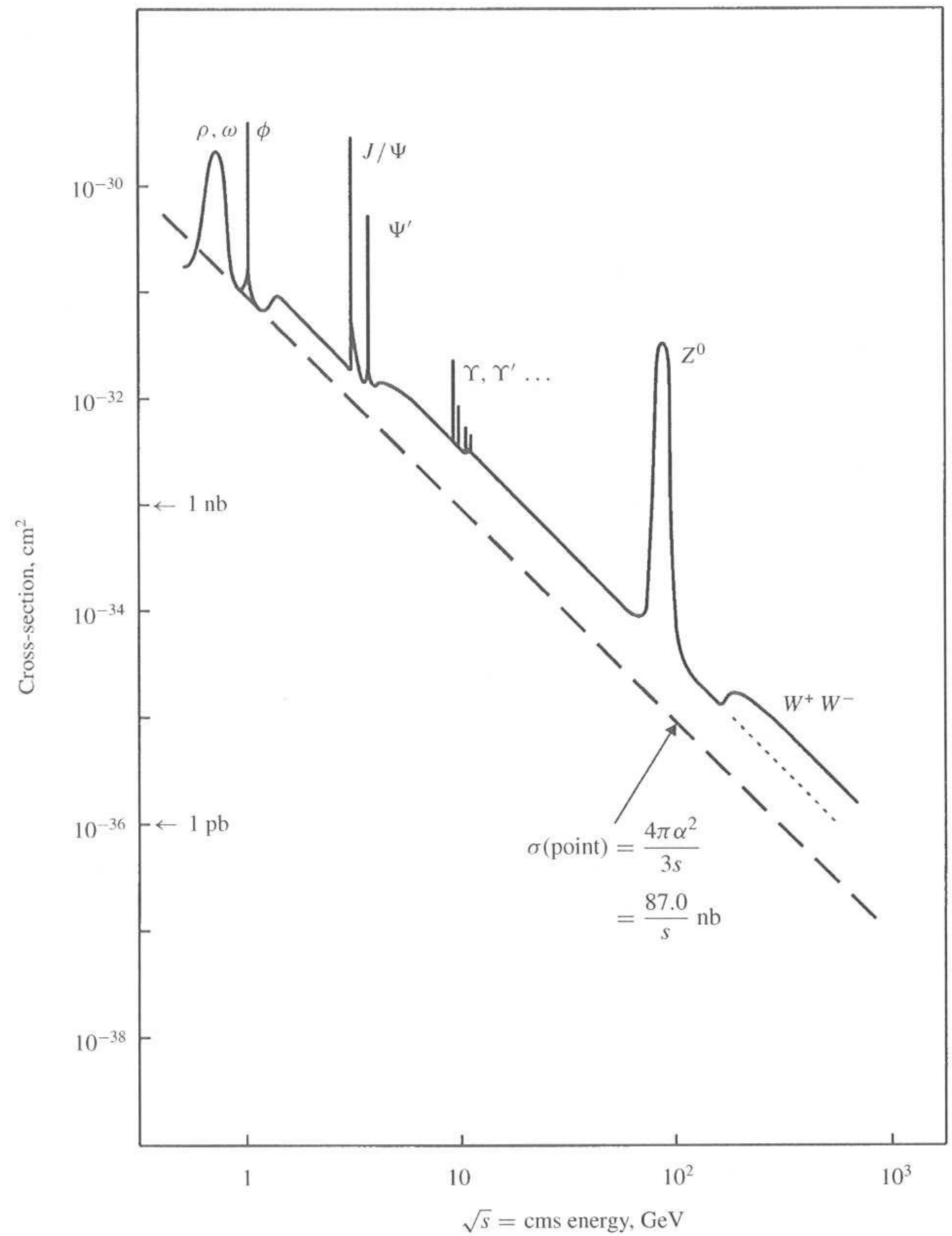
The observed asymmetries gave:

$$\sin^2\theta_W = 0.22 \pm 0.02$$

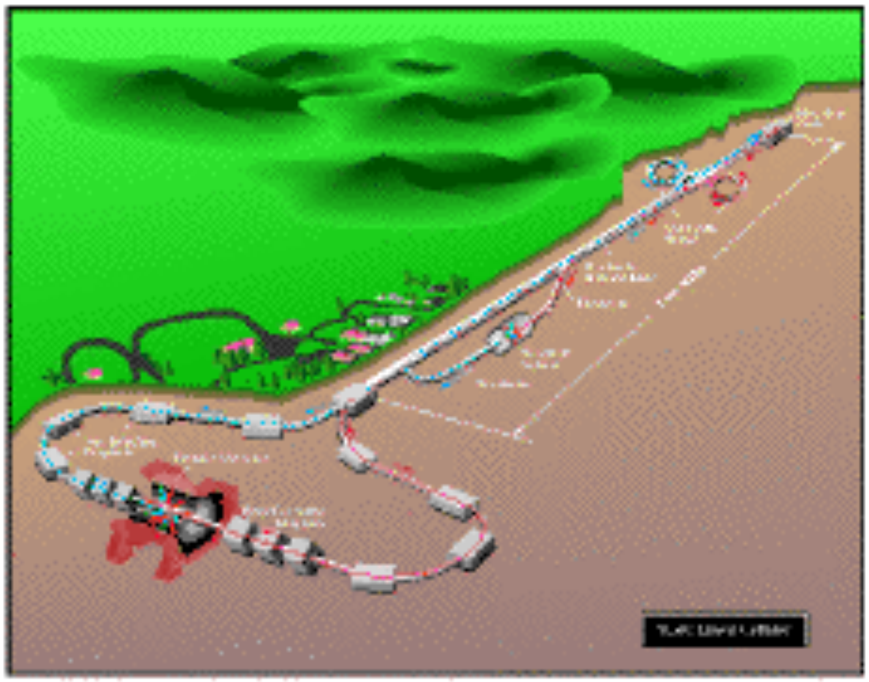
Observations on the Z Resonance

LEP and SLC have produced the Z^0 in electron-positron collisions and measured the properties.





Observations on the Z Resonance



SLC at SLAC

LEP at CERN

Observations on the Z Resonance

Key measurement of the Z^0 at SLC and LEP

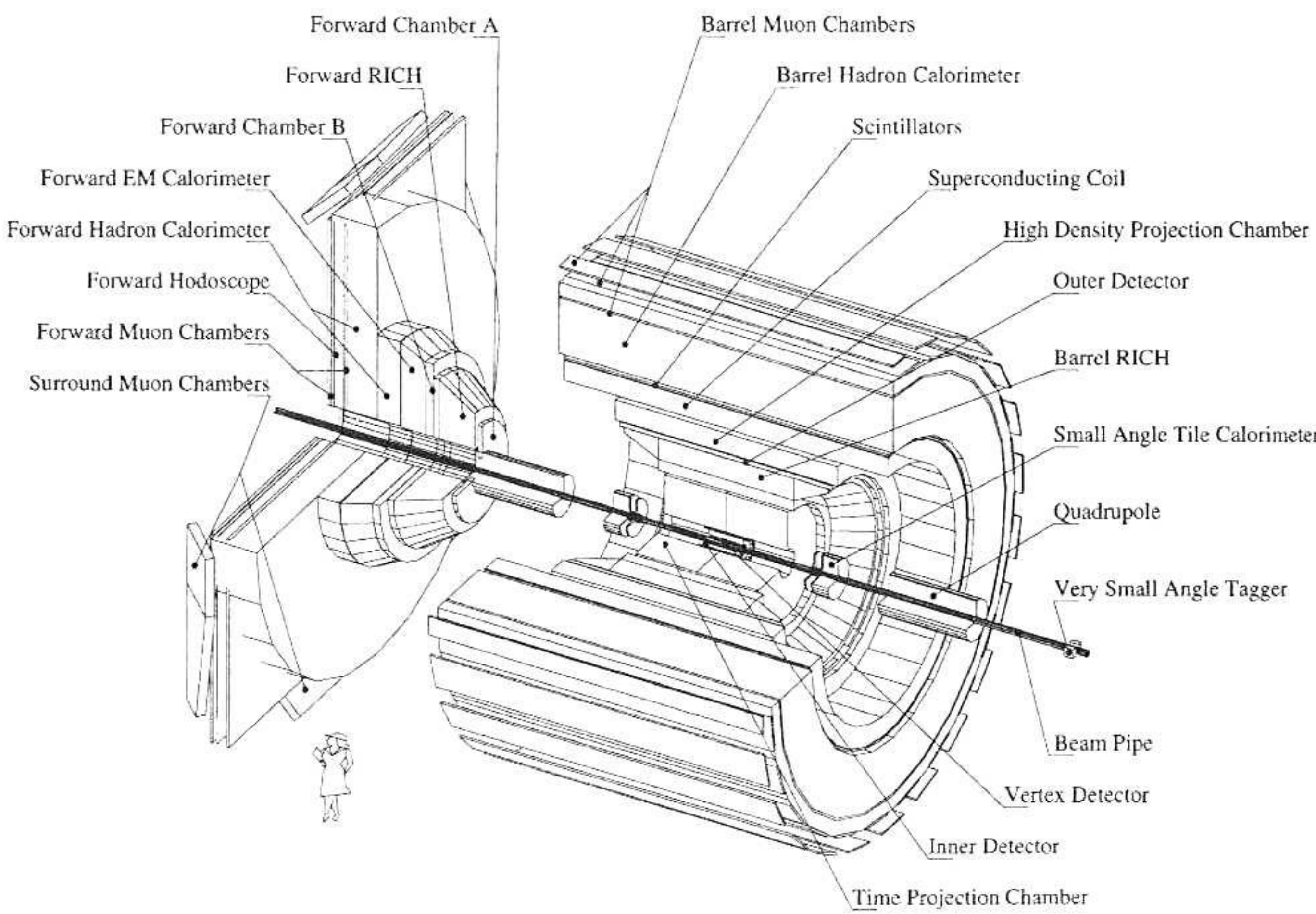
Z_0 mass
line shape
branching ratios to leptons and quarks
angular asymmetries
cross-section asymmetries with longitudinally polarized beams

SLC Experiments
- Mark II (initial)
- SLD (replaced Mark II)

