N.B. Vacuum phase (α) Calculation

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1 Calculation Vacuum phase CP Violation

In this section, we perform a calculation within the minimal Nelson-Barr framework of [1]. Our objective is to find the value of the phase α. Let us start by first considering the Lagrangian of the theory:

- Vector-like quark pair, D_L, D_R
- Single Neutral Complex Scalar S

The couplings of the theory go as so:

$$\mathcal{L} = \mu D_L D_R + \left(g_i S + \tilde{g}_i S^*\right) \bar{u}_i D_R + y^u_{ij} \tilde{H} Q_i \bar{u}_j + y^d_{ij} H Q_i \bar{d}_j + \ldots$$

We enforce CP invariance in the Lagrangian and introduce an additional $\mathbb{Z}_2$ and $U(1)$ symmetry. Under this symmetry, all Standard Model fields remain unchanged, while the new fields D_L, $D_R = d^4_R$, and S are odd.

In this model, the scalar sector has a single spontaneously broken U(1) symmetry which is broken by more than one spurion (S^2 and S^4) [3]. We wish to find the phase α for which the vacuum breaks CP invariance. We begin with the most general potential of that stated in Bento [1].

1.1

$$V_x = V_0(\varphi, S) + \left(\mu^2 + \lambda_1 S^* S + \lambda_2 \varphi^\dagger \varphi\right)\left(S^2 + S^{*2}\right)$$

Which we can express more simply as

$$V_x = V_\varphi + V_S + V_{\varphi,S}$$

Where, if one distributes through (1.1), the following values are found:

$$\begin{aligned} V_\varphi &= \rho \varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2, \\ V_S &= S^* S[a_1 + b_1 S^* S] + \left(S^2 + S^{*2}\right)\left(a_2 + b_2 S^* S\right) + b_3\left(S^4 + S^{*4}\right), \\ V_{\varphi,S} &= \varphi^\dagger \varphi[cc_1\left(S^2 + S^{*2}\right) + c_2 S^* S]. \end{aligned}$$

This is a lot but with some work, we can get some important information. Let's start by finding the phase α

$$\rho \varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2 + S^* S\left(a_1 + b_1 S^* S\right) + \ldots + V_{\varphi,S}$$

Start by using the VEVS $\varphi = V_\varphi e^{i\alpha}$, $S = V_S e^{i\alpha}$. Also, let $\varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2 = \gamma_4$

$$= \gamma_4 + V_S^2[a_1 + b_1 V_S^2] + \left(V_S^2 e^{2i\alpha} + V_S^2 e^{-2i\alpha}\right)\left(a_2 + b_2 V_S^2\right) + b_3 V_S^4[e^{4i\alpha} + e^{-4i\alpha}] + V_\varphi^2\left(c_1\left(a V_S\left(e^{2i\alpha} + e^{-2i\alpha}\right) + c_2 V_S\right)\right)$$

$$= \gamma_4 + V_S^2[a_1 + b_1 V_S^2] + V_S^2\left(e^{2i\alpha} + e^{-2i\alpha}\right)\left(a_2 + b_2 V_S^2\right) + b_3 V_S^4[e^{4i\alpha} + e^{-4i\alpha}] + \left(a_2 + b_2 V_S^2\right)\left(e^{2i\alpha} e^{-2i\alpha}\right.$$

Group like-terms

$$= \gamma_4 + V_S^2[a_1 + b_1 V_S^2] + [V_S^2\left(e^{2i\alpha} + e^{-2i\alpha}\right)\left(a_2 + b_2 V_S^2\right) + \left(a_2 + b_2 V_S^2\right)\left(e^{2i\alpha} e^{-2i\alpha}\right] + b_3 V_S^4[e^{4i\alpha} + e^{-4i\alpha}]$$

$$= \gamma_4 + V_S^2[a_1 + b_1 V_S^2] + [V_S^2\left(a_2 + b_2 V_S^2\right) + \left(a_2 + b_2 V_S^2\right)]\left(e^{2i\alpha} + e^{-2i\alpha}\right) + b_3 V_S^4\left(e^{4i\alpha} + e^{-4i\alpha}\right)$$

