

Physics 663

Particle Physics Phenomenology

April 9, 2002

Accelerators

- History
- Two Principles
 - Electrostatic
 - Cockcroft-Walton
 - Van de Graaff and tandem Van de Graaff
 - Transformers
 - Cyclotron
 - Betatron
 - Linear Induction Accelerator
- Phase Stability
- Synchrocyclotron
- Synchrotron
- Strong focusing
- High-impedance Microwave Devices
- Superconducting Technology

References:

Donald H. Perkins, Introduction to High Energy Physics, Fourth Edition
Bergstrom and Goobar, Cosmology and Particle Astrophysics
E.J.N.Wilson, “Physics of Accelerators”

History

- 1931 - Van de Graaff
- 1932 - Cockroft-Walton
- 1932 - cyclotron
 - Lawrence and Livingston
- 1940 - betatron
 - Wideroe/Kerst
- 1944/45 - phase stability
 - McMillan and Veksler
- 1950/52 -strong focusing
 - Christofilos/ Courant, Livingston, and Snyder

Two Principle Approaches

- Electrostatic
 - particles traverse a difference in electric potential
- or
- Transformer
 - high-current, low-voltage circuit element used to supply energy to a high-voltage, low-current accelerating path

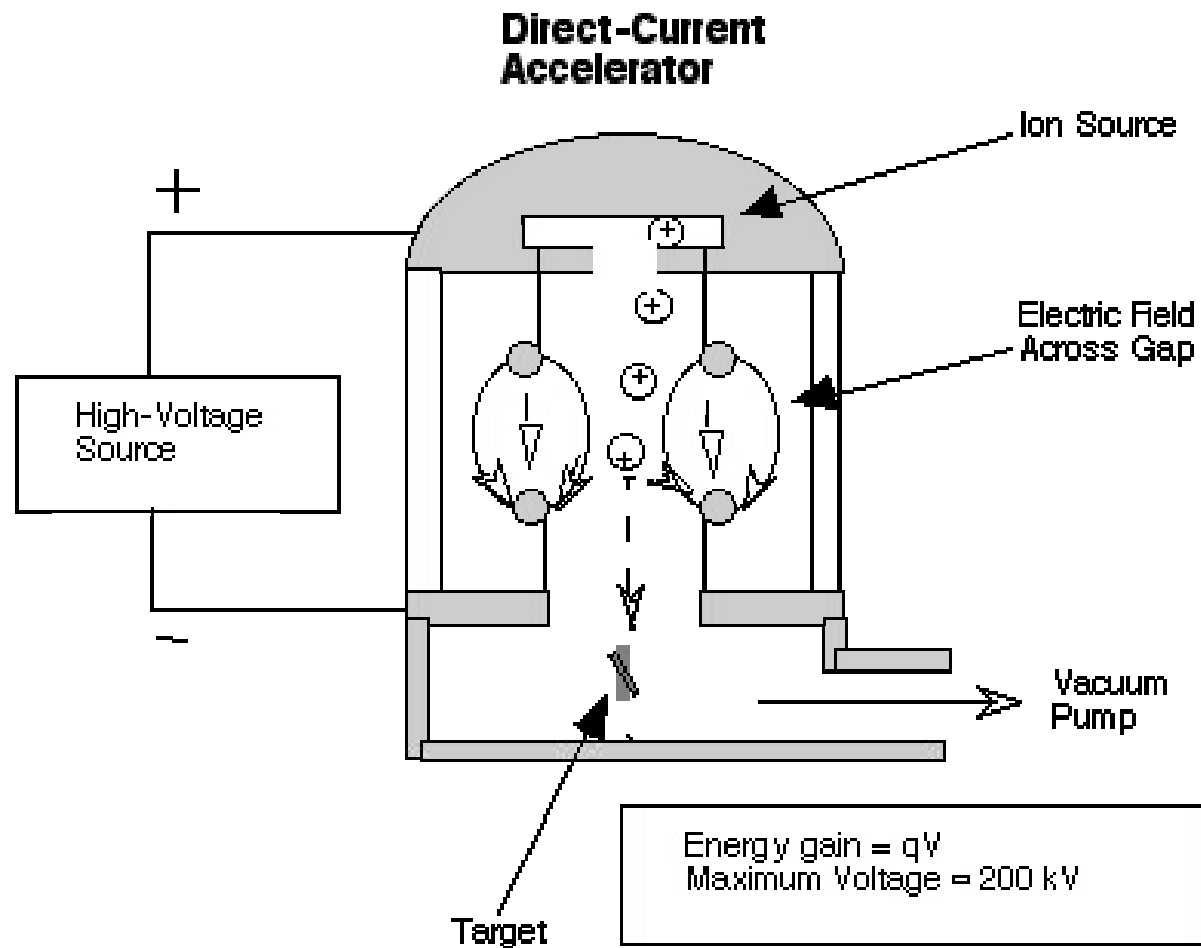
W.K.H. Panofsky and M. Breidenbach, Rev. Mod. Phys. 71, S122