LEP Experiments
- ALEPH
- DELPHI
- L3
- OPAL

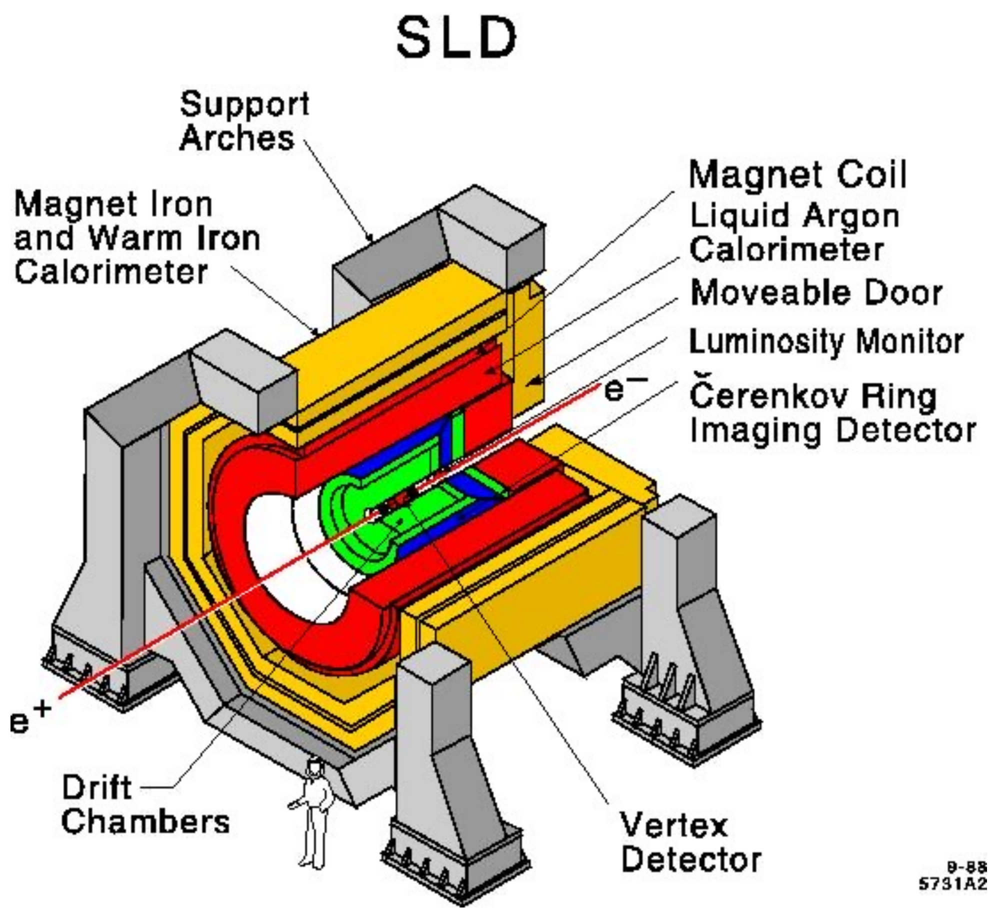
Observations on the Z Resonance

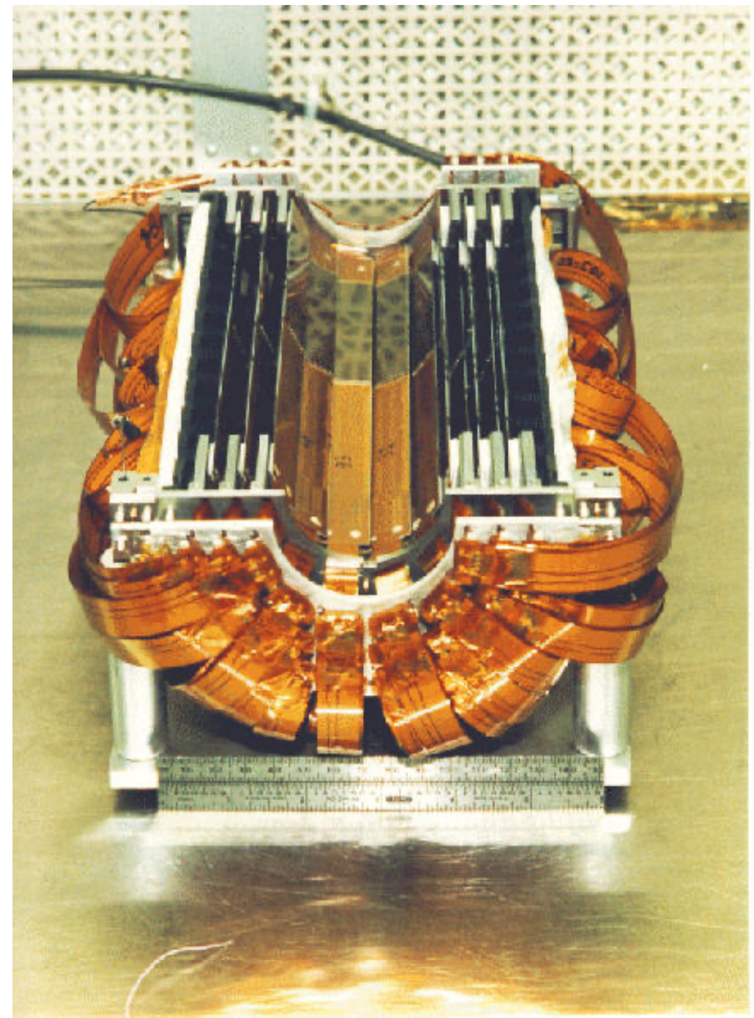
DELPHI at LEP



Observations on the Z Resonance

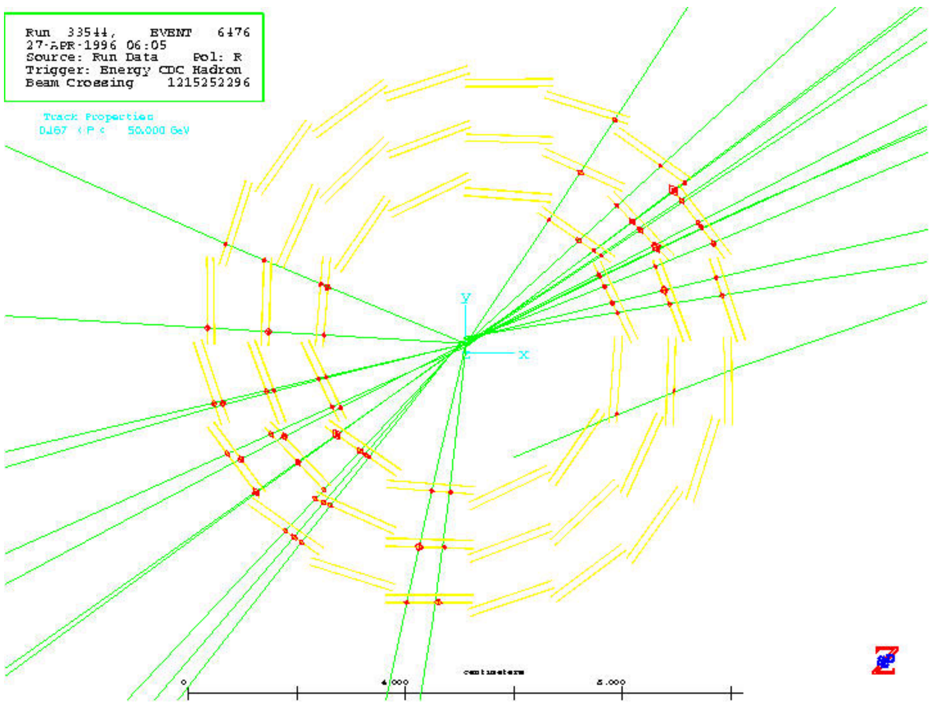
SLD at SLC

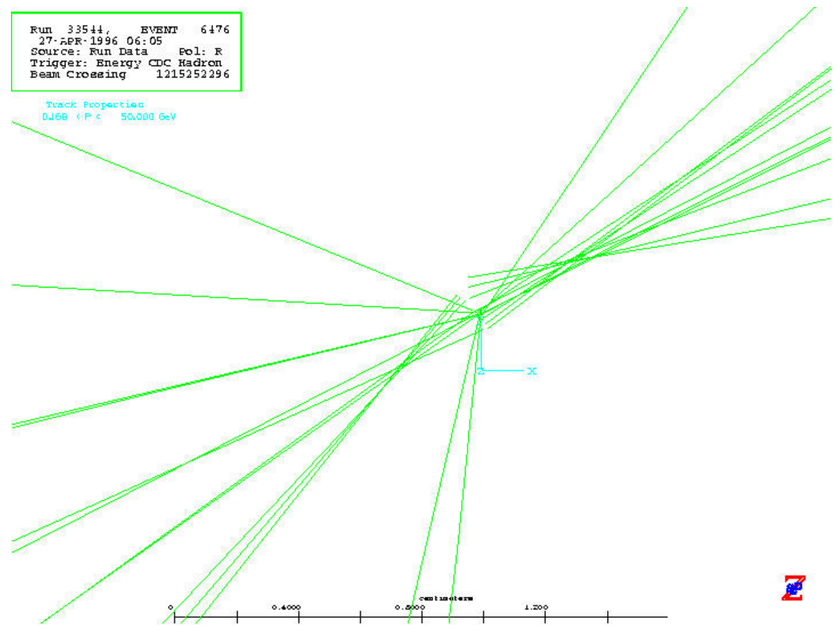




Observations on the Z Resonance

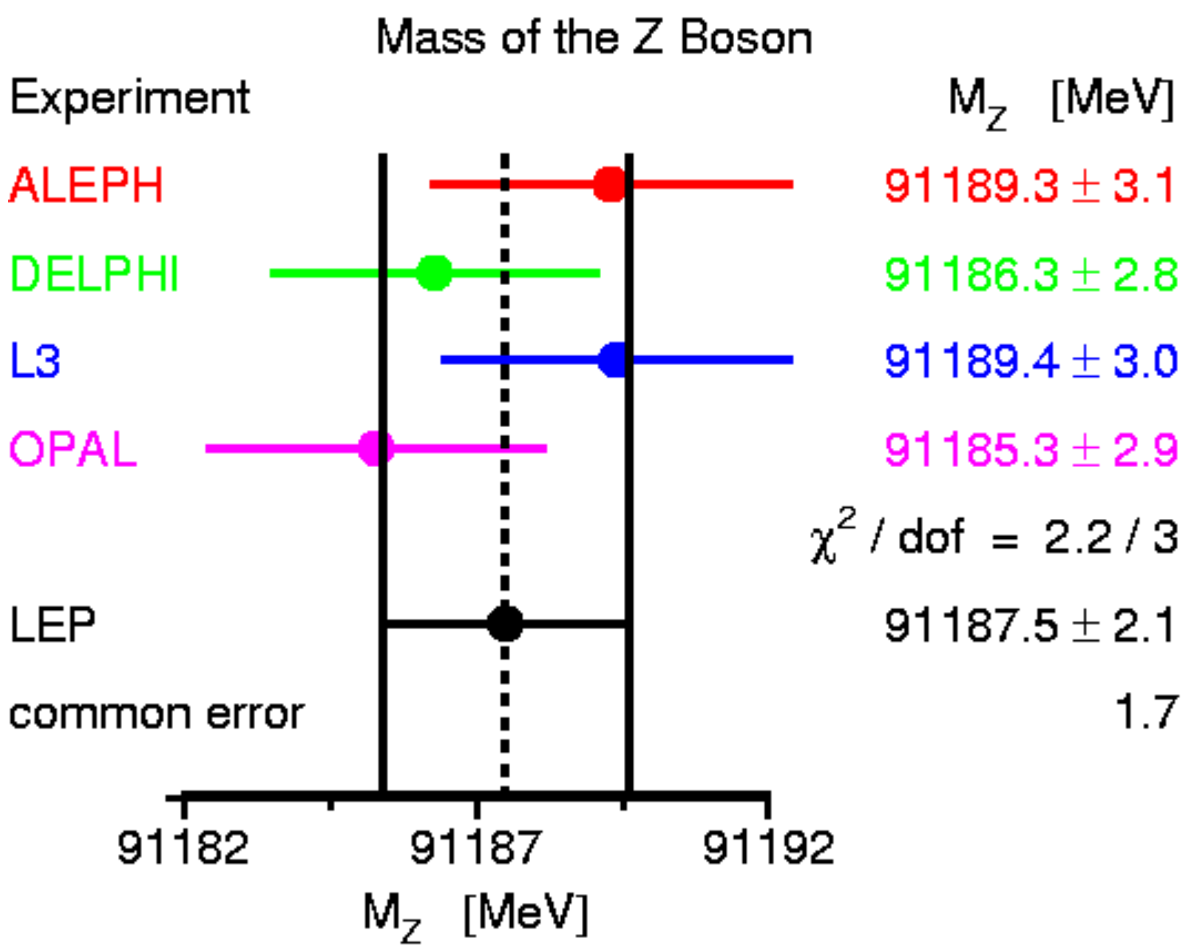
SLD at SLC





Observations on the Z Resonance

Mass of the Z^0



Observations on the Z Resonance

Total and partial widths of the Z^0

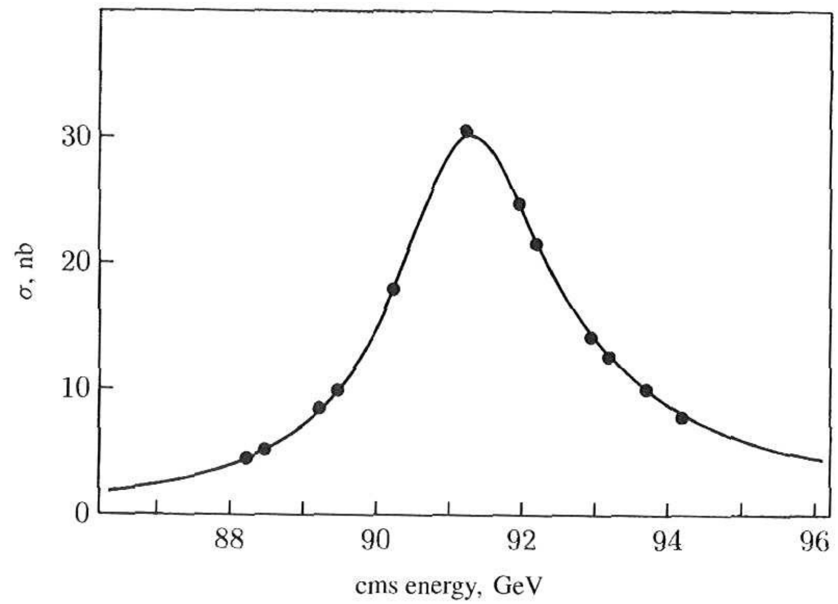
$$\sigma = \frac{4\pi\lambda\!\!\!^{-2}(2J+1)}{(2s+1)^2}\frac{\Gamma_e\Gamma/4}{[(E-E_0)^2+\Gamma^2/4]}$$

$$\Gamma(\text{partial}) = \frac{GM_Z^3\rho}{6\pi\sqrt{2}}(c_A^2+c_V^2)F$$

$Z^0 \to \nu\bar{\nu}$, $F = 1$

$Z^0 \to l\bar{l}$, $F = (1 + 3\alpha/4\pi)$

$Z^0 \to Q\bar{Q}$, $F = 3(1 + \alpha_s/\pi)$



$\Gamma_{\nu\bar{\nu}} = 0.166$ GeV

$\Gamma_{l\bar{l}} = 0.084$ GeV

$\Gamma_{u\bar{u}} = \Gamma_{c\bar{c}} = 0.29$ GeV

$\Gamma_{d\bar{d}} = \Gamma_{s\bar{s}} = \Gamma_{b\bar{b}} = 0.38$ GeV

Observations on the Z Resonance

Total and partial widths of the Z^0

$$\Gamma_{\nu\bar{\nu}} = 0.166 \text{ GeV}$$

$$\Gamma_{l\bar{l}} = 0.084 \text{ GeV}$$

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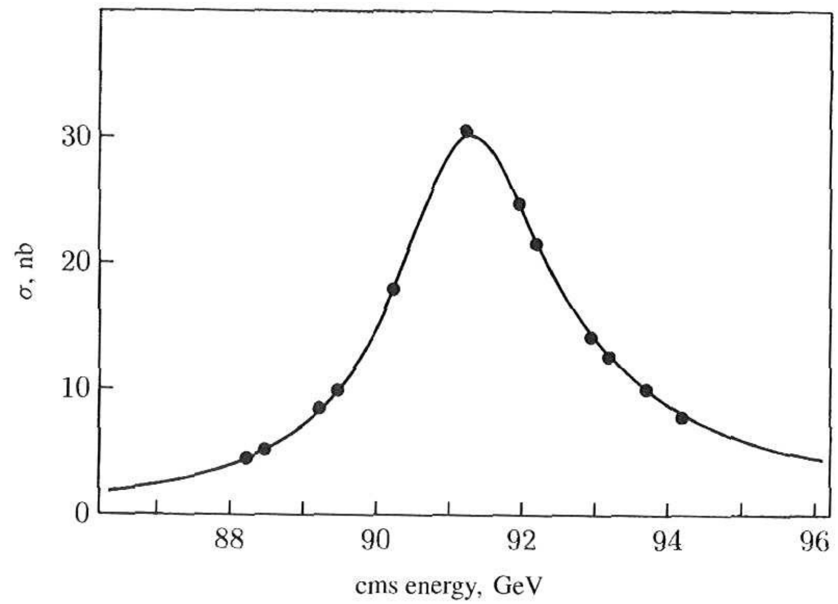


$$\Gamma_{\text{total}}(\text{calculated}) = 2.49 \text{ GeV}$$

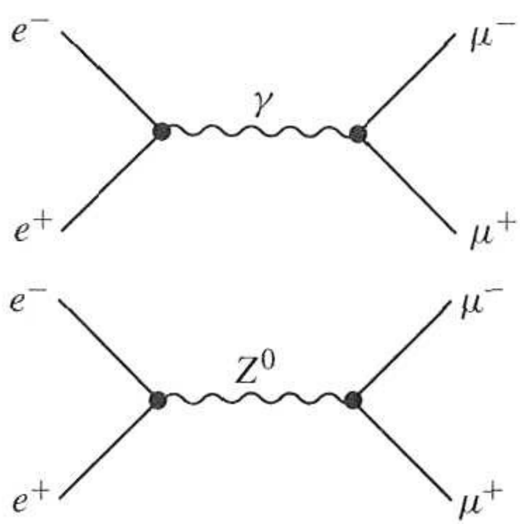
$$\Gamma_{\text{total}}(\text{observed}) = 2.50 \text{ GeV}$$

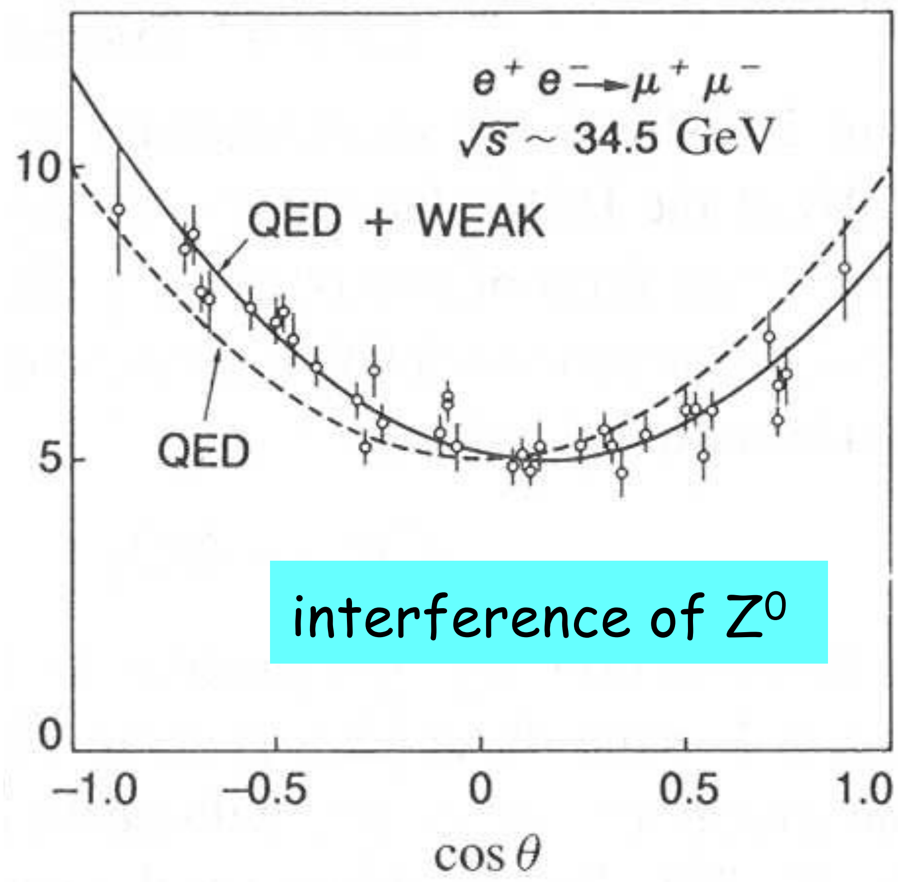
$N_\nu = 2.99 \pm 0.02$

Observations on the Z Resonance

Forward-backward Asymmetries

Even well below the Z^0 resonance the interference between photon exchange and Z^0 exchance results in a forward backward asymmetry





interference of Z^0

$f \sim a_{wk} a_{em} / a_{em}^2 \sim Gs/(4\pi\alpha) \sim 10^{-4}s$ (interference)

Observations on the Z Resonance

Forward-backward Asymmetries

the asymmetry below the Z

$$d\sigma/d\Omega = (d\sigma/d\Omega)_{\text{QED}} + (d\sigma/d\Omega)_{\text{interf.}} + (d\sigma/d\Omega)_{\text{weak}}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\alpha^2/s \qquad\qquad Gs/\alpha \;\; (\alpha^2/s) \qquad\qquad G^2 s$$

$$A_{FB} = \frac{F - B}{F + B} \simeq \frac{Gs}{\alpha}$$

$$A_{FB} = -\frac{3c_A^2}{4\sqrt{2}\pi}\frac{Gs}{\alpha}$$

Dependence is on c_A, and therefore is not sensitive to $\sin^2\theta_W$

Observations on the Z Resonance

Forward-backward Asymmetries

on the Z^0 resonance $A^0_{FB} = 3/4 \quad A^0_e \, A^0_f$

$$A^0 = \frac{2\, c_V c_A}{c_V^2 + c_A^2} = \frac{2\,(I_3 - 2Q\sin^2\theta_W)\; I_3}{(I_3 - 2Q\sin^2\theta_W)^2 + I_3^2}$$

$$A_e \equiv \frac{2(1 - 4\sin^2\theta_W)}{1 + (1 - 4\sin^2\theta_W)^2}$$

Fermion	A^0_f	A^0_{FB}
ν_e, ν_μ, ν_τ	1	0.12
e, μ, τ	0.16	0.02
u, c, t	0.67	0.08
d, s, b	0.93	0.11

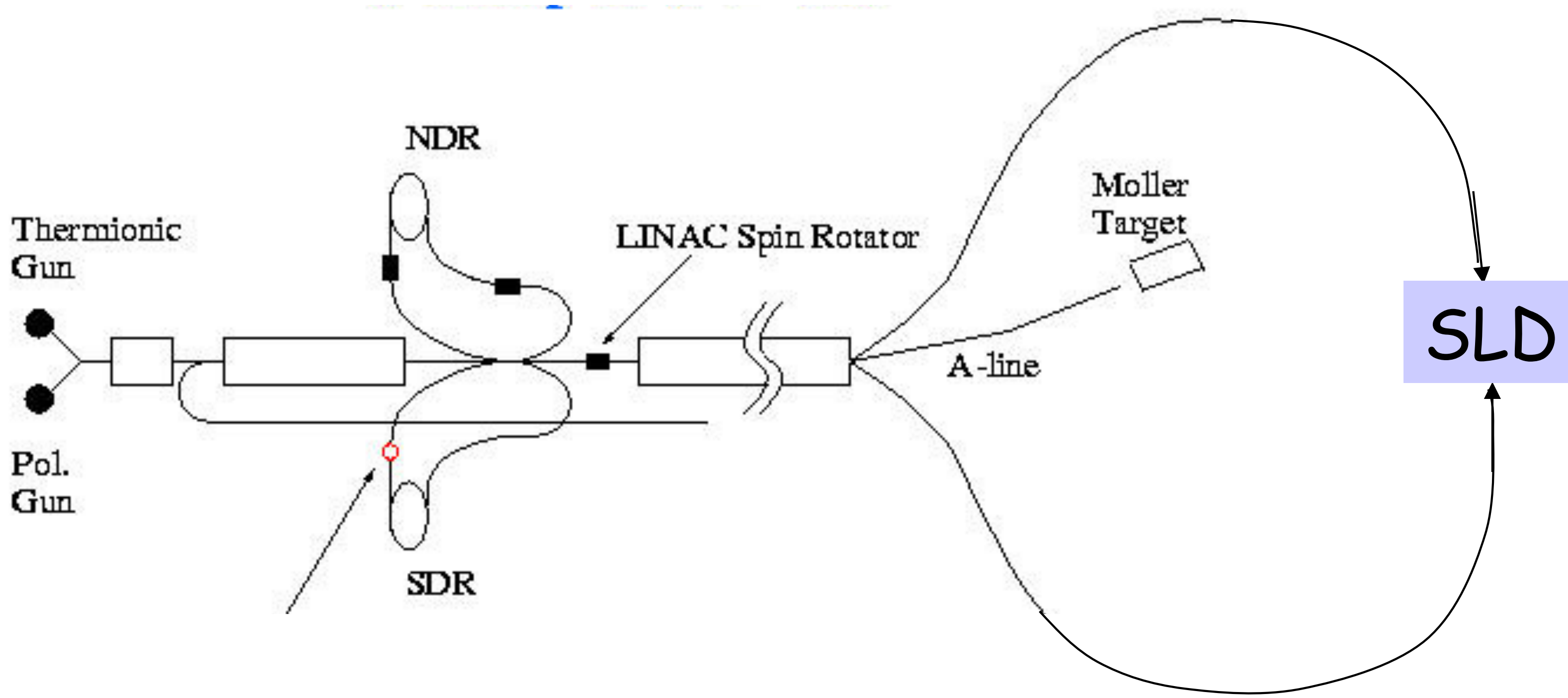
Observations on the Z Resonance

Left-right and Polarization Asymmetries

SLD at SLC

$$A^0_{LR} = \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\right) = A^0_e$$