We only care about terms with α. Hence,

let $\gamma_4 + V_S^2[a_1 + b_1 V_S^2] = c$ let $V_S^2\left(a_2 + b_2 V_S^2\right) + \left(a_2 + b_2 V_S^2\right) = b$

Then our expression becomes,

1.2

$$V_x = c + b\left(e^{2i\alpha} + e^{-2i\alpha}\right) + a\left(e^{4i\alpha} + e^{-4i\alpha}\right)$$

This has simple form which we will certainly stash for later use. Now, our objective is to find the phase α. Let us minimize V.

$$min\,(V) = \frac{\partial}{\partial\alpha} V_x = 0$$

$$min\,(V) = \frac{\partial}{\partial\alpha}[c + b\left(e^{2i\alpha} + e^{-2i\alpha} + \left(e^{4i\alpha} + e^{-4i\alpha}\right)\right] = 0$$

$$0 = \frac{\partial}{\partial\alpha}[c + b\left((\cos 2\alpha + i\sin 2\alpha) + \cos -2\alpha + i\sin -2\alpha\right) + (\cos 4\alpha + i\sin 4\alpha) + \cos -4\alpha + i\sin -4\alpha] = 0$$

Notice, eg $i\sin -4\alpha = -i\sin 4\alpha$ and $\cos -4\alpha = \cos 4\alpha$, hence we have cancellations

$$\begin{aligned}
0 &= \frac{\partial}{\partial\alpha}\left[c + b\left(\cos 2\alpha + i\sin 2\alpha + \cos 2\alpha - i\sin 2\alpha\right) + (\cos 4\alpha + i\sin 4\alpha) + \cos 4\alpha - i\sin 4\alpha\right],\\
0 &= \frac{\partial}{\partial\alpha}\left[c + b\left(\cos 2\alpha + i\sin 2\alpha\right) + \cos 2\alpha - i\sin 2\alpha + (\cos 4\alpha + i\sin 4\alpha) + \cos 4\alpha - i\sin 4\alpha\right],\\
0 &= \frac{\partial}{\partial\alpha}\left[c + b\cos 2\alpha + \cos 2\alpha + \cos 4\alpha + \cos 4\alpha\right] = 0,\\
0 &= \frac{\partial}{\partial\alpha}\left[c + b\cdot 2\cos 2\alpha + 2\cos 4\alpha\right],\\
0 &= \frac{\partial}{\partial\alpha}\left(b\cdot 2\cos 2\alpha\right) + \frac{\partial}{\partial\alpha}\left(a\cdot 2\cos 4\alpha\right),\\
0 &= 4b\sin 2\alpha + 8a\sin 4\alpha,\\
0 &= b\sin 2\alpha + 4a\sin 4\alpha,\\
0 &= b\sin 2\alpha + 4a\left(2\cos 2\alpha\sin 2\alpha\right),\\
0 &= \sin 2\alpha\left(b + 4a\cdot 2\cos 2\alpha\right),\\
0 &= b + 4a\cdot 2\cos 2\alpha,\\
\frac{-b}{8a} &= \cos 2\alpha,\\
\cos 2\alpha &= \frac{-b}{8a},\\
2\alpha &= \arccos\frac{-b}{8a},\\
\alpha &= \frac{1}{2}\arccos\frac{-b}{8a}.
\end{aligned}$$

For this value of α the vacuum breaks general CP invariance.
(*section with minimizing Scalar potential*) (*section with minimizing Higgs potential*)

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References

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[3] Howard E. Haber and Ze'ev Surujon, *Group-theoretic condition for spontaneous CP violation*, Physical Review D, vol. 86, no. 7, Article 075007, American Physical Society (APS), 2012, http://dx.doi.org/10.1103/PhysRevD.86.075007.