Electrostatic

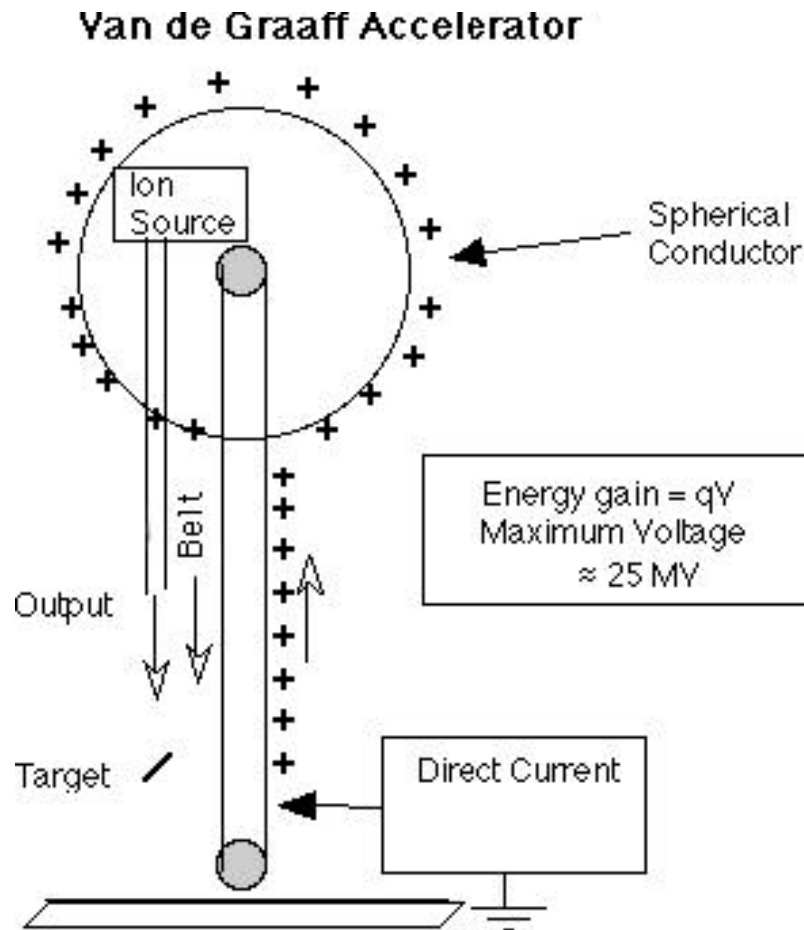
- The limitation of the electrostatic approach comes from the ultimate breakdown of high voltages
 - Cockroft-Walton (1932)
 - charge capacitors in parallel and connect in series
 - Van de Graaff (1931)
 - charges sprayed onto moving belt and removed inside a high-voltage electrode

W.K.H. Panofsky and M. Breidenbach, Rev. Mod. Phys. 71, S122

Cockcroft-Walton



Van de Graaff

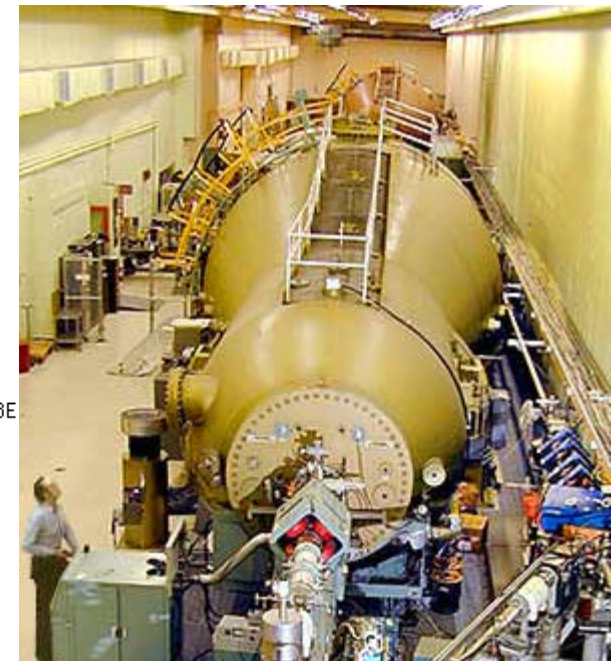
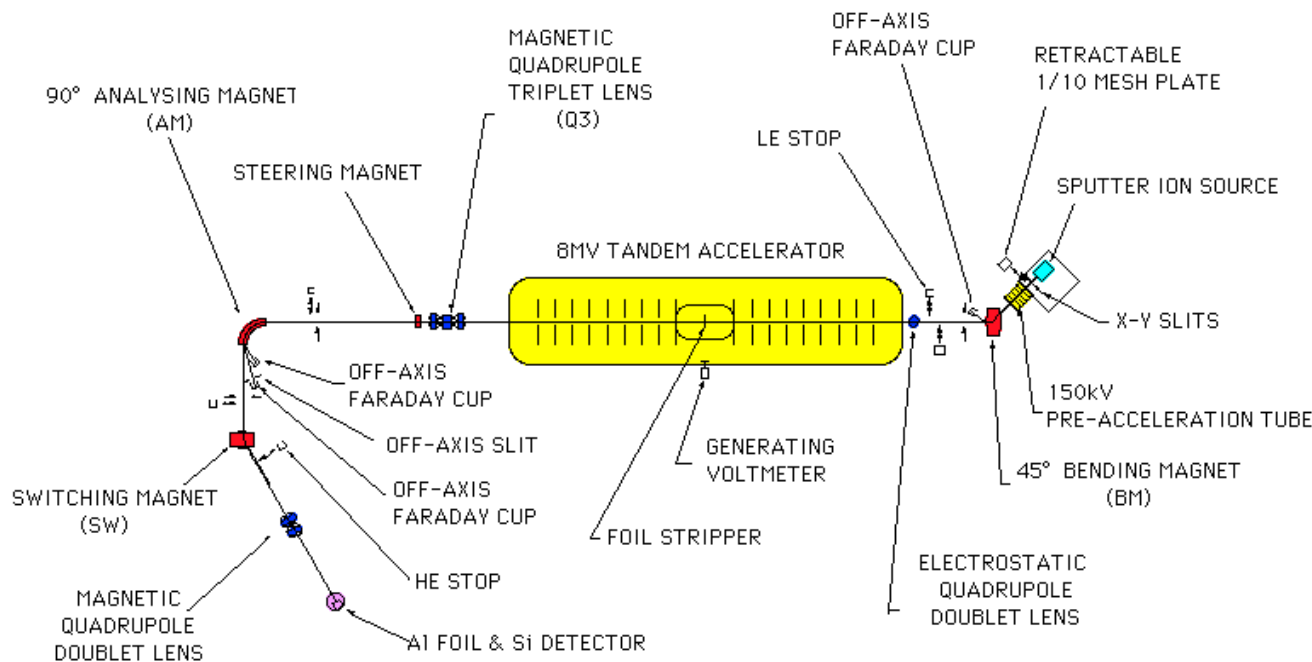