A_{LR} is a direct measurement of A_e^0



Observations on the Z Resonance

Actually we don't measure A_{LR} directly,

$A^{meas}_{LR} = P_e A_{LR} = P_e A_e$

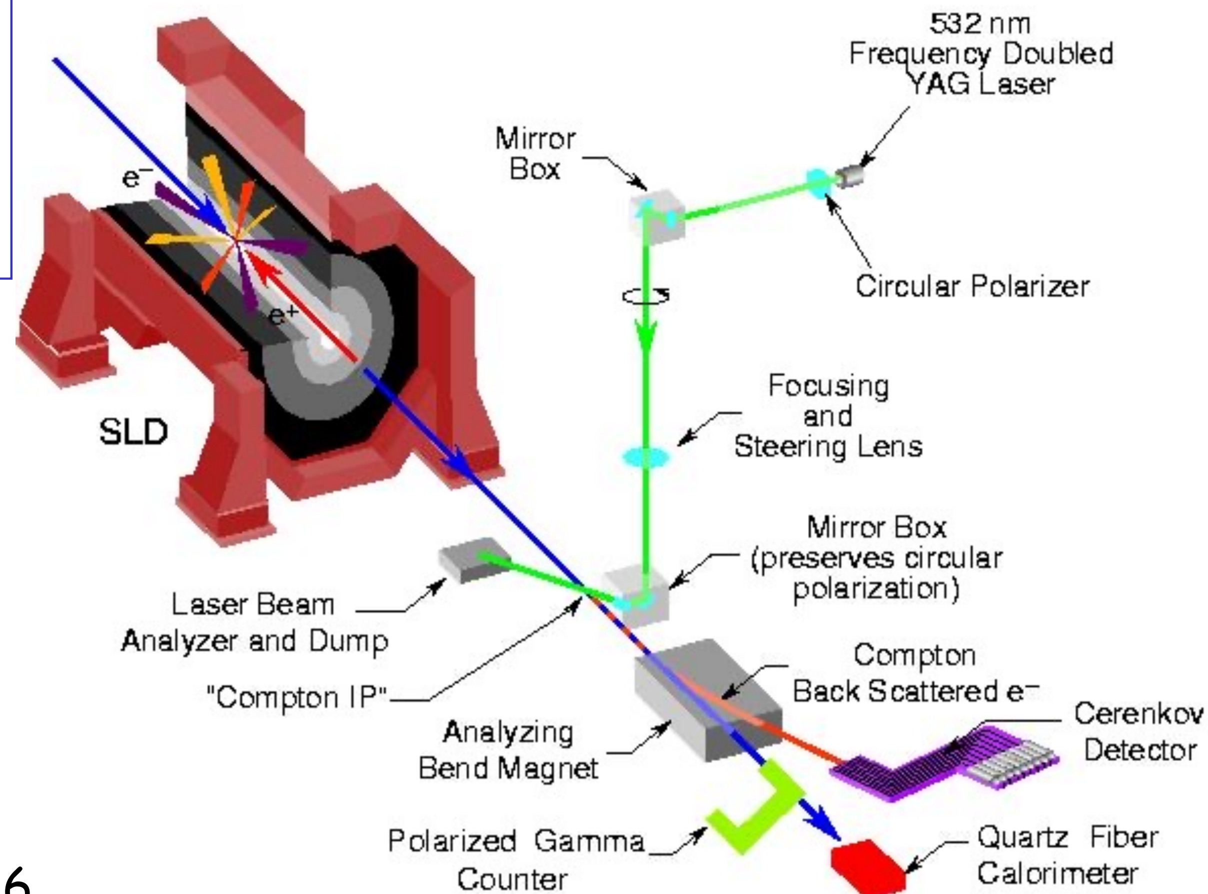
so we must know P_e

$$A_e \equiv \frac{2(1-4\sin^2\theta_W)}{1+(1-4\sin^2\theta_W)^2}$$

$A_{LR} = 0.1513 \pm 0.0021$

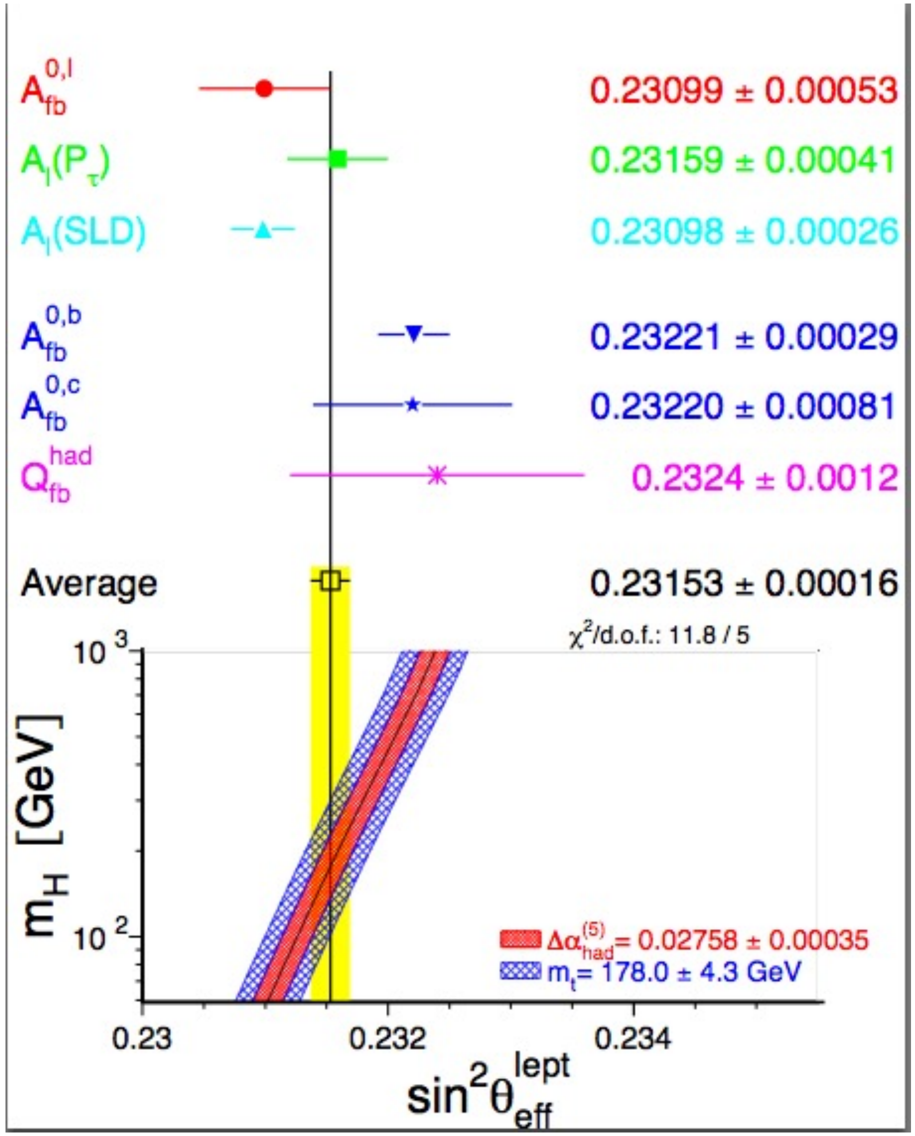
$\sin^2\theta_{Weff} = 0.23098 \pm 0.00026$

SLD at SLC



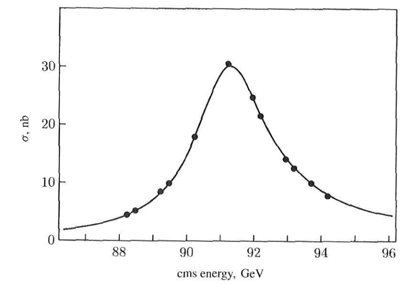
Observations on the Z Resonance

Summary

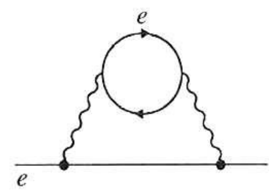


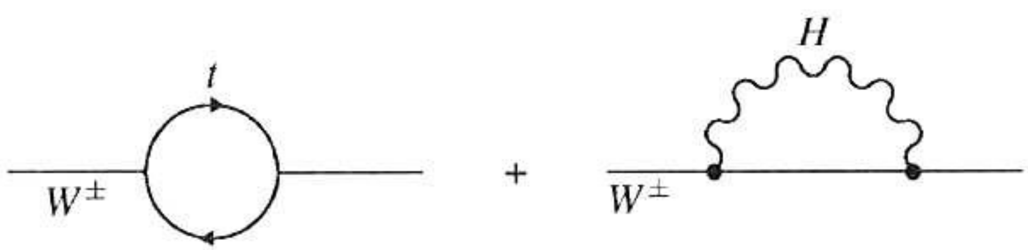
Fits to the Standard Model and Radiative Corrections

- The Standard Model can only be tested precisely after the effects of radiative corrections have been accounted for
- Initial state radiation of real photons prior to collision distorts the Breit-Wigner resonance shape



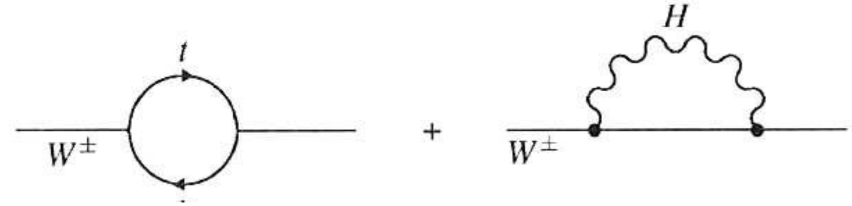
- Other corrections:
  - virtual photon emission ⇒ running of α_{EM}
  - also, virtual gluon emission
  - loop diagrams of virtual top quark and Higgs





Fits to the Standard Model and Radiative Corrections

- The Loop diagrams affect the W and Z differently, leading to an effect on the ratio of the neutral- to charged-current couplings



$$M_{Z^0}^2 = \frac{M_W^2}{\rho \cos^2 \theta_W}$$

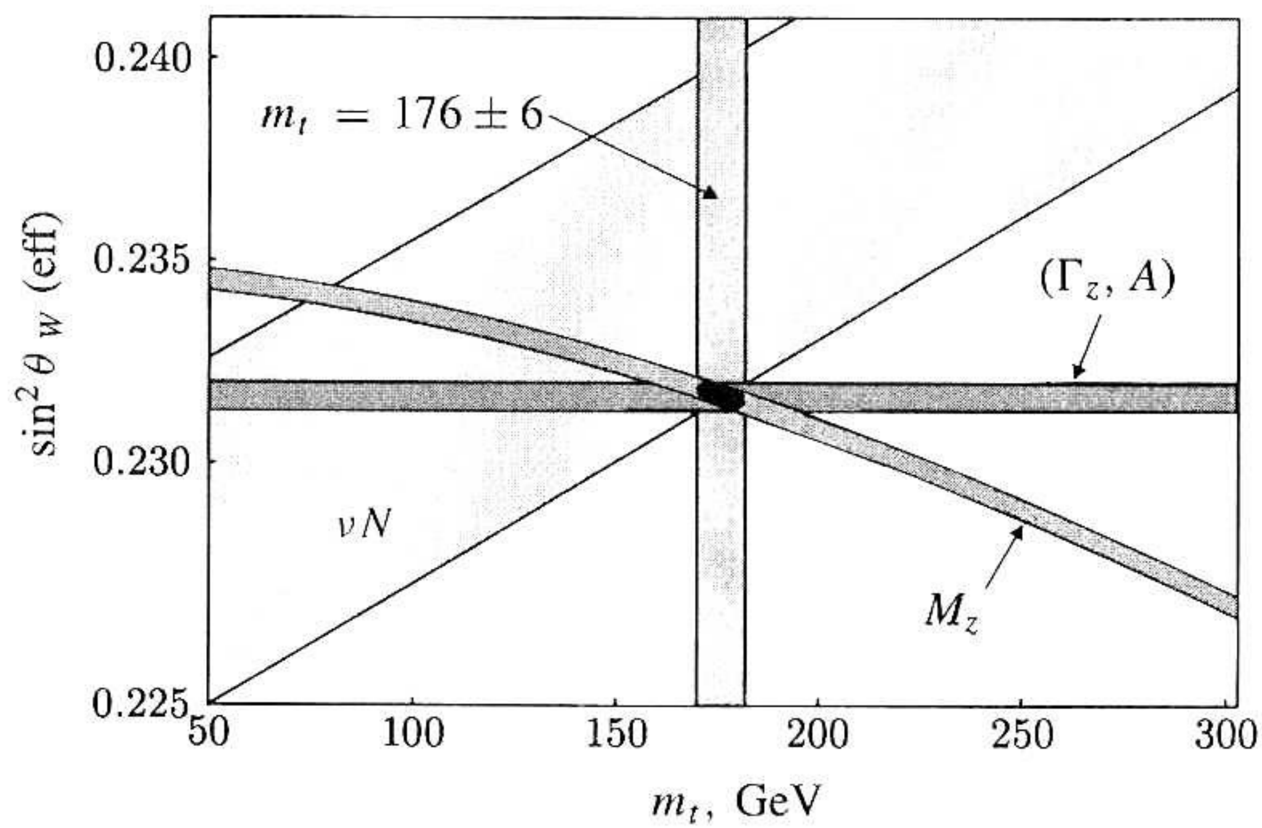
$$\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}$$

$$\sin^2 \theta_W(\text{eff}) \simeq \sin^2 \theta_W (1 + \cot^2 \theta_W \Delta\rho)$$

- Now the measured value of $\sin^2\theta_W$ will depend on the input parameters of the model
- Normally we can take M_Z, M_W, α, α_S as known and allow variation in m_t and m_H

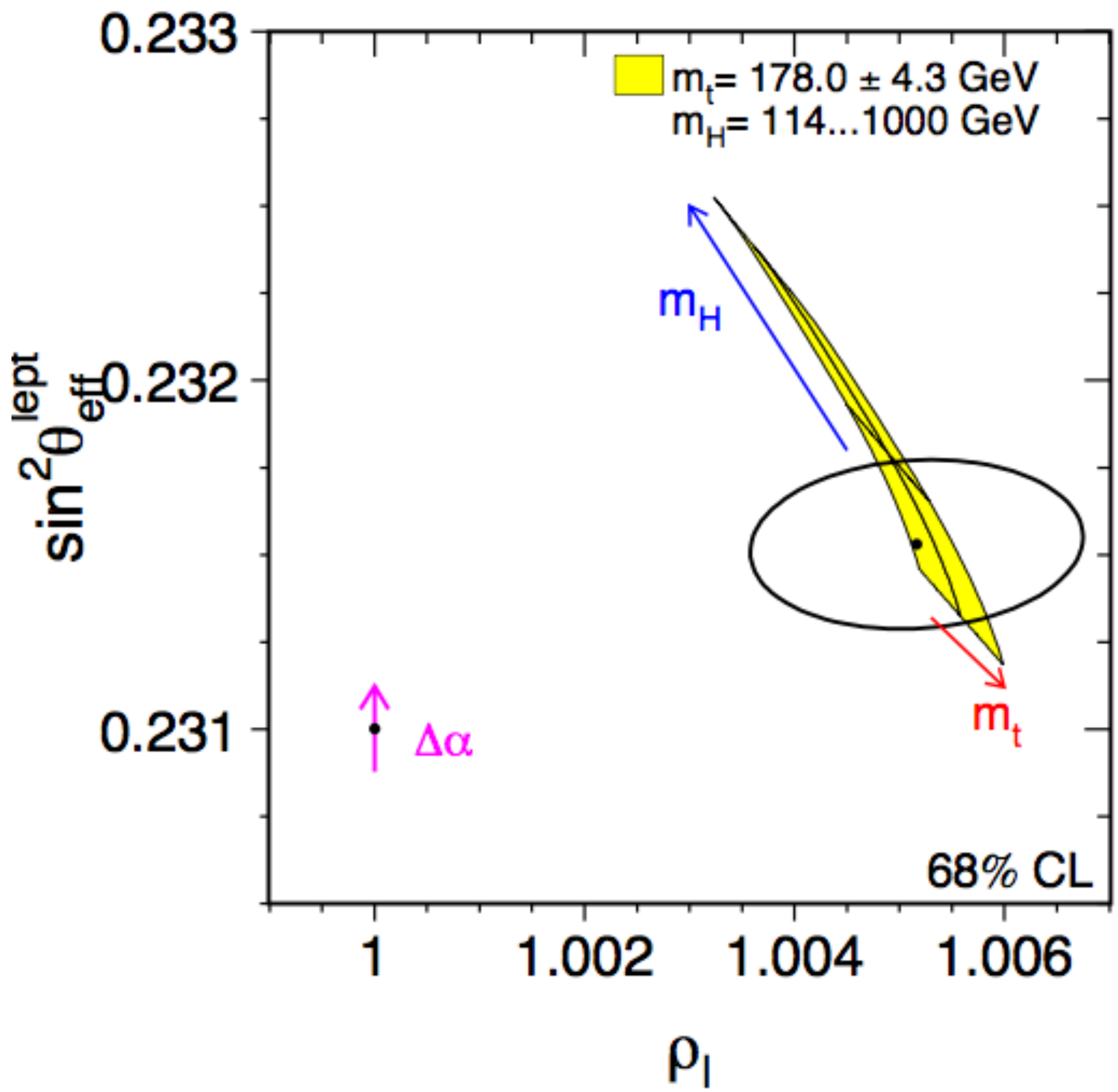
Fits to the Standard Model and Radiative Corrections

- Fitted values of $\sin^2\theta_{Weff}$ as a function of the top quark mass, for M_H = 300 GeV (1996)



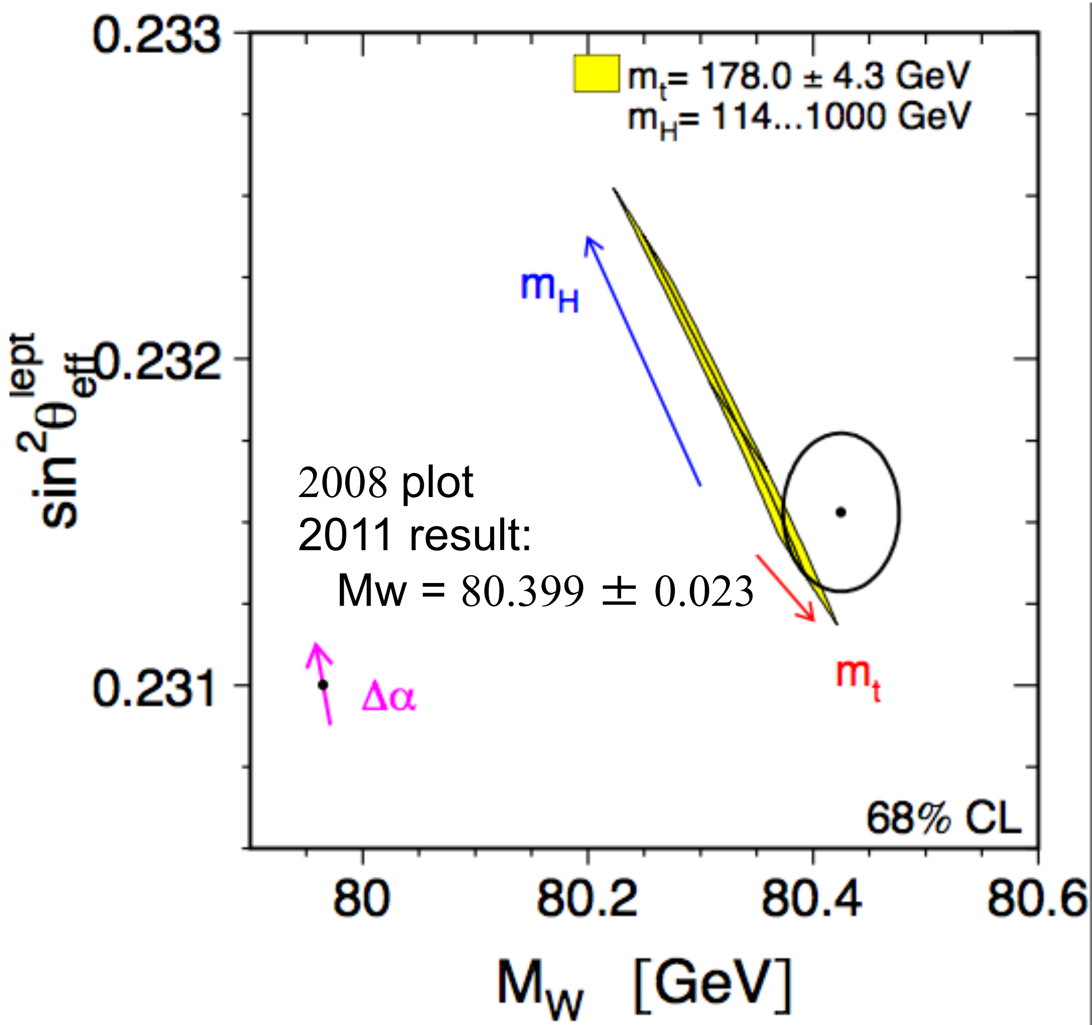
Fits to the Standard Model and Radiative Corrections

- $\sin^2\theta_{Weff}$ and ρ as a function of the top quark mass (m_t) and the Higgs mass (m_H).
- The ellipse represents the measurements

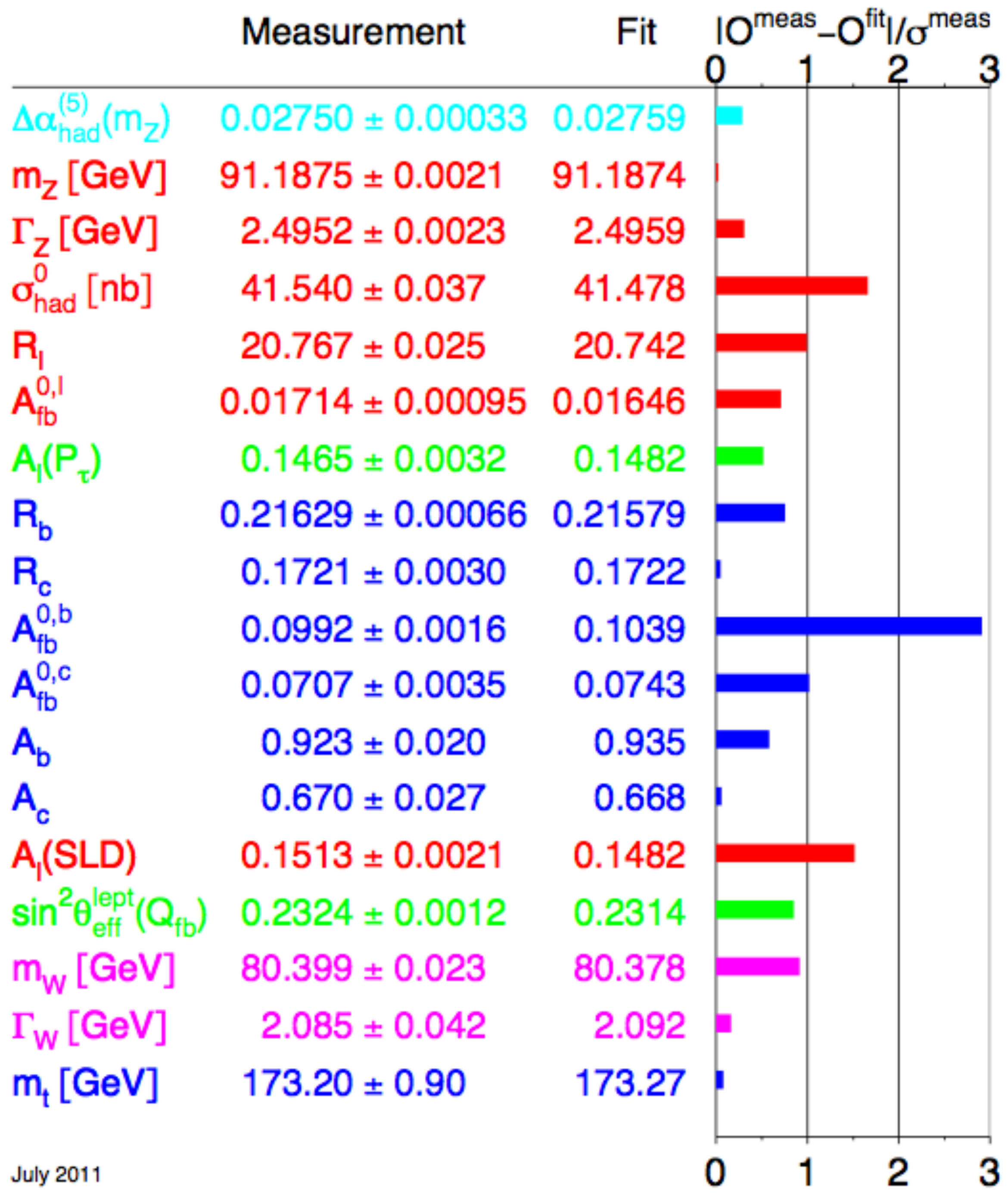


Fits to the Standard Model and Radiative Corrections

- $\sin^2\theta_{Weff}$ as a function of the top quark mass (m_t), the Higgs mass (m_H), and the W mass (M_w).
- The ellipse represents the measurements

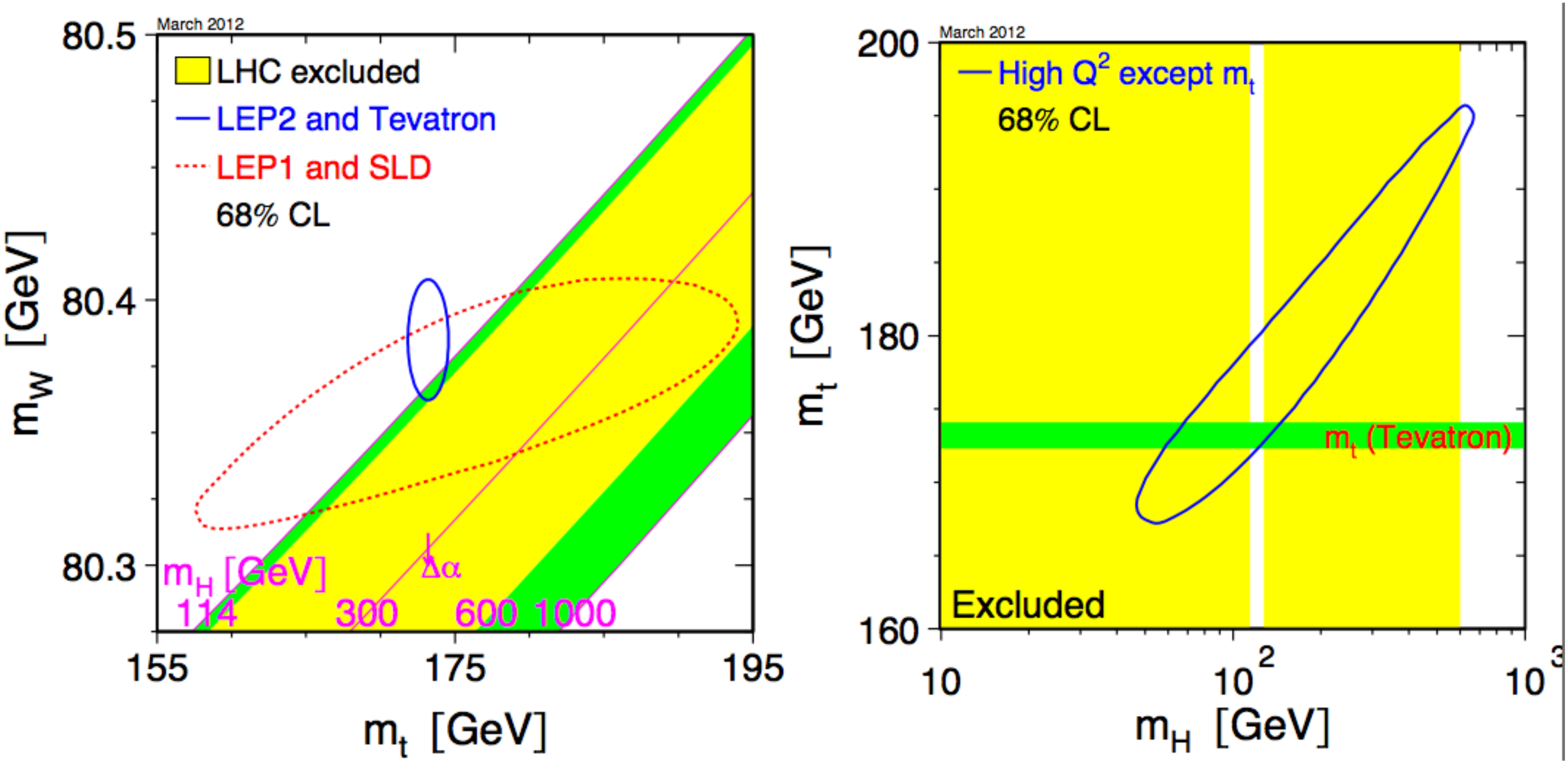


Fits to the Standard Model and Radiative Corrections

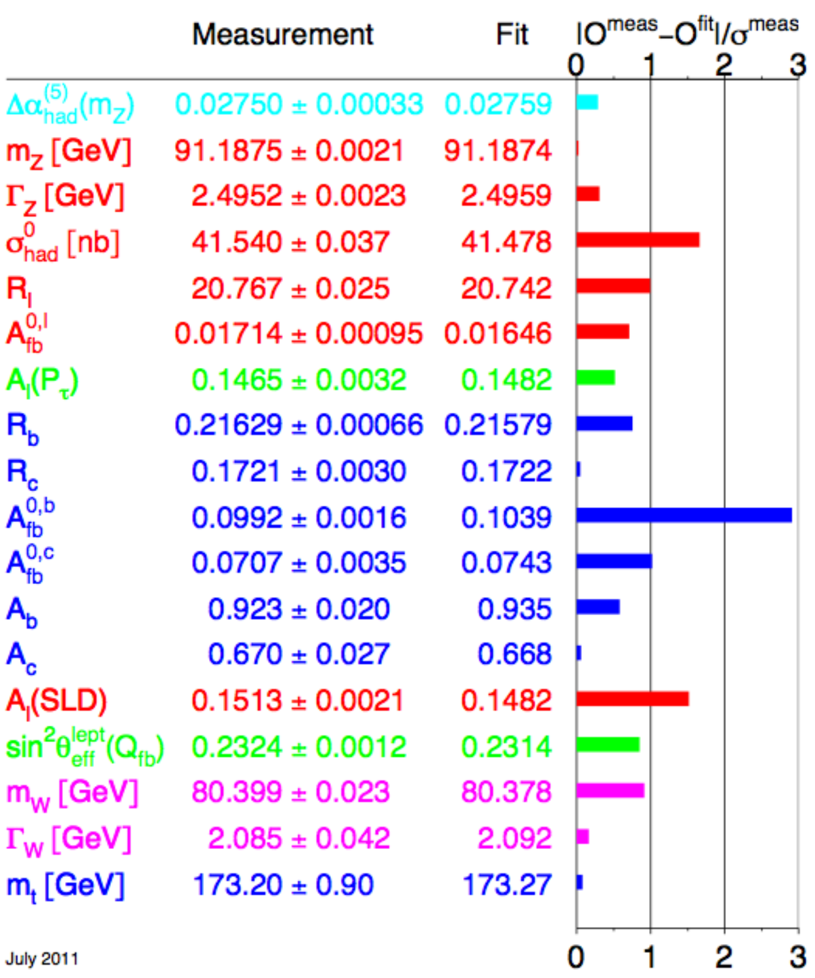


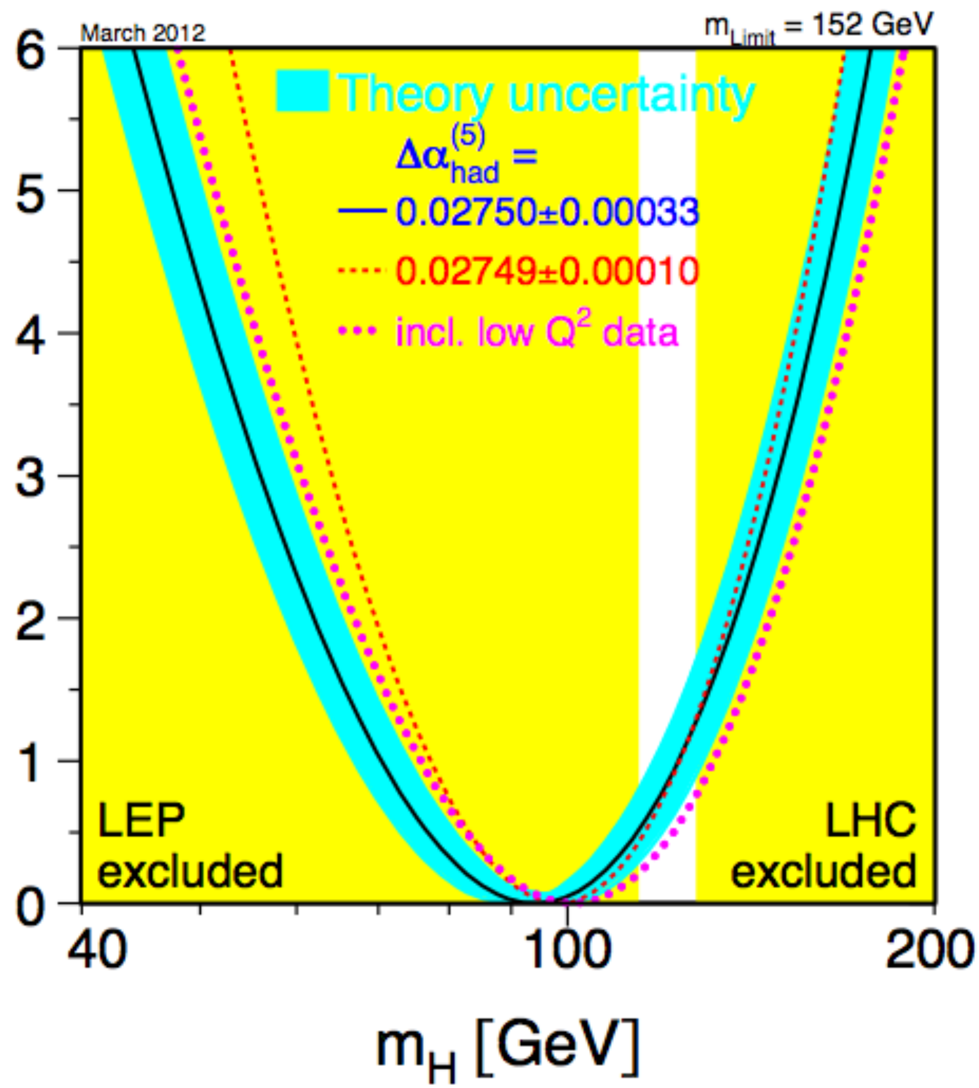
J. Brau

Fits to the Standard Model and Radiative Corrections



Fits to the Standard Model and Radiative Corrections



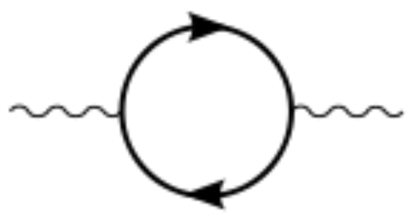


Theoretical Estimation of the Radiative Corrections

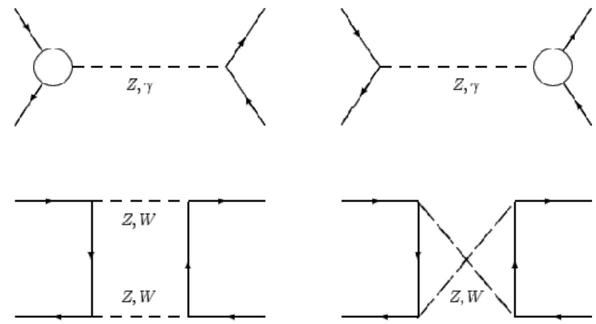
PHYSICAL REVIEW D VOLUME 46, NUMBER 1 1 JULY 1992

Estimation of oblique electroweak corrections

Michael E. Peskin and Tatsu Takeuchi



vacuum polarizations
(**oblique corrections**)



vertex corrections and box diagrams
(**direct corrections**)

Radiative corrections due to physics beyond the standard model

- appear dominantly through vacuum polarizations (**oblique corrections**)
- vertex corrections and box diagrams (**direct corrections**) can be neglected

Theoretical Estimation of the Radiative Corrections

PHYSICAL REVIEW D VOLUME 46, NUMBER 1 1 JULY 1992

Estimation of oblique electroweak corrections

Michael E. Peskin and Tatsu Takeuchi
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 9 December 1991)

We review the general analysis of the contributions of electroweak vacuum-polarization diagrams to precision experiments. We first review the representation of these contributions by three parameters S, T, and U and discuss the assumptions involved in this reduction. We then discuss the contributions to these parameters from various models of new physics. We show that S can be computed by a dispersion relation, and we use this technique to estimate S in technicolor models of the Higgs sector. We discuss the reliability and the gauge invariance of this estimate. Finally, we present the limits on S and T imposed by current experimental results.