Developed
in the
30's



Tandem Van de Graaff

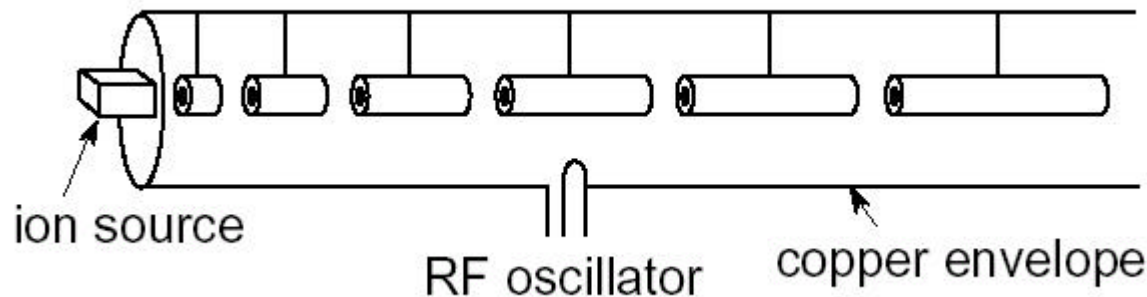
- Introduced in the 50's
 - accelerate negative ions, strip, and accelerate positive ions



Transformer

- Electrostatic approaches fail above about 10 MeV
- Linear Accelerator
- Cyclotron
 - orbital period of nonrelativistic particles circulating in a uniform magnetic field is independent of energy
- Betatron
 - electrons become relativistic at moderate energies, and cyclotron fails

RF Linear Accelerator

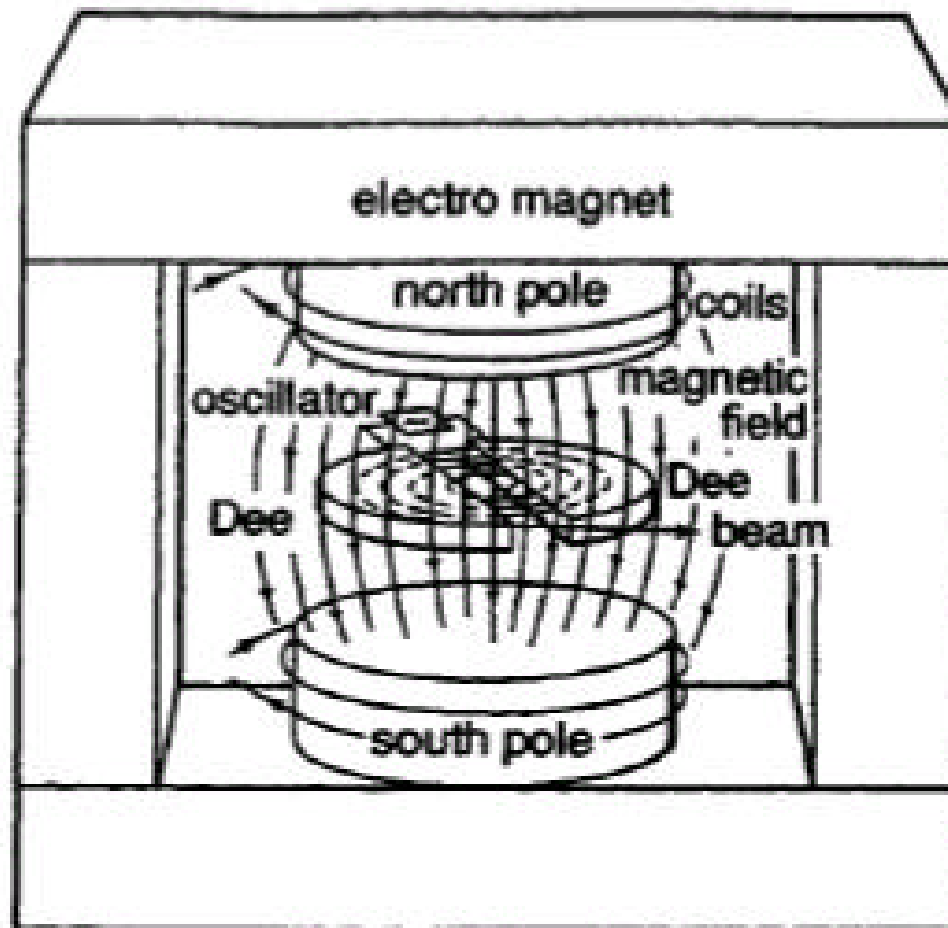


- The early accelerators inspired the linear accelerator, where adding stages can increase energy reach
- Copper lined tubes in which oscillating fields are excited by radio transmitter (“drift tubes”)
- But in order to increase beam energy, length must be expanded
- Circular accelerators were invented to make higher energy possible in a more compact device