- **T** measures the difference between the new physics contributions of neutral and charged current processes at low energies (*i.e.*, sensitive to isospin violation).
- **S (S+U)** describes new physics contributions to neutral (charged) current processes at different energy scales.
- **U** is only constrained by the W boson mass and its total width. In addition U is generally small in new physics models. Therefore, the STU parameter space can often be projected down to a two-dimensional parameter space in which the experimental constraints are easy to visualise.

Theoretical Estimation of the Radiative Corrections

Peskin/Takeuchi input:

M_t = 150 GeV
m_H = 1000 GeV
$e^2 = 4\pi/129$
$\sin^2\theta_w$ = .2337
α_s = 0.12

$$\frac{m_W}{m_Z} = 0.8787 - (3.15\times10^{-3})S + (4.86\times10^{-3})T + (3.70\times10^{-3})U\ ,$$

$$\Gamma_Z = 2.484 - (9.58\times10^{-3})S + (2.615\times10^{-2})T\ \text{(GeV)}\ ,$$

$$\Gamma_{l^+l^-} = 0.0835 - (1.91\times10^{-4})S + (7.83\times10^{-4})T\ \text{(GeV)}\ ,$$

$$\Gamma_{u\bar{u}} = 0.2962 - (1.92\times10^{-3})S + (3.67\times10^{-3})T\ \text{(GeV)}\ ,$$

$$\Gamma_{d\bar{d}} = 0.3823 - (1.72\times10^{-3})S + (4.20\times10^{-3})T\ \text{(GeV)}\ ,$$

$$\Gamma_{b\bar{b}} = 0.3779 - (1.72\times10^{-3})S + (4.20\times10^{-3})T\ \text{(GeV)}\ ,$$

$$\Gamma_{\text{had}} = 1.7348 - (9.00\times10^{-3})S + (1.993\times10^{-2})T\ \text{(GeV)}\ ,$$

$$R_Z = \Gamma_{\text{had}}/\Gamma_{l^+l^-} = 20.78 - (5.99\times10^{-2})S + (4.24\times10^{-2})T\ ,$$

$$s_*^2(m_Z^2) = 0.2337 + (3.59\times10^{-3})S - (2.54\times10^{-3})T\ ,$$

$$A_{LR} = -P_\tau = 0.1297 - (2.82\times10^{-2})S + (2.00\times10^{-2})T\ ,$$

$$A_{\text{FB}}^b = 0.0848 - (1.97\times10^{-2})S + (1.40\times10^{-2})T\ ,$$

$$A_{\text{FB}}^l = 0.0126 - (6.72\times10^{-3})S + (4.76\times10^{-3})T\ ,$$

$$g_L^2 = 0.3001 - (2.67\times10^{-3})S + (6.53\times10^{-3})T\ ,$$

$$g_R^2 = 0.0302 + (9.17\times10^{-4})S - (1.94\times10^{-4})T\ ,$$

$$R_\nu = 0.3126 - (2.32\times10^{-2})S + (6.46\times10^{-2})T\quad (r = 0.383)\ ,$$

$$R_{\bar{\nu}} = 0.3824 - (2.77\times10^{-3})S + (6.03\times10^{-3})T\quad (r = 0.371)\ ,$$

$$Q_W({}^{133}_{55}\text{Cs}) = -73.31 - 0.790S - 0.011T\ ,$$

Theoretical Estimation of the Radiative Corrections

Peskin/Takeuchi input:

M_t = 150 GeV, m_H = 1000 GeV, $e^2 = 4\pi/129$, $\sin^2\theta_w = .23$, $\alpha_s = 0.12$

$$s_*^2(m_Z^2) = 0.2337 + (3.59\times10^{-3})S - (2.54\times10^{-3})T$$

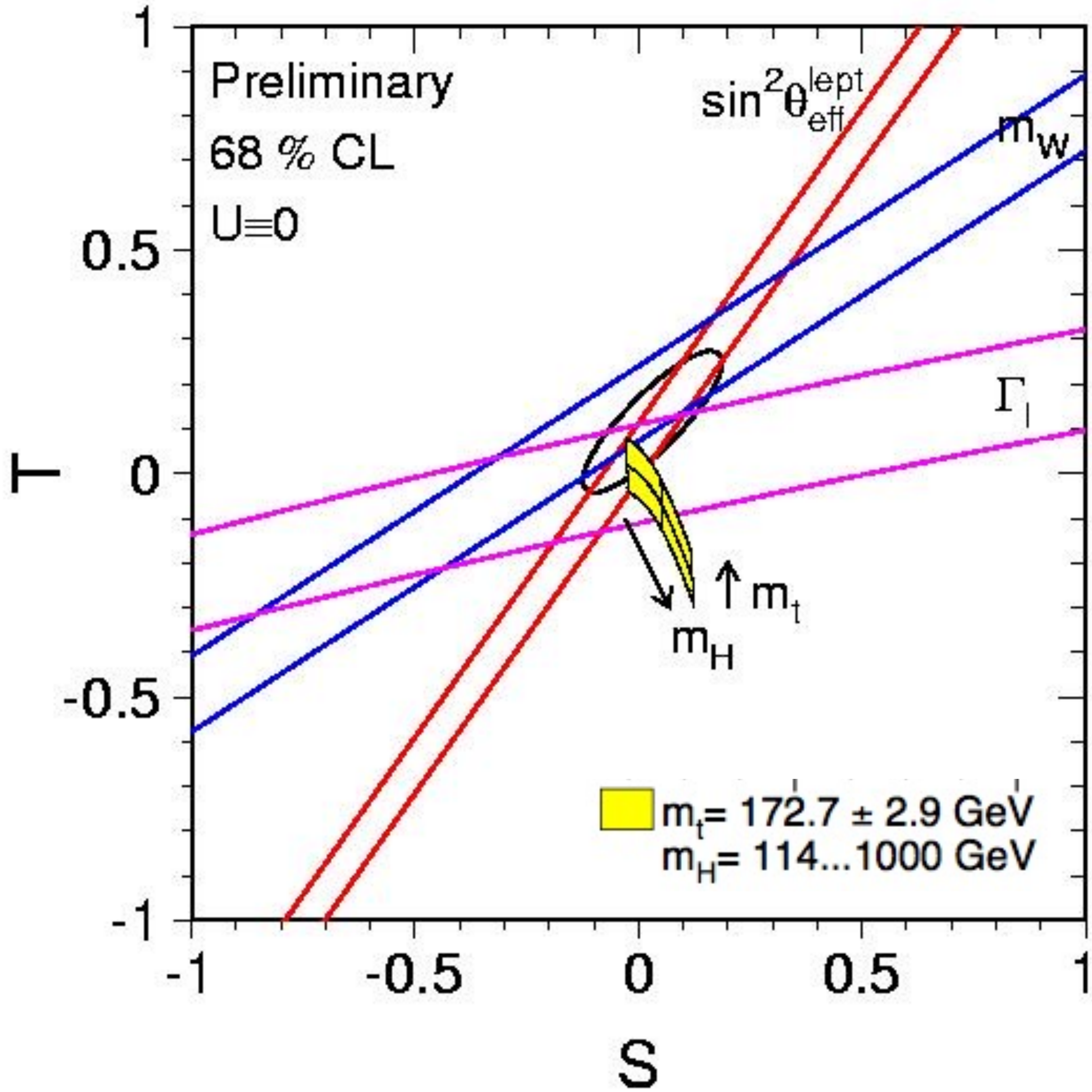
$$T = 92.0 - 393.7\, s^2 + 1.41\, S$$
$$= 393.7\,(0.2337 - s^2) + 1.41\, S$$

$$\frac{m_W}{m_Z} = 0.8787 - (3.15\times10^{-3})S + (4.86\times10^{-3})T$$
$$+ (3.70\times10^{-3})U\ ,$$

$$T = -\,180.8 + 205.8\, m_W/\, m_Z + 0.65\, S$$
$$= 205.8\,(m_W/\, m_Z - 0.8787) + 0.65\, S$$

Theoretical Estimation of the Radiative Corrections

http://arxiv.org/pdf/hep-ex/0509008v3.pdf



Theoretical Estimation of the Radiative Corrections

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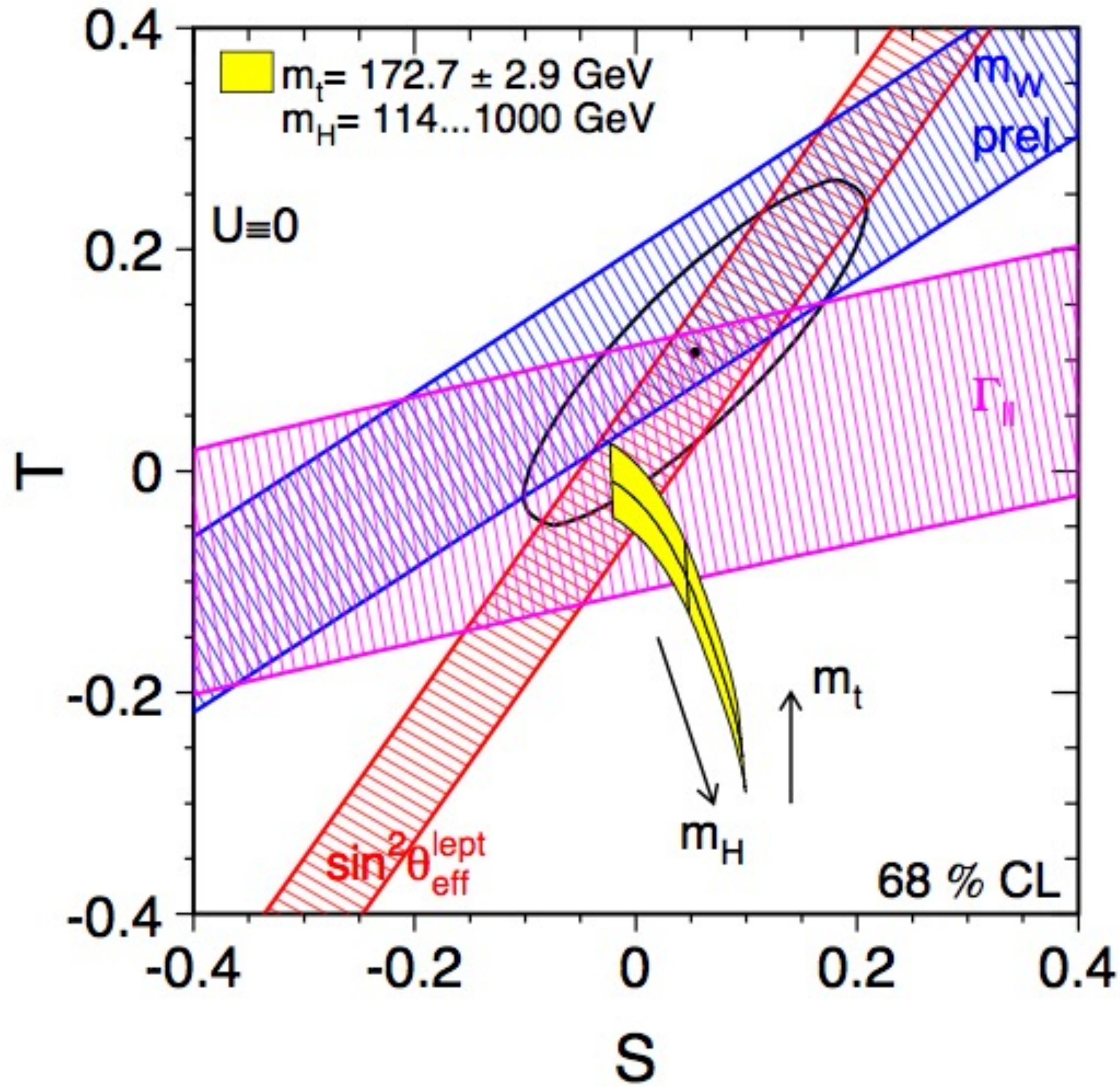
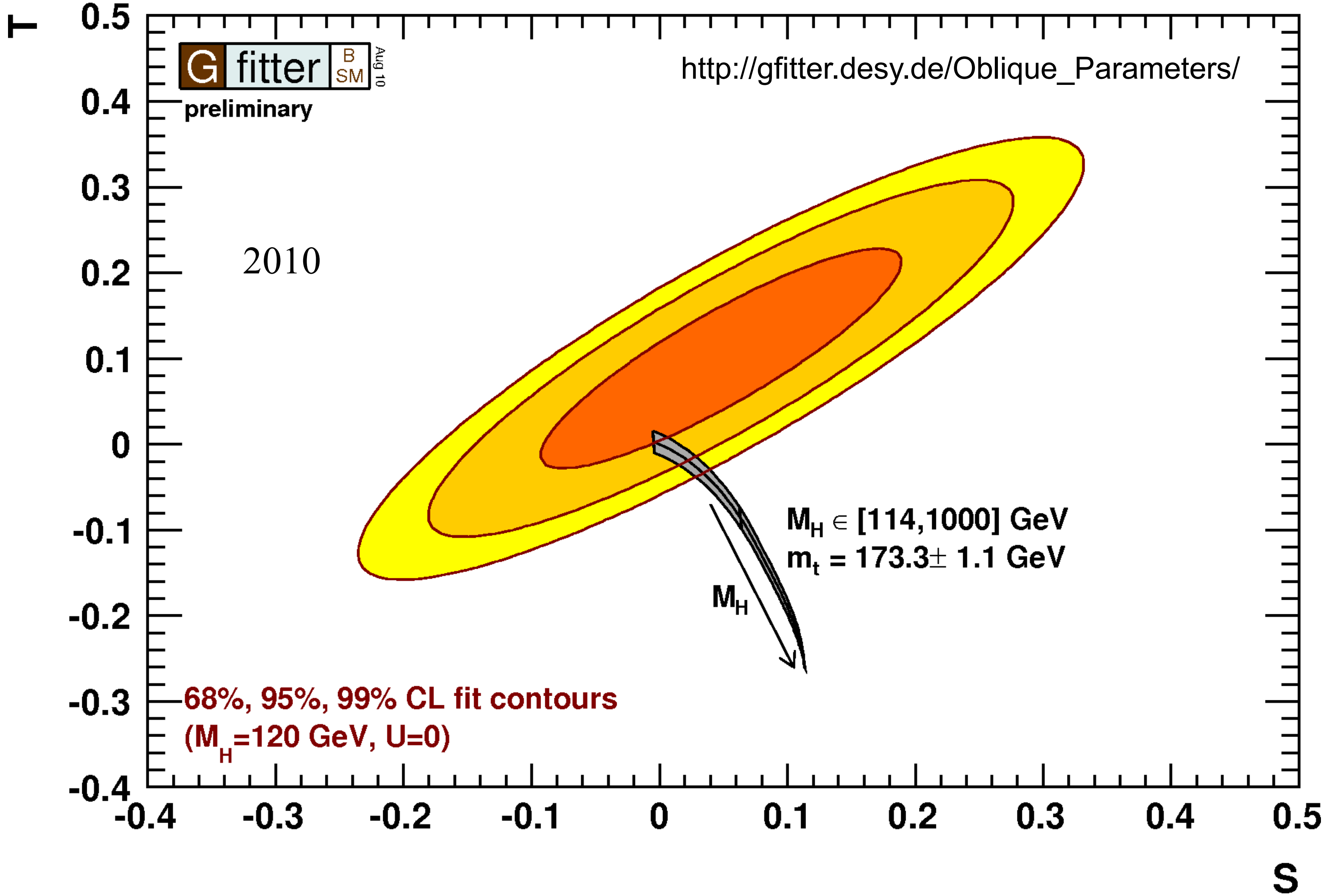
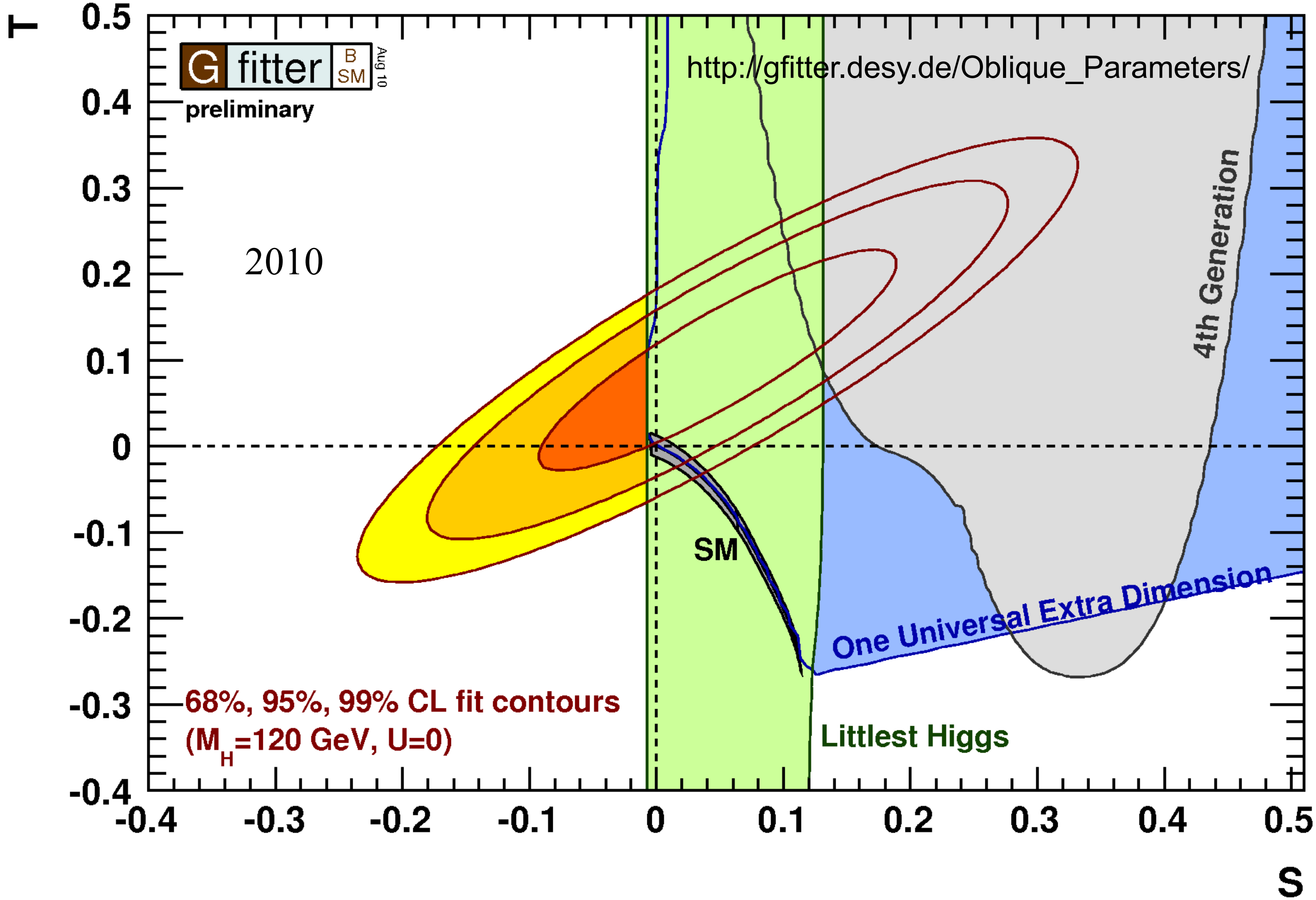


Theoretical Estimation of the Radiative Corrections



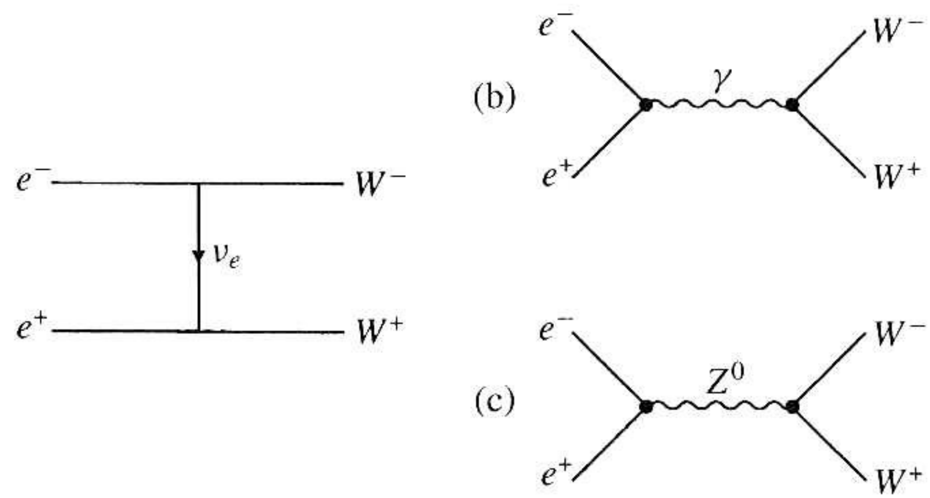
Theoretical Estimation of the Radiative Corrections



W Pair Production

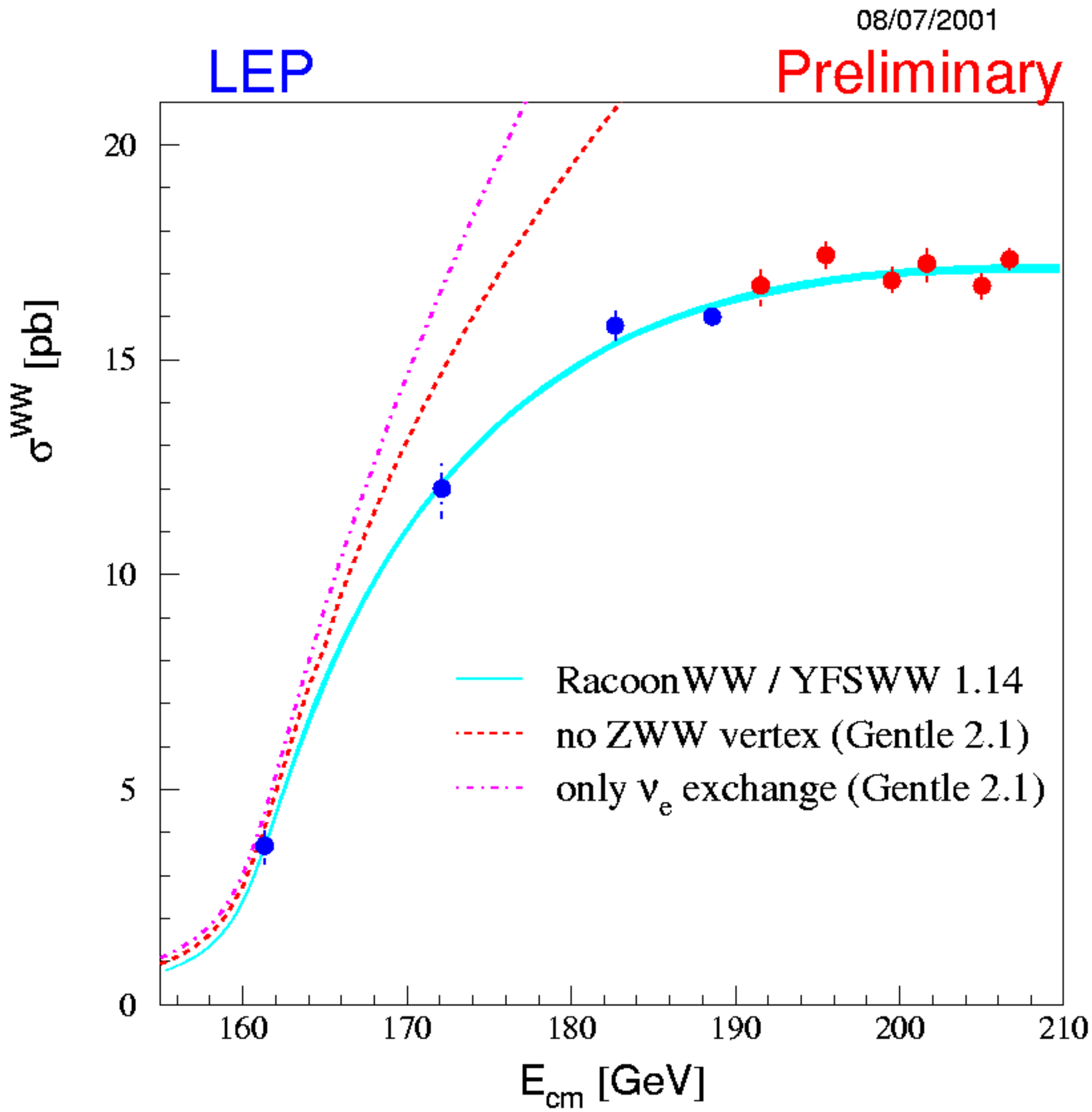
In 1996, the energy of LEP at CERN was increased above the W pair production threshold to begin the LEP2 program

Each of these processes is individually divergent with s

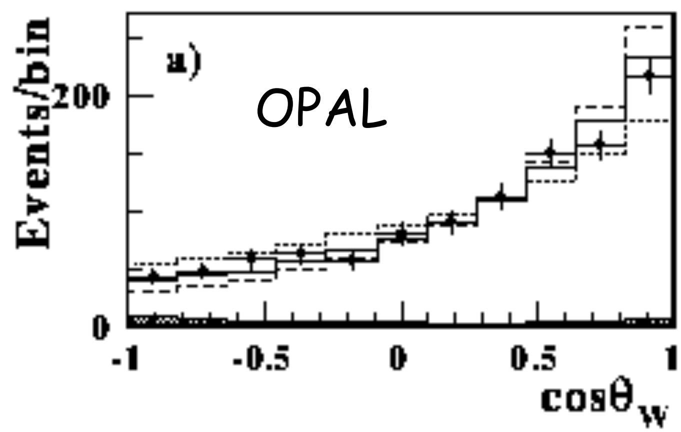


When combined, with the couplings given by the Standard Model, the total cross section remains finite, and falls as ln(s)/s

W Pair Production

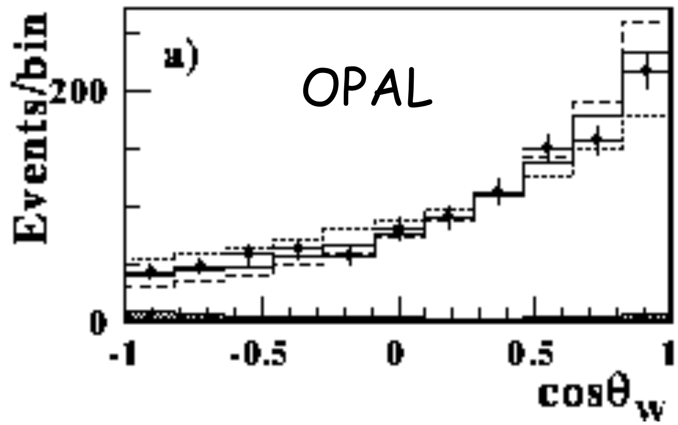


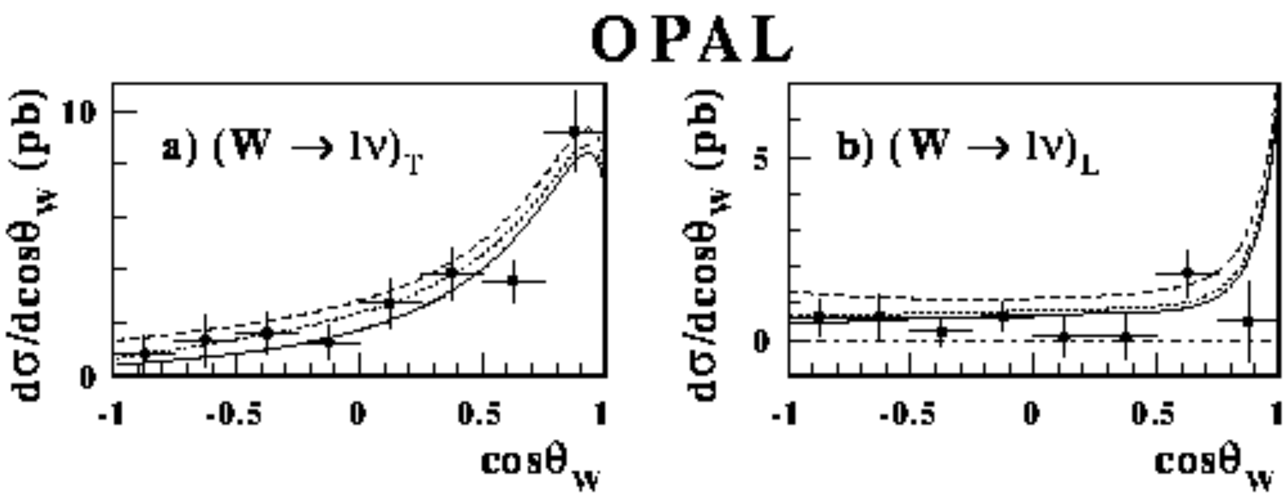
The angular distributions are a good test of the Standard Model, and are in good agreement



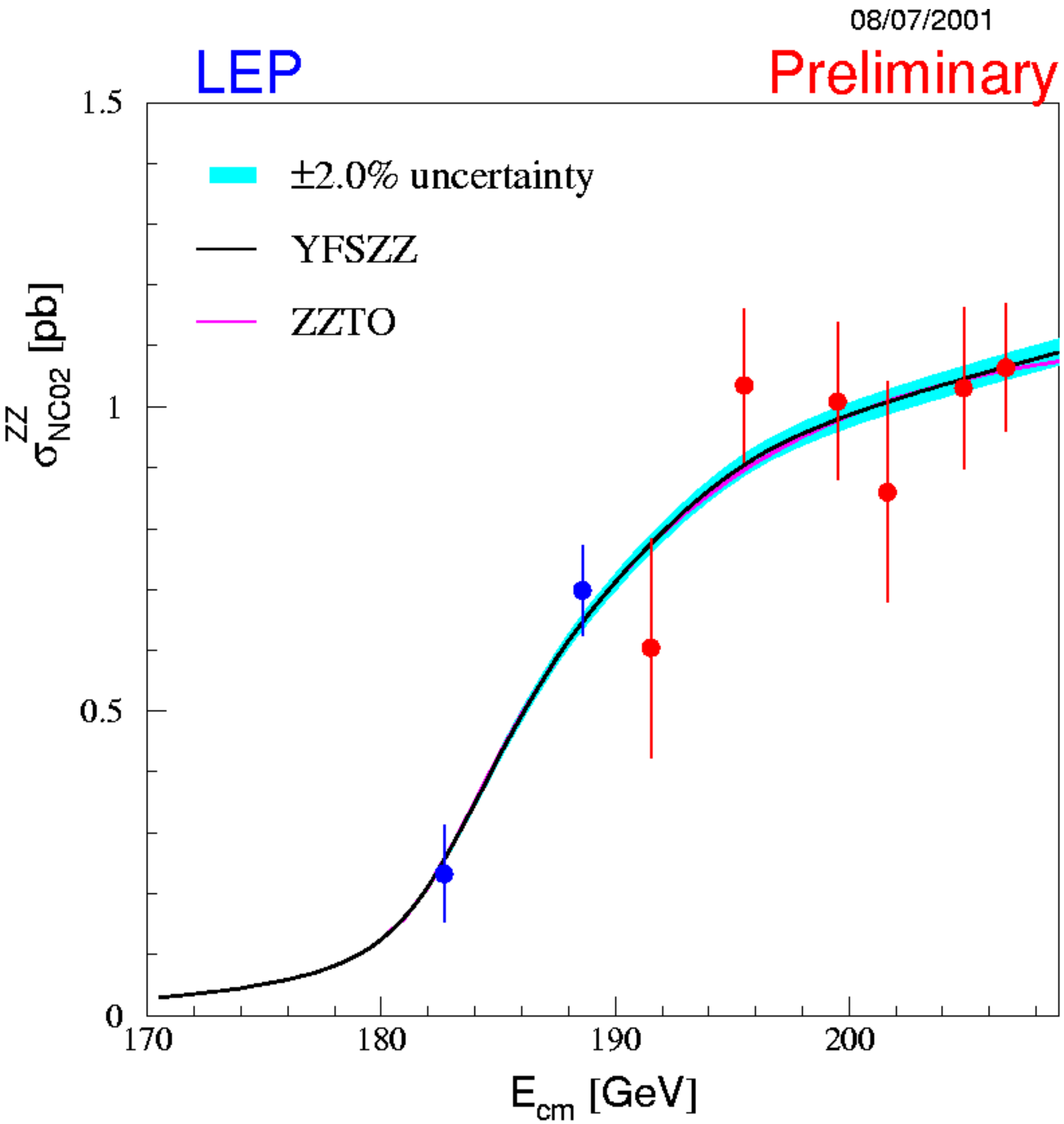
W Pair Production

The angular distributions are a good test of the Standard Model, and are in good agreement





Z Pair Production



Spontaneous Symmetry Breaking and the Higgs Mechanism

- The open issue of Electroweak symmetry breaking is:

  How is the symmetry broken?

- In other words, how do we move from the underlying Lagrangian:

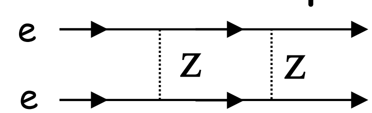
$$L = g\mathbf{J}_\mu \cdot \mathbf{W}_\mu + g' J_\mu^Y B_\mu$$

  to one in which the EM field remains massless and the weak neutral current acquires mass

- The standard model solution to this problem is the local gauge symmetry, and the Higgs mechanism

Spontaneous Symmetry Breaking and the Higgs Mechanism

- The Higgs mechanism was invented to explain how the symmetry could be broken, endow the weak bosons with mass, and preserve the massless photon
- One problem was that the Weinberg-Salam model is dealing with massless bosons. Arbitrarily adding mass for the W and Z spoils gauge invariance and leads to divergences



- Can some underlying principle do it naturally?
  - The Higgs mechanism
- Suppose there is a scalar field filling space that is self interacting.
  - The most general Lagrangian for this field is