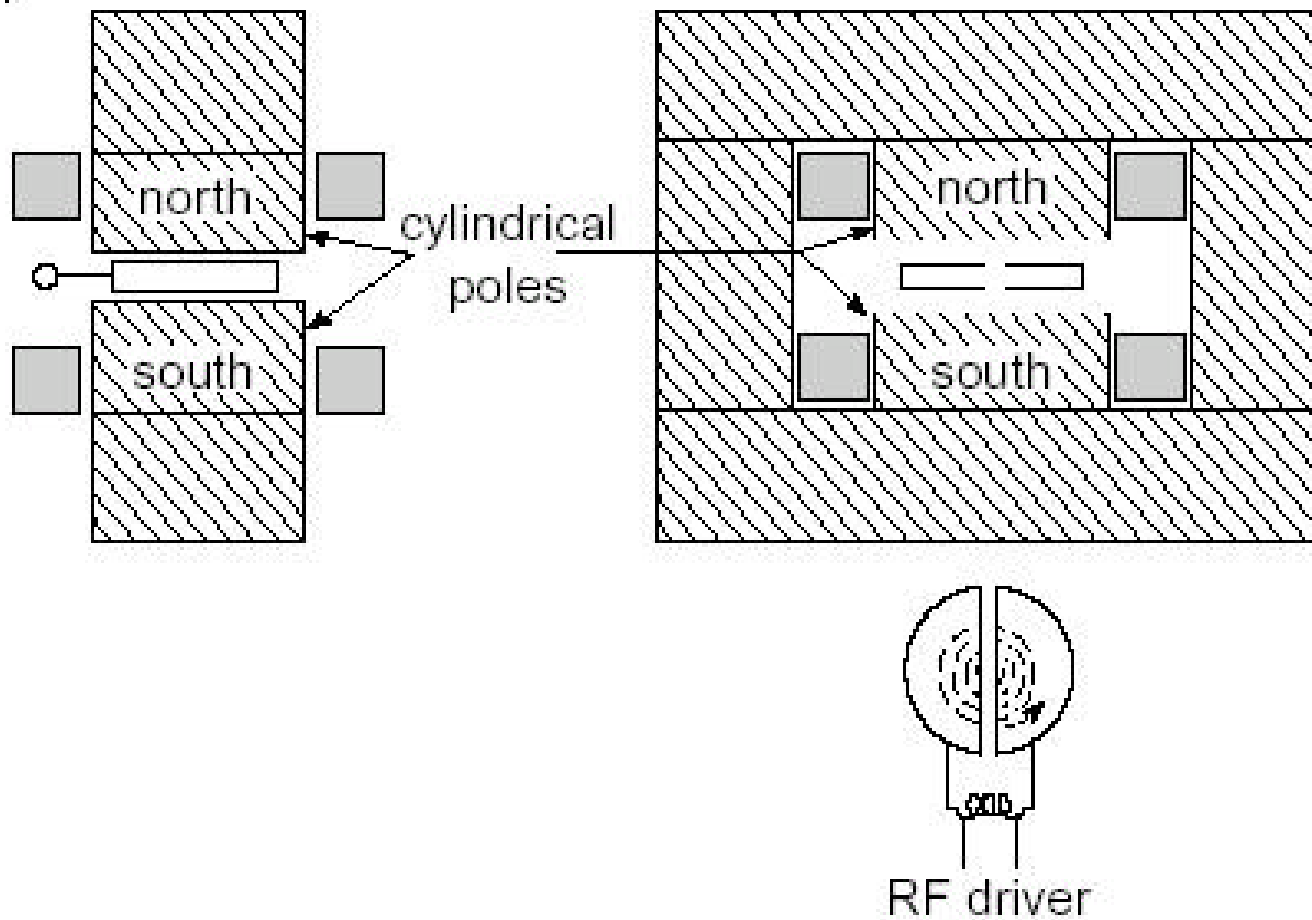
Cyclotron

- Orbital period of nonrelativistic particles circulating in a uniform magnetic field is independent of energy
$$f/2\pi = qB / m \approx 10^8 \text{ B(Tesla)}$$
radio frequency
- Match the revolution frequency with a RF voltage across a gap
- Focussing by small radial decrease in magnetic field
 - results in decrease in orbital frequency
- 184-inch cyclotron at Berkeley for deuteron energies above 100 MeV

Cyclotron

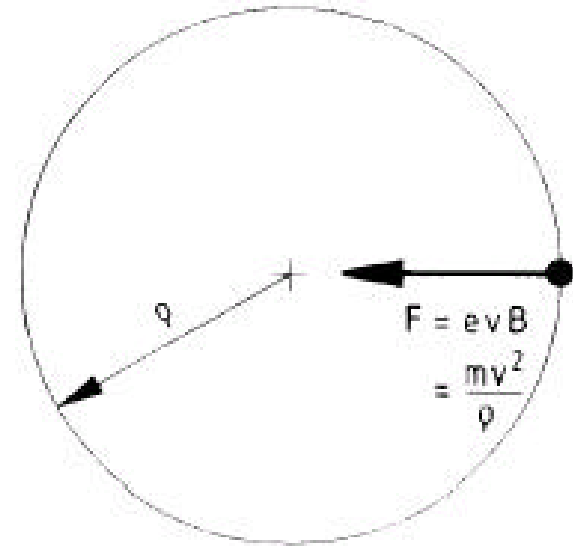


Cyclotron



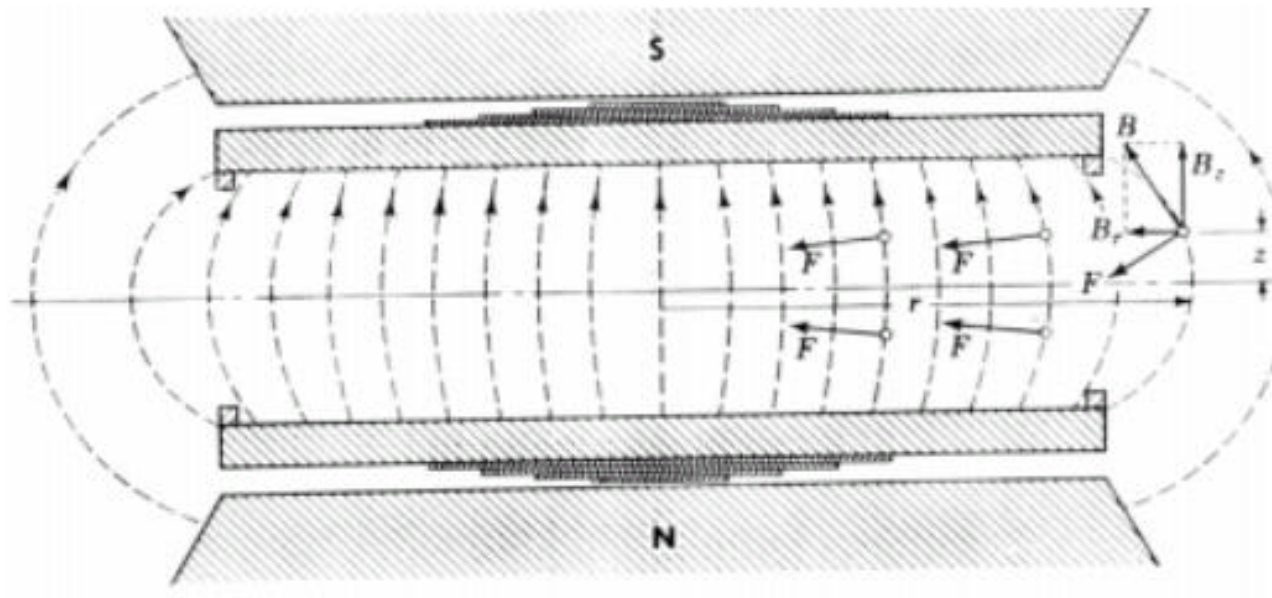
Cyclotron

- Nonrelativistic acceleration
- Particles spiral out in uniform magnetic field, B
- Balance of forces makes radius proportional to the velocity
- This means the period of rotation, $P = 2 \pi \rho / v$, is a constant
- Focusing of orbits is needed to achieve the best currents
- Shims do this (see next page)
- The constancy of period breaks down for kinetic energies of about $5\% mc^2$



Cyclotron Focussing

- Shims lead to increased focussing
 - both vertical and horizontal
 - discovered more intense fields



Horizontal Focusing

-

“Physics of Accelerators” E.J.N.Wilson

Horizontal focusing

If we examine the equation above for the radial magnetic force we can substitute, for all particle of the same velocity v_0

$$\rho \dot{\theta} = v_0$$

and write

$$F_r = \frac{d}{dt} \left(m \frac{d\rho}{dt} \right) - \frac{mv_0^2}{\rho} = ev_0 B_0$$

and change from the independent variable, t , to s , the distance measured along the reference orbit.

$$\frac{d}{dt} = v_0 \frac{d}{ds}$$

and use the notation for small deviations from the reference orbit parameters $\Delta B_z = B_z - B_0$

$$x = \rho - \rho_0$$

The next two steps assume that x is small compared with ρ_0

$$\frac{1}{mv_0} \frac{d}{ds} \left(mv_0 \frac{d(\rho_0 + x)}{ds} \right) - \frac{1}{(\rho_0 + x)} = \frac{e}{mv_0} (B_0 + \Delta B)$$

$$\text{and substitute } p_0 = mv_0 \frac{1}{mv_0} \frac{d}{ds} \left(p_0 \frac{dx}{ds} \right) - \frac{x}{\rho_0^2} = \frac{1}{\rho_0} \frac{\Delta B_z}{B_0}$$

We can already see a second order differential equation rather similar to a harmonic oscillator beginning to take shape. We must now find some suitable function to describe $\Delta B_z(x)$. There is a convenient expansion, a Taylor series, which describes the variation of the field about a point on the reference orbit

$$B_z = B_0 + \frac{\partial B_z}{\partial x} x + \frac{1}{2!} \frac{\partial^2 B_z}{\partial x^2} x^2 + \dots$$

$$\text{Define linear coefficient. - normalised gradient } k = - \frac{1}{(B_0 \rho_0)} \frac{\partial B_z}{\partial x}$$

$$\text{Substitute in differential equation: } \frac{1}{p_0} \frac{d}{ds} \left(p_0 \frac{dx}{ds} \right) + \left(\frac{1}{\rho_0^2} - k \right) x = 0$$

Betatron

- Electrons become relativistic at moderate energies, and the cyclotron concept fails (period no longer constant)
- Energy of electrons in circular orbit increased by the induced E field from an increasing flux in a central iron core
- Particles are kept in single circular orbit (not spirally out as in cyclotron)
- Limited at 300 MeV due to radiation losses which cannot be compensated

Synchrocyclotron

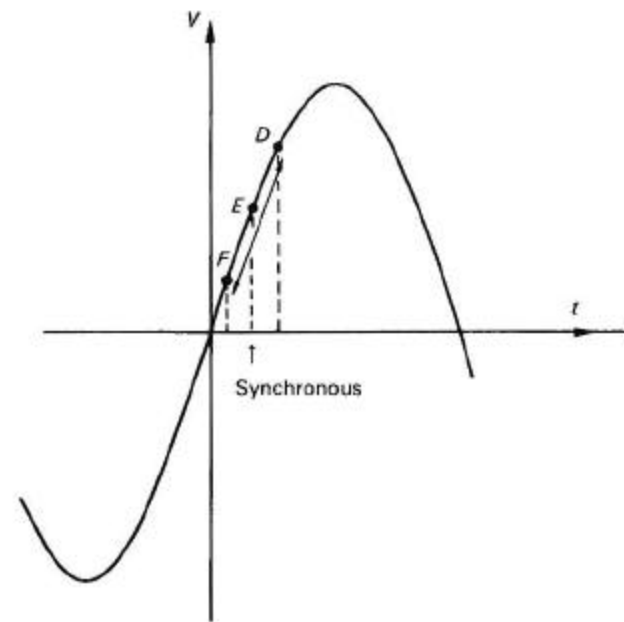
- Problem: how to overcome the changing orbital period
- One solution: add radial magnetic field gradient
 - new problem: destroys vertical focusing
- Second solution: change the RF frequency as particles circulate
 - another problem: particles must now be accelerated in bursts, not continuously
- Synchrocyclotron
 - Static magnetic field
 - RF frequency decreases to match the revolution frequency as function of energy

Synchrotron

- Rising magnetic field keeps particles in circular orbit at a constant radius
- Accelerating field achieved with fields inside hollow cylinder
- Guide field achieved by ring of individual magnets
- Principal in use in all current high energy circular accelerators
 - invented during World War II during the uranium enrichment project