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

kinetic energy → $\frac{1}{2}(\partial_\mu \phi)^2$

mass → $\frac{1}{2}\mu^2\phi^2$

self-interaction → $\frac{1}{4}\lambda\phi^4$

Spontaneous Symmetry Breaking and the Higgs Mechanism

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4$$

- Consider the minimum in the potential energy of the Lagrangian:

$$V = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4$$

- minimum at $\partial V/\partial\phi = \phi(\mu^2 + \lambda\phi^2) = 0$

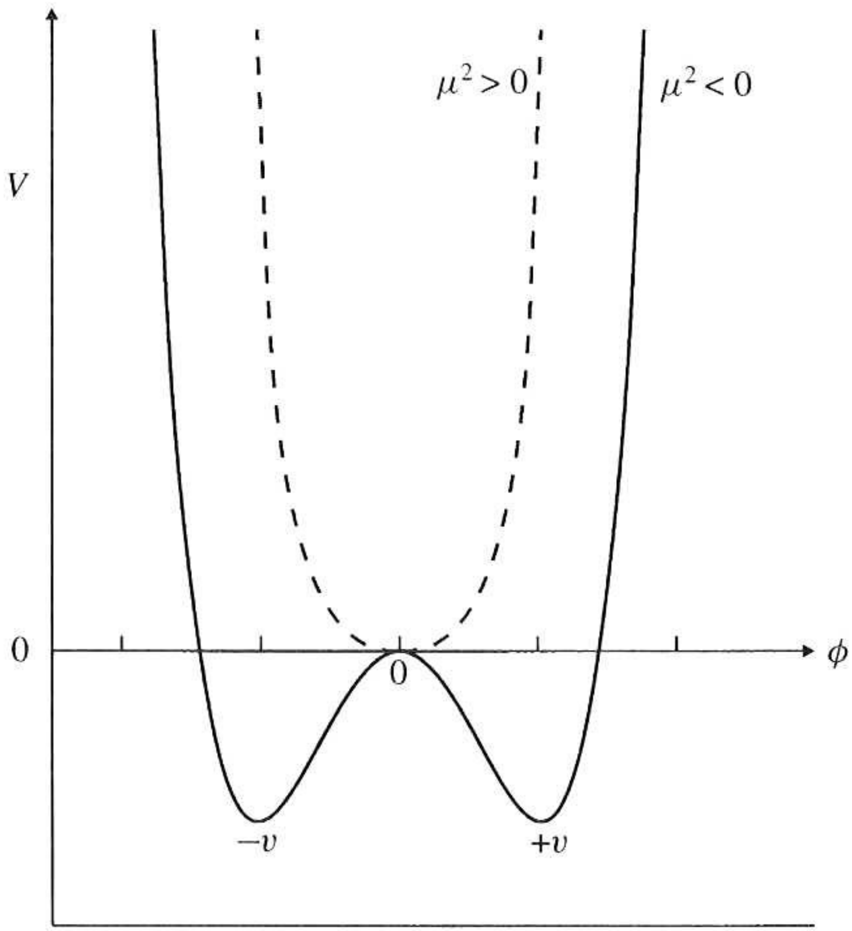
- depends on sign of μ^2

  $\mu^2 > 0$ $\quad \phi = \phi_{\min}$ when $\phi = 0$

  $\mu^2 < 0$ $\quad \phi = \phi_{\min}$ when $\phi = \pm v = \pm\sqrt{\dfrac{-\mu^2}{\lambda}}$

non-zero vacuum expectation value

Spontaneous Symmetry Breaking and the Higgs Mechanism



$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4$$

$$V = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4$$

- For $\mu^2 < 0$ there are two minima: +v and -v
- Expand the field about the minimum v (or -v)

$$\phi = v + \sigma(x)$$

- Now the Lagrangian energy density becomes

$$L = \tfrac{1}{2}(\partial_\mu \sigma)^2 - \lambda v^2 \sigma^2 - \left(\lambda v \sigma^3 + \tfrac{1}{4}\lambda\sigma^4\right) + \text{constant}$$

since $\mu^2 = -\lambda v^2$

by randomly choosing +v or -v we have **Spontaneous Symmetry Breaking**

Spontaneous Symmetry Breaking and the Higgs Mechanism

$$L = \tfrac{1}{2}(\partial_\mu \sigma)^2 - \lambda v^2 \sigma^2 - \left(\lambda v \sigma^3 + \tfrac{1}{4}\lambda \sigma^4\right) + \text{constant}$$

kinetic energy

mass

new

self-interaction

- Look at the mass term
  - form is $^1/_2$ (mass)^2 (field)^2
  - so (mass)^2 is $2\lambda v^2$

$$m = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \qquad v = \pm\sqrt{\frac{-\mu^2}{\lambda}}\ \text{e}$$

What about the masses of the gauge bosons (W^+, W^-, and Z^0)?

Spontaneous Symmetry Breaking and the Higgs Mechanism

- Local gauge invariance in QED makes the interaction (or the Langrangian energy density) invariant under arbitrary local phase transformations and introduces the EM field:

$$\psi(x) \rightarrow e^{ie\theta(x)}\psi(x)$$

- if the field is also transformed

$$A_\mu \rightarrow A_\mu + \partial_\mu \theta(x)$$

- This occurs automatically, if the derivative ∂_μ in the Lagrangian is replaced by a covariant derivative:

$$D_\mu = \partial_\mu - ieA_\mu$$

- So this is the well-known local gauge symmetry in QED, U(1);
  what about SU(2)xU(1)?

Spontaneous Symmetry Breaking and the Higgs Mechanism

- Consider the gauge symmetries of the weak hypercharge [U(1)] and the weak isospin [SU(2)] interactions
- Weak hypercharge behaves under gauge transformations as electric charge since both are U(1)
- Weak isospin will be invariant under a rotation in weak isospin space

$$\psi \to e^{ig\boldsymbol{\tau}\cdot\boldsymbol{\Lambda}}\psi \qquad \text{(recall U(1): } \psi(x) \to e^{ie\theta(x)}\psi(x)\text{)}$$

Λ is an arbitrary vector in isospin space

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \tau_2 = \begin{bmatrix} 0 & -\imath \\ \imath & 0 \end{bmatrix} \qquad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Spontaneous Symmetry Breaking and the Higgs Mechanism

- With weak isospin invariant under a rotation in weak isospin space

$$\psi \rightarrow e^{ig\boldsymbol{\tau}\cdot\boldsymbol{\Lambda}}\psi$$

- To preserve the interaction we must introduce a massless isovector field, $\mathbf{W}_\mu$, containing charged and neutral components

- This leads to the covariant derivative of SU(2)

$$D_\mu = \partial_\mu - ig\boldsymbol{\tau}/2\cdot\mathbf{W}_\mu$$

$$\mathbf{W}_\mu \rightarrow \mathbf{W}_\mu + \partial_\mu\boldsymbol{\Lambda} - g\boldsymbol{\Lambda}\times\mathbf{W}_\mu$$

Warning – this is different from Perkins

QED

$$D_\mu = \partial_\mu - ieA_\mu$$

$$A_\mu \rightarrow A_\mu + \partial_\mu\theta(x)$$

Spontaneous Symmetry Breaking and the Higgs Mechanism

- Adding the weak hypercharge [U(1)] leads to the covariant derivative

$$D_\mu = \partial_\mu - ig\boldsymbol{\tau}/2 \cdot \mathbf{W}_\mu - ig'YB_\mu$$

- This covariant derivative can now be substituted into the Lagrangian

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4$$
$$L = \tfrac{1}{2}(D_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4$$

- Expand the first term and the following terms are found

$$\tfrac{1}{2}(gv/2)^2\, W_\mu^+ W_\mu^- \quad \Rightarrow \quad M_W = (gv/2)$$
$$\tfrac{1}{8}(v)^2\,(g^2+g'^2)\, Z_\mu^2 \quad \Rightarrow \quad M_Z = \tfrac{1}{2}\, v\, \sqrt{g^2+g'^2}$$
$$0\;\; A_\mu^2 \quad \Rightarrow \quad M_\gamma = 0$$

Note – $Y_\phi = 1/2$

$v = 246$ GeV

Homework assignment: show these M_W and M_Z relationships result

Spontaneous Symmetry Breaking and the Higgs Mechanism

- The Electroweak field Lagrangian terms:

$$L = g\mathbf{J}_\mu \cdot \mathbf{W}_\mu + g' J_\mu^Y B_\mu$$

- The Higgs field Lagrangian terms:

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4$$

- The covariant derivative:

$$D_\mu = \partial_\mu - ig\boldsymbol{\tau}/2 \cdot \mathbf{W}_\mu - ig' Y B_\mu$$

- The Higgs Lagrangian becomes:

$$\mathcal{L}_\Phi = \left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right) - \mu^2 \Phi^\dagger \Phi - \lambda\left(\Phi^\dagger \Phi\right)^2$$

Note – in this formulation, the factor of 2 has been embedded in the definition of Φ .

Spontaneous Symmetry Breaking and the Higgs Mechanism

$$\mathcal{L}_\Phi = \left(D_\mu\Phi\right)^\dagger\left(D_\mu\Phi\right) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

The Higgs field is now a doublet since D_μ is.

$$\Phi = \begin{bmatrix}\varphi^+ \\ \varphi^0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4\end{bmatrix}$$

The minimum in the potential occurs at:

$$\frac{dV}{d\left(\Phi^\dagger\Phi\right)} = 0 \quad \Rightarrow \quad \mu^2 + 2\lambda\left(\Phi^\dagger\Phi\right) = 0$$

$$\left(\Phi^\dagger\Phi\right)_{\min} = -\frac{\mu^2}{2\lambda}$$

$$\Phi^\dagger\Phi = \tfrac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2\right).$$

Spontaneous Symmetry Breaking and the Higgs Mechanism

$$\mathcal{L}_\Phi = \left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right) - \mu^2 \Phi^\dagger \Phi - \lambda\left(\Phi^\dagger \Phi\right)^2$$

$$\Phi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{bmatrix}$$

The minimum: we <u>choose</u> $\varphi_1 = \varphi_2 = \varphi_4 = 0$

$$\Phi_{\min} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v \end{bmatrix} \qquad \left(\Phi^\dagger \Phi\right)_{\min} = -\frac{\mu^2}{2\lambda} \equiv v^2/2$$

So $Y_\phi = Q - I_3 = 1/2$

Expand around the minimum:

$$\Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

$$D_\mu \Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} i\frac{g}{\sqrt{2}} W_\mu^+ (v + H) \\ \left(\partial_\mu - i\frac{1}{2}(g\cos\theta_W + g'\sin\theta_W) Z_\mu\right)(v + H) \end{bmatrix}$$

Note – this expression uses the convention

Spontaneous Symmetry Breaking and the Higgs Mechanism

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger(D_\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

$$= \tfrac{1}{2}\partial_\mu H\partial^\mu H + \tfrac{1}{4}g^2(v^2 + 2vH + H^2)W_\mu^+W^{-\mu} + \tfrac{1}{8}(g^2 + g'^2)(v^2 + 2vH + H^2)Z_\mu Z^\mu$$

$$-\frac{\mu^2 v^2}{4} + \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4$$

Identify the Z and W mass terms:

$$\tfrac{1}{2}m_Z^2 Z_\mu Z^\mu \quad \Rightarrow \quad m_Z = \tfrac{1}{2}(g^2 + g'^2)^{1/2}v \equiv \tfrac{1}{2}\frac{g}{\cos\theta_W}v$$

$$m_W^2 W_\mu^+ W^{-\mu} \quad \Rightarrow \quad m_W = \tfrac{1}{2}gv \quad \left(\Rightarrow m_Z = \frac{m_W}{\cos\theta_W}\right)$$

$$-\tfrac{1}{2}m_H^2 H^2 \quad \Rightarrow \quad m_H = \sqrt{-2\mu^2}$$

Spontaneous Symmetry Breaking and the Higgs Mechanism

$^{1}/_{2}(gv/2)^2\, W_\mu^+ W_\mu^-$	$\Rightarrow$	$M_W = (gv/2)$
$^{1}/_{8}(v)^2\,(g^2+g'^2)\, Z_\mu^2$	$\Rightarrow$	$M_Z = {}^{1}/_{2}\, v\, \sqrt{g^2+g'^2}$
$0\ \ A_\mu^2$	$\Rightarrow$	$M_\gamma = 0$

$v = 246$ GeV

Homework assignment:
show M_W and M_Z yield this

Spontaneous Symmetry Breaking and the Higgs Mechanism

- The Higgs mechanism also endows the fermions with mass
- The full Lagrangian has terms coupling all the fermions to the Higgs field

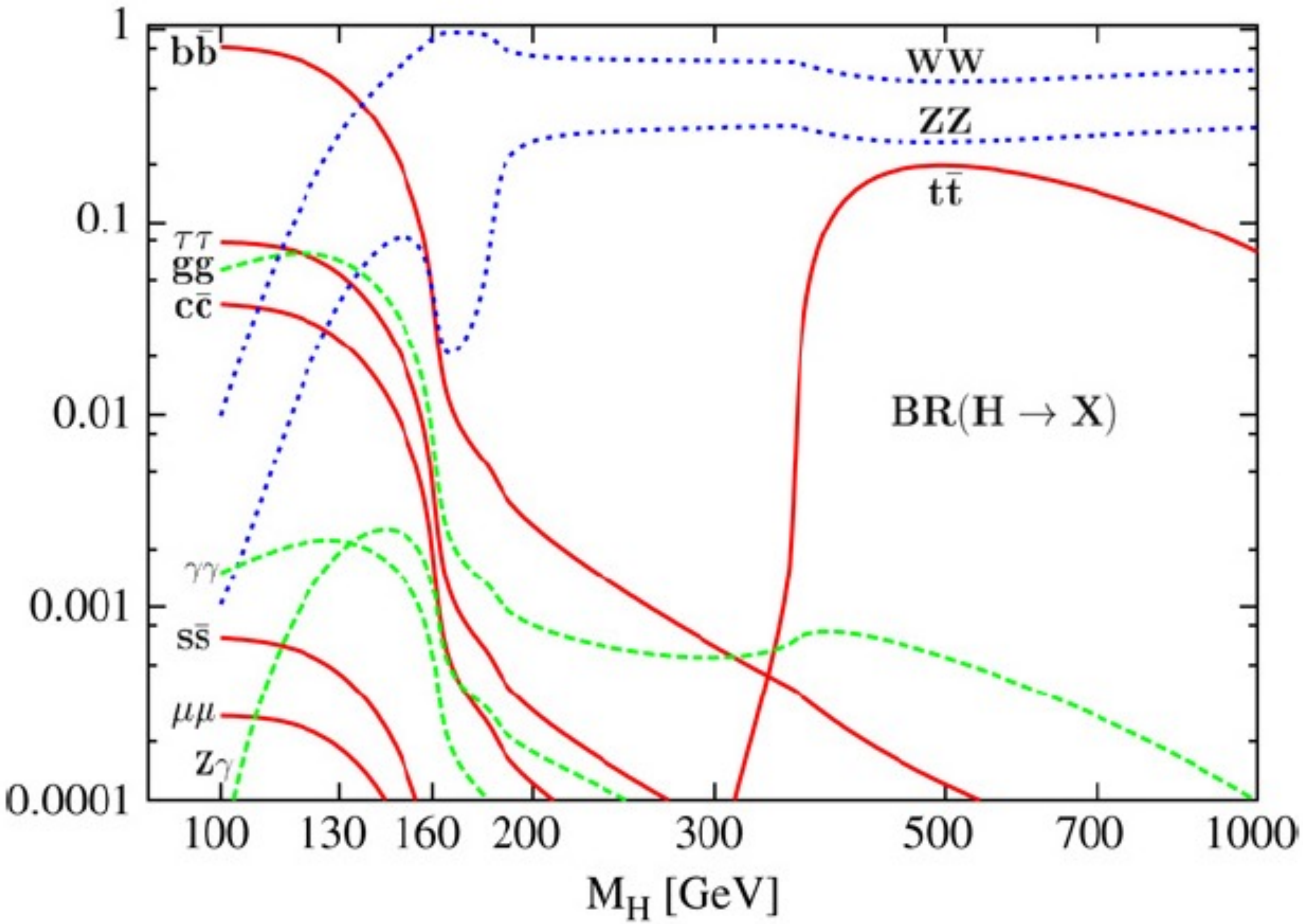
$$L = m_f \, e \, \bar{e} \; - \underbrace{\frac{m_f}{v}}_{\text{Yukawa coupling}} e \, \bar{e} \, H$$

- electron: $m_e/v = 2 \times 10^{-6}$
- top quark: $m_t/v = 0.7$

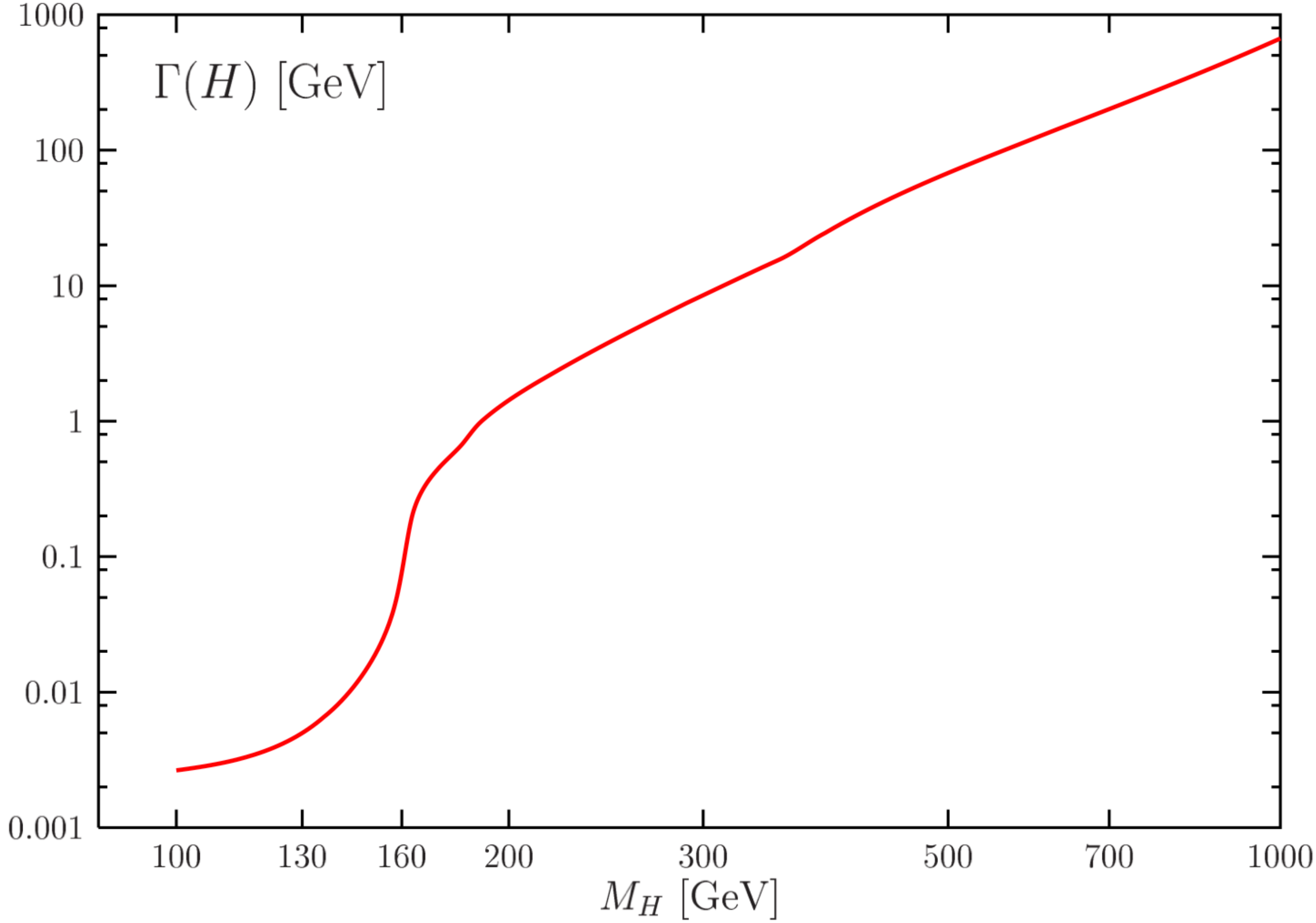
The size of the top quark mass seems much more natural than the mass of the lighter fermions given the value of v = 246 GeV