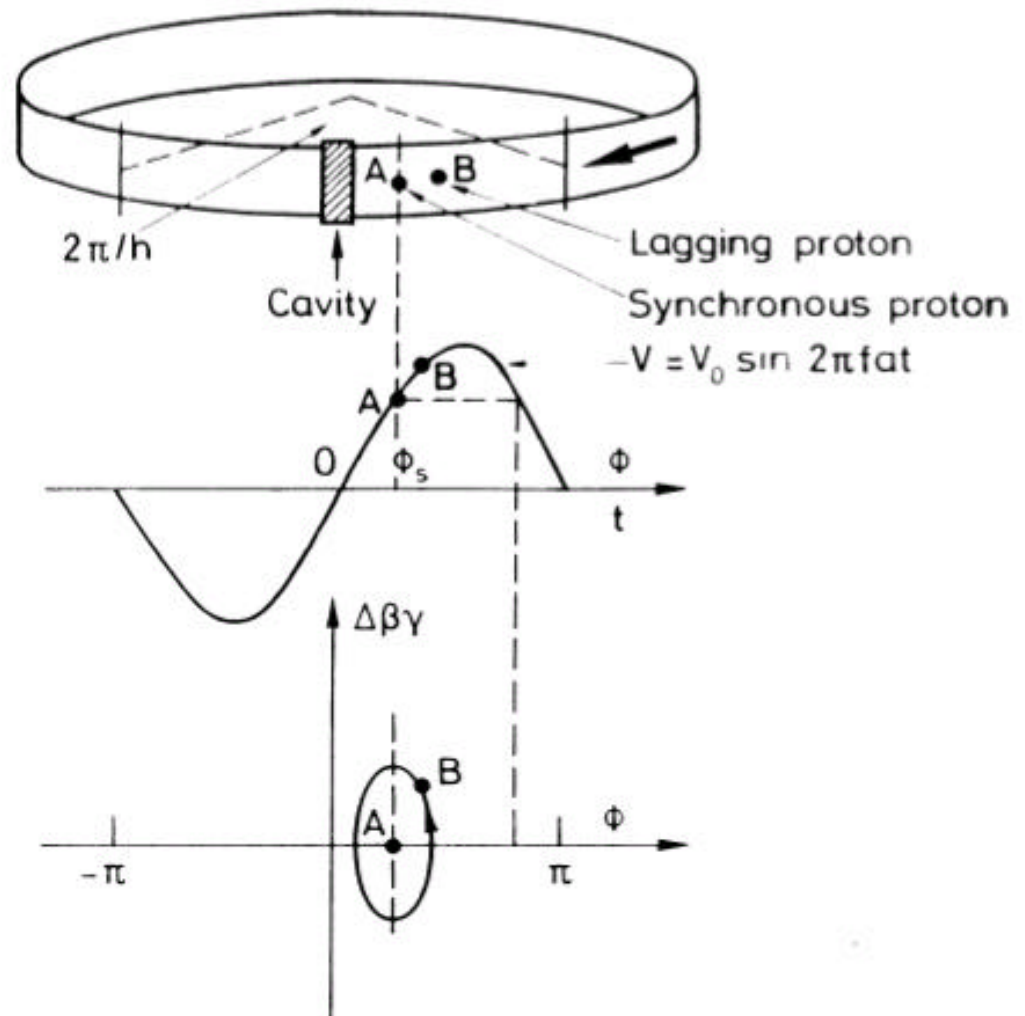
Phase Stability

- In circular accelerator, phase stably locked by synchronizing the phase of the RF voltage (rising or falling) as particle crosses an accelerating gap
- Synchrotron oscillations about a stable phase results
 - synchrotron oscillations
(longitudinal oscillations)



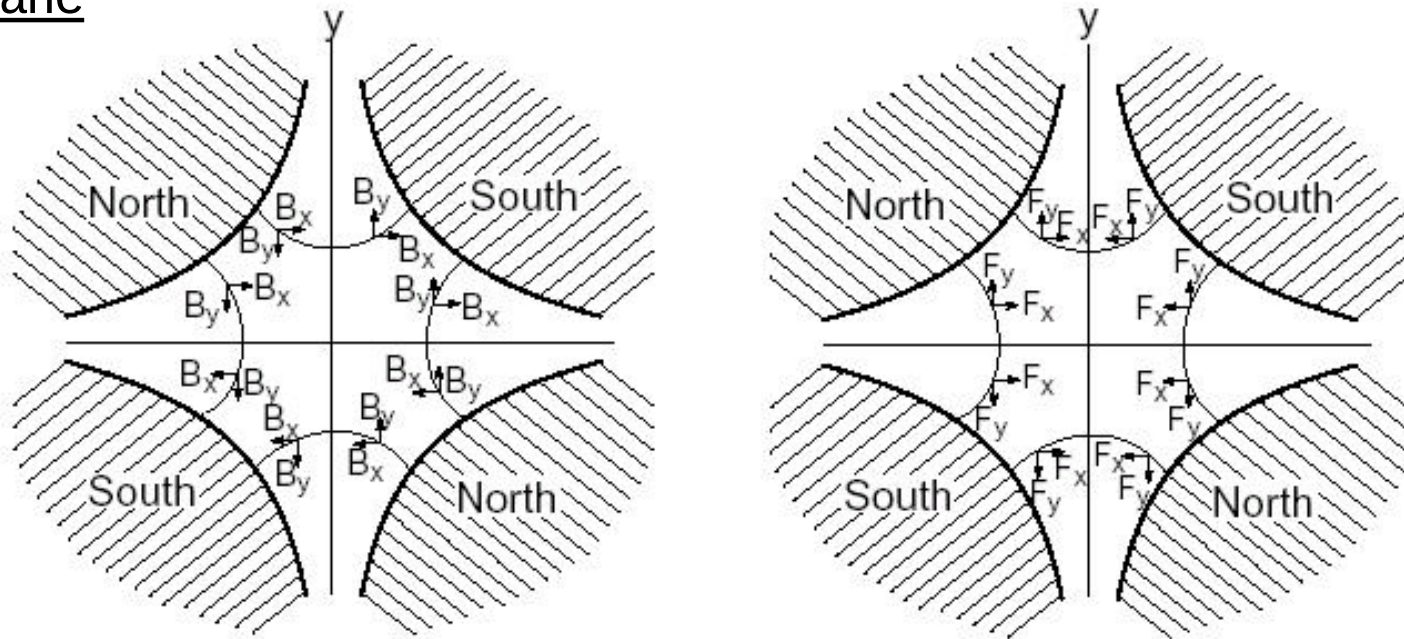
Phase Stability

- Particles are trapped in an “RF bucket”



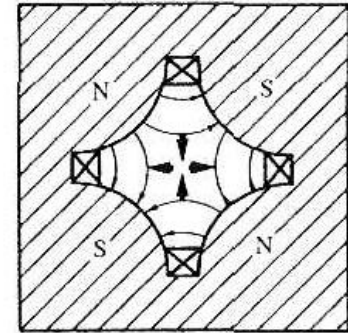
Quadrupole

- The principal focusing element used today in synchrotrons is the quadrupole magnet
- The quadrupole shown below would focus in the horizontal plane positive particles coming out or negative particles going in. Such a quadrupole is defocusing in the vertical plane



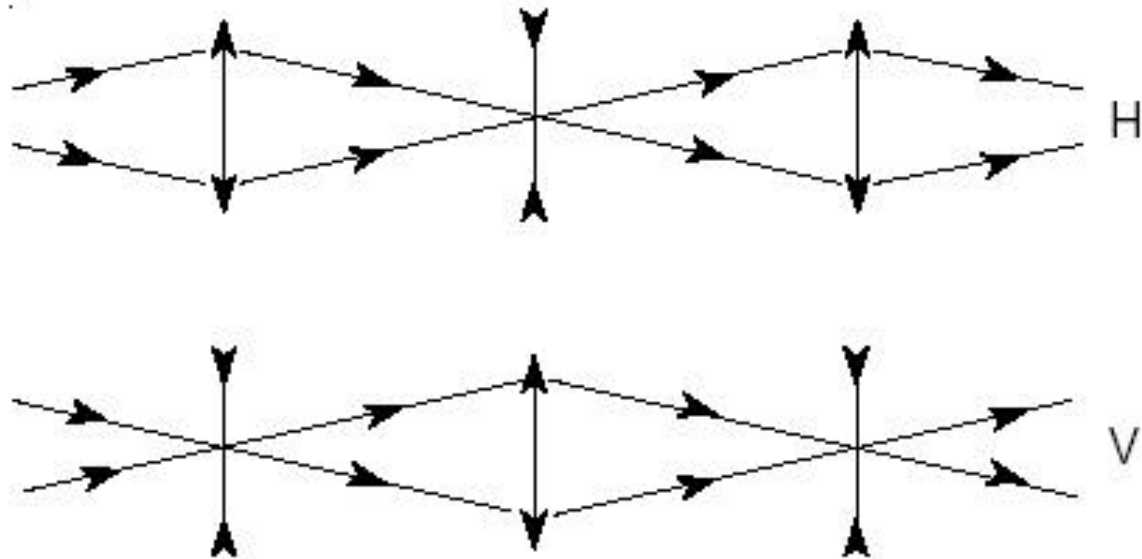
Strong focusing

- Alternating diverging and focusing lenses separated by finite distance results in net focusing
- Magnetic quadrupole focuses in one plane and defocuses in orthogonal plane
- Alternating quadrupoles focus in both planes
 - much stronger than focussing of solenoids or radial magnetic gradients (dipoles)
 - decreases aperture required for stability
 - greatly extends energy range of acceleration



Strong Focusing

- Ray diagrams showing the contained trajectory in an alternating gradient optical system
- An alternating pattern of lenses which are convex in one plane and concave in the other will transport rays which pass through the centres of defocusing lenses. The upper diagram shows the horizontal motion and the lower shows the vertical



Betatron Oscillations

21.3.1. *Betatron oscillations:*

Present-day high-energy accelerators employ alternating gradient focussing provided by quadrupole magnetic fields [1]. The equations of motion of a particle undergoing oscillations with respect to the design trajectory are

$$x'' + K_x(s)x = 0 \quad , \quad y'' + K_y(s)y = 0 \quad , \quad (21.5)$$

with

$$x' \equiv dx/ds \quad , \quad y' \equiv dy/ds \quad (21.6)$$

$$K_x \equiv B'/(B\rho) + \rho^{-2} \quad , \quad K_y \equiv -B'/(B\rho) \quad (21.7)$$

$$B' \equiv \partial B_y / \partial x \quad . \quad (21.8)$$

The independent variable s is path length along the design trajectory. This motion is called a *betatron* oscillation because it was initially studied in the context of that type of accelerator. The functions K_x and K_y reflect the transverse focussing—primarily due to quadrupole fields except for the radius of curvature, ρ , term in K_x for a synchrotron—so each equation of motion resembles that for a harmonic oscillator but with spring constants that are a function of position. No terms relating to synchrotron oscillations appear, because their time scale is much longer and in this approximation play no role.