Higgs Couplings

A. Djouadi / Physics Reports 457 (2008) 1–216



Higgs Width



1000
100
10
1
0.1
0.01
0.001
Γ(H) [GeV]
100
130
160
200
300
500
700
1000
M_H [GeV]

Higgs Production and Detection

- The isospin doublet of scalar Higgs particles in the minimal Standard Model yields one real particle to be found

- Four real components of the new fields are reduced to one when three are "eaten" by the massless W and Z to produce W and Z mass

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} (\phi_1 + i\phi_2)/\sqrt{2} \\ (\phi_3 + i\phi_4)/\sqrt{2} \end{pmatrix}$$

I	$Y = Q - I_3$
$+1/2$	$+1/2$
$-1/2$	$+1/2$

- The mass of the remaining physical neutral boson is unknown

- Limits on the mass can be determined

Higgs Production and Detection

- Upper limit on the Higgs mass

  consider $\Gamma_H \sim G M_H^3$

- The Higgs must be weakly coupled $\Rightarrow \Gamma_H < M_H$

$$M_H < G^{-1/2} < (10^5 \text{ GeV})^{1/2} \approx 300 \text{ GeV}$$

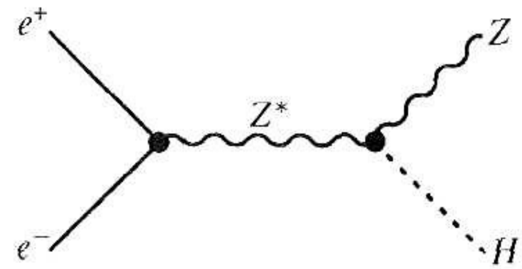
- We also have a unitarity limit on WW scattering

$$M_H < (8\pi\sqrt{2}/3)^{1/2} G^{-1/2} \simeq 1 \text{ TeV}$$

Higgs Production and Detection at LEP

- The Higgs boson might be produced in an electron-positron collider

$$e^+e^- \to H^0 Z^0$$
$$H^0 \to b\bar{b}, \tau\bar{\tau}, \ldots$$
$$Z^0 \to Q\bar{Q}, l\bar{l}, \nu\bar{\nu}$$

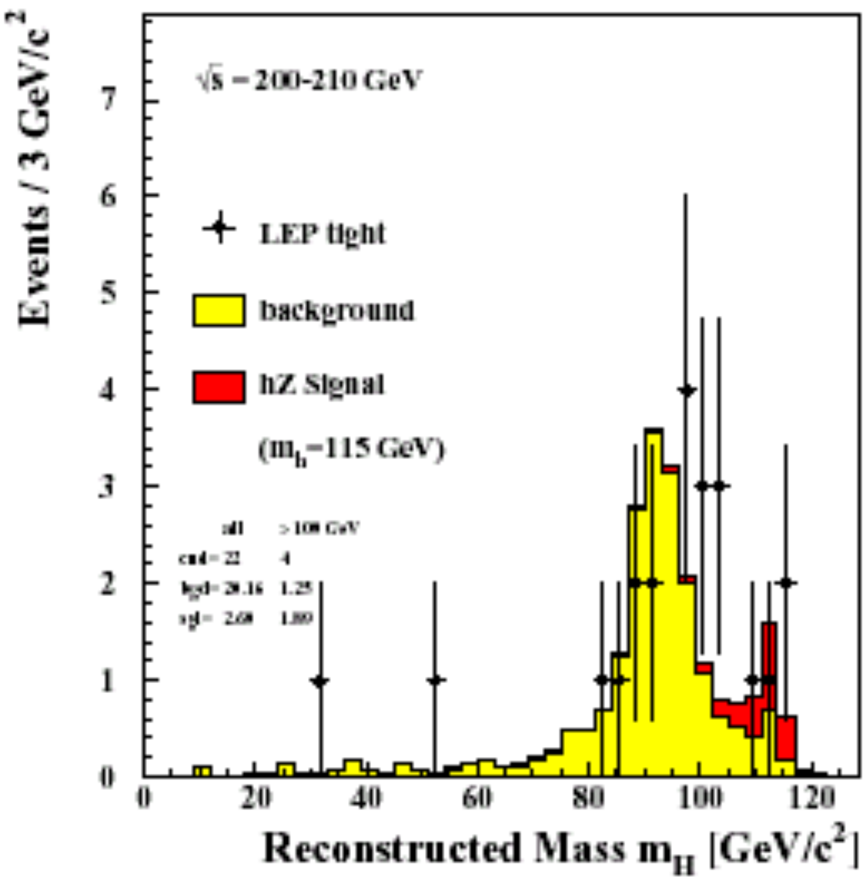


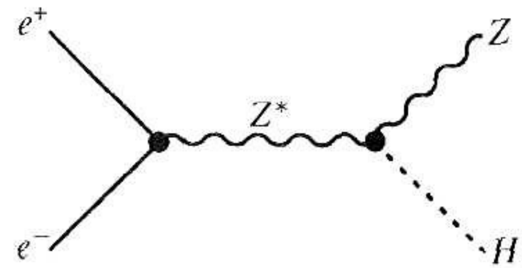
- The Higgs couples proportionally to mass, so it should decay preferentially to the heaviest possible quark or lepton

- In an electron-positron collider, the Higgs signal would show up dramatically recoiling from a Z decay

Higgs Production and Detection at LEP

LEP results



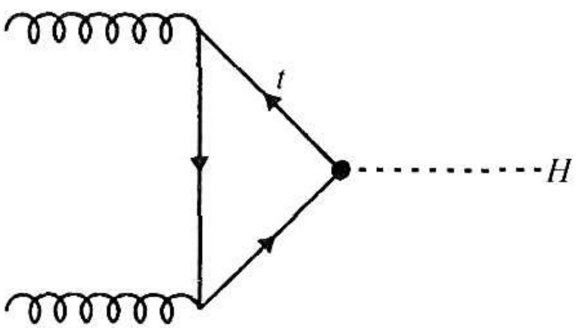


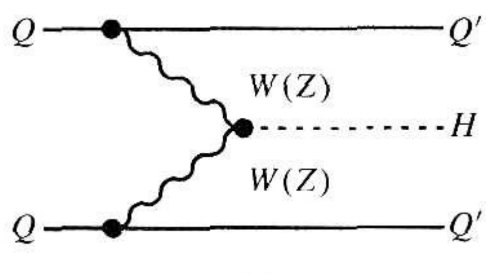
$M_H > 114$ GeV/c^2
(95% CL)

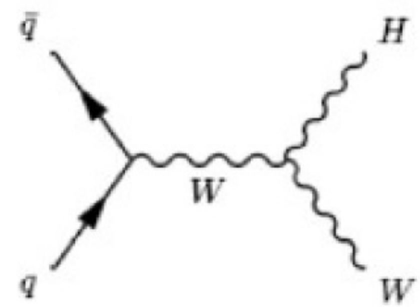
Higgs Production and Detection

- Following LEP, the Higgs search moved to
  - Fermilab TeVatron Collider
  - and to the Large Hadron Collider at CERN

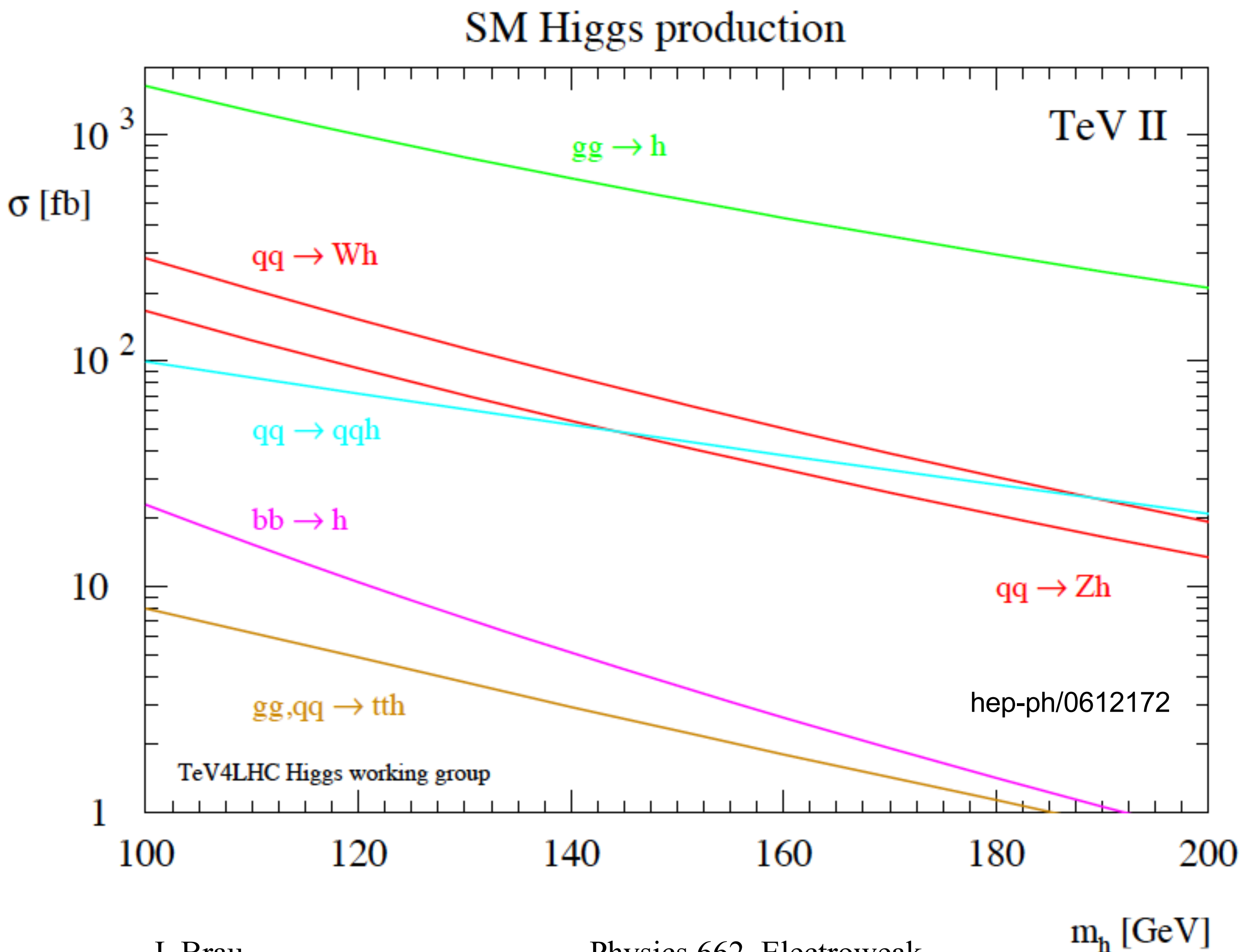
- Hadron colliders



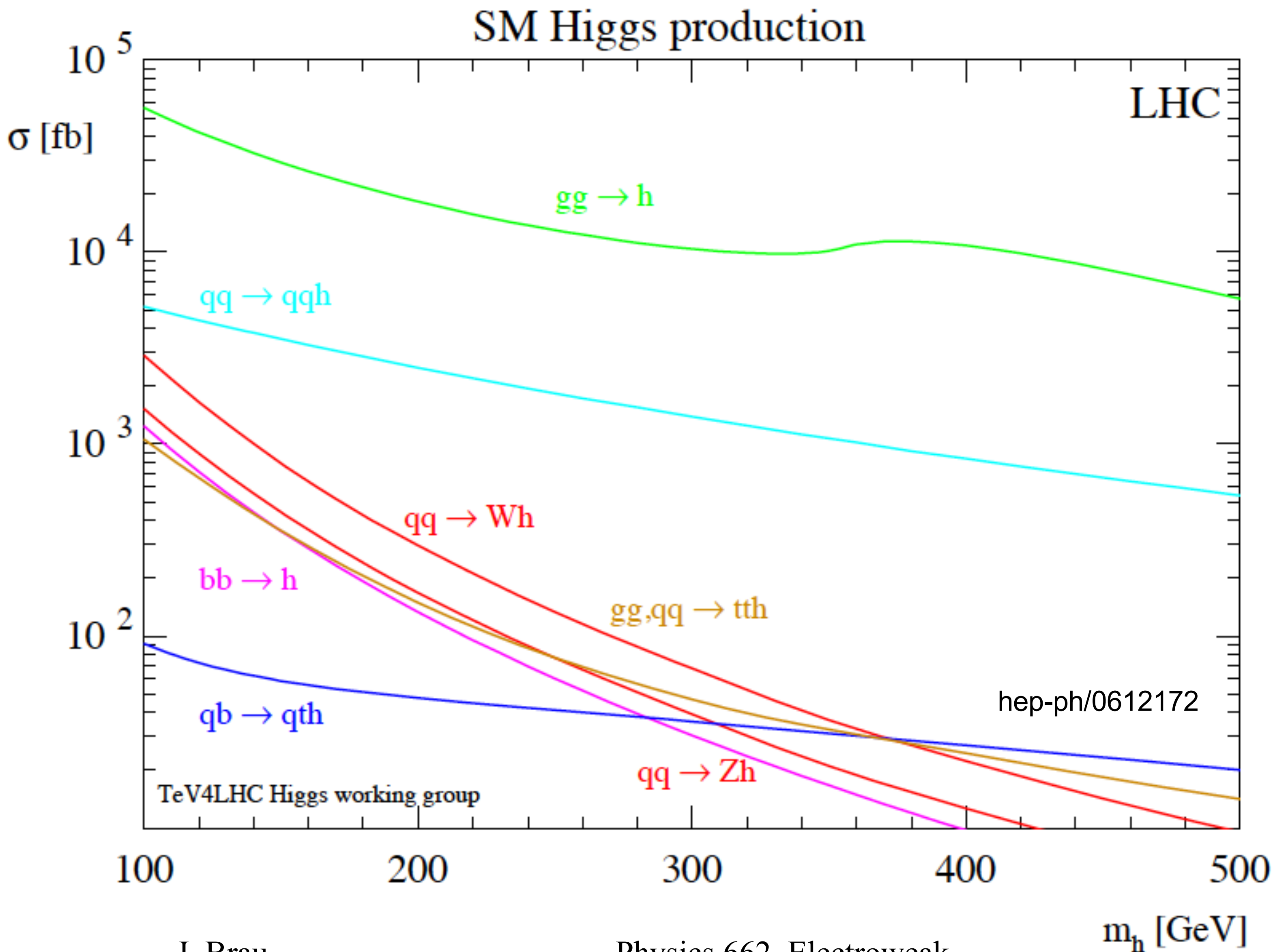




Hadron Collider Cross sections

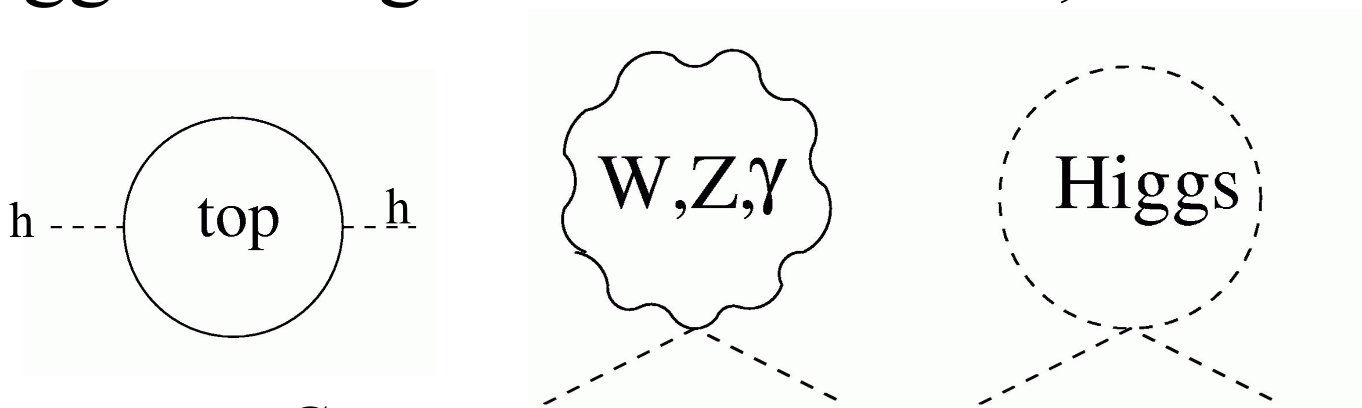


Hadron Collider Cross sections



Light Scalars Are Unnatural

- Higgs mass grows with cut-off, Λ



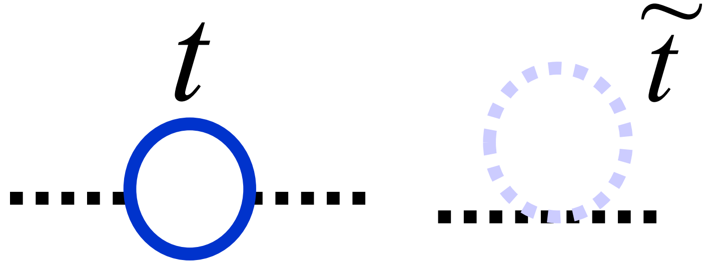
$$\delta M_h^2 = \frac{G_F}{4\sqrt{2}\pi^2}\Lambda^2\left(6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2\right)$$

$$= -\left(\frac{\Lambda}{0.7\ \mathrm{TeV}}\ 200\ \mathrm{GeV}\right)^2$$

$M_h \leq 200$ GeV requires large cancellations

SUSY….Our favorite model*

- Quadratic divergences cancelled automatically if SUSY particles at TeV scale
- Cancellation result of ***supersymmetry***, so happens at every order



$$\delta M_h^2 \approx (\ldots.) G_F \Lambda^2 (M_t^2 - M_{\tilde{t}}^2)$$

✭ Inspires: 18,115 papers with title supersymmetry or supersymmetric!

Higgs Production and Detection

SUSY Higgs

- Single Higgs doublet is replaced with two Higgs doublets
- 8 fields (4 complex) provide 3 "eaten" fields to endow mass to W,Z and leave 5 Higgs fields
- Two vev's – $\bar{v}$, v

  $\tan\beta = \bar{v}/v$

Two scalar (CP even) neutral particles:	h^0 H^0
One pseudoscalar (CP odd) neutral	A^0
Two charged scalars	H^+ H^-