from Particle Data Group

Betatron Oscillations

These equations have the form of Hill's equation and so the solution in one plane may be written as

$$x(s) = A\sqrt{\beta(s)} \cos(\psi(s) + \delta), \quad (21.9)$$

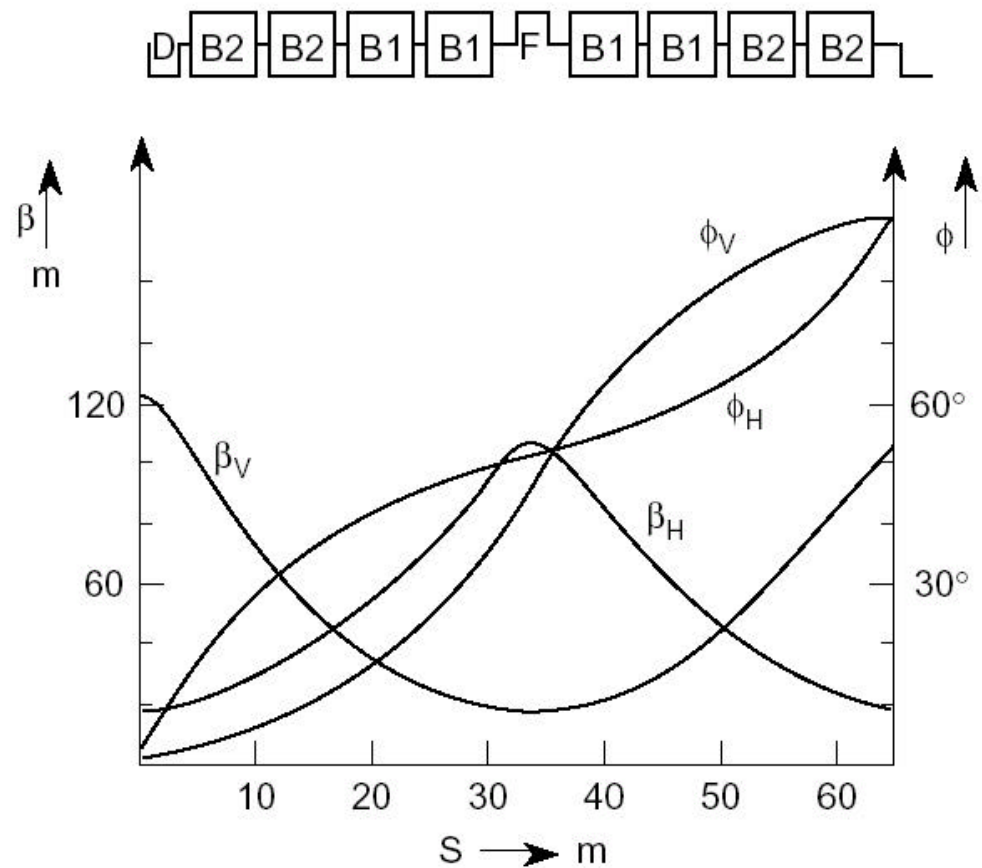
where A and δ are constants of integration and the phase advances according to $d\psi/ds = 1/\beta$. The dimension of A is the square root of length, reflecting the fact that the oscillation amplitude is modulated by the square root of the amplitude function. In addition to describing the envelope of the oscillation, β also plays the role of an 'instantaneous' λ . The wavelength of a betatron oscillation may be some tens of meters, and so typically values of the amplitude function are of the order of meters rather than on the order of the beam size. The beam optics arrangement generally has some periodicity and the amplitude function is chosen to reflect that periodicity. As noted above, at the interaction point a small value of the amplitude function is desired, and so the focussing optics is tailored in the neighborhood to provide a suitable β^* .

The number of betatron oscillations per turn in a synchrotron is called the *tune* and is given by

$$\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta} . \quad (21.10)$$

FODO cells

- Quadrupole magnets alternate with a the lattice of bending magnets
- Structure is called FODO.
- The envelope of oscillations follows the function $\beta(s)$
- β has the dimensions of length, but is not the physical beam size.



One cell of the CERN SPS representing 1/108 of the circumference.

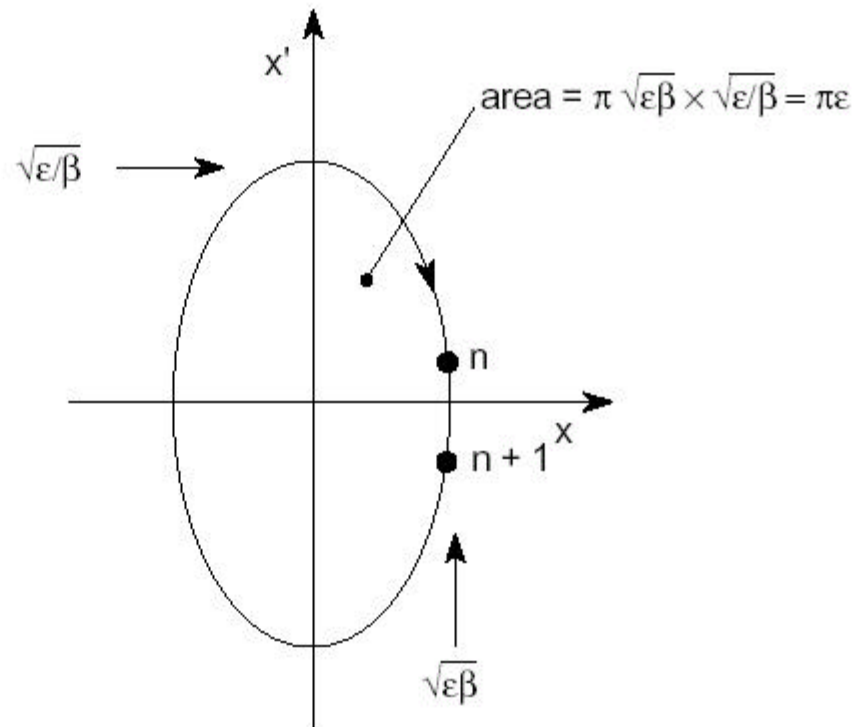
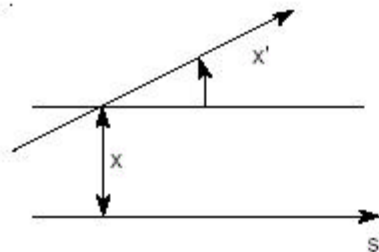
Particle trajectories

- The coordinates of particles in the beam will fall in the range:

$$-\sqrt{\beta\epsilon} \leq x \leq \sqrt{\beta\epsilon}$$

$$-\sqrt{\epsilon/\beta} \leq x' \leq \sqrt{\epsilon/\beta}$$

- β is a property of the accelerator
- ϵ is a property of the beam



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