Hierarchical Bayesian Inference for Compact Binary Black Holes

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Introduction

The Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors have revealed, in great detail, the Universe's immense abundance for Gravitational Wave Sources. These waves are born from immense distortions in space and time, created by the largest rotating objects in our universe: Black Holes.

Observations of these gravitational wave sources have greatly shifted our understanding of Black Hole Physics; including their formation history as well as novel measurements of cosmological parameters. These observations also greatly exposed their tendencies; that is, the way in which they behave themselves. Here, we present our "Analysis from Analyses," across a catalog of 125 Black Hole Events (GWTC-3).

Research Question

Polynomial splines have been applied successfully across several different areas of GW astronomy (namely noise & detector calibrations), but rarely in the Hierarchical case. In this investigation, we deploy the use of Basis Spline (B-spline) models in hopes to illustrate how one can more efficiently represent mass, spin, and redshift distributions of merging binaries in GWTC-3. We constrain a few parameters, then ask: to what extent do our results, distributions over all observed events, explain the underlying formation channels of Binary Black Holes.

Methods

For a comprehensive data-driven characterization of Binary Black Holes (plural), one must employ Bayesian inference as their working framework. As more and more gravitational-wave events are detected, it is increasingly more interesting to employ the Hierarchical case.

A primary aim of Bayesian inference is to construct a posterior distribution:

$$p(\theta \mid d)$$

However, in order to probe the properties of an ensemble of events, we make the prior for θ conditional on a set of "hyper-parameters" Λ:

$$\pi(\theta|\Lambda)$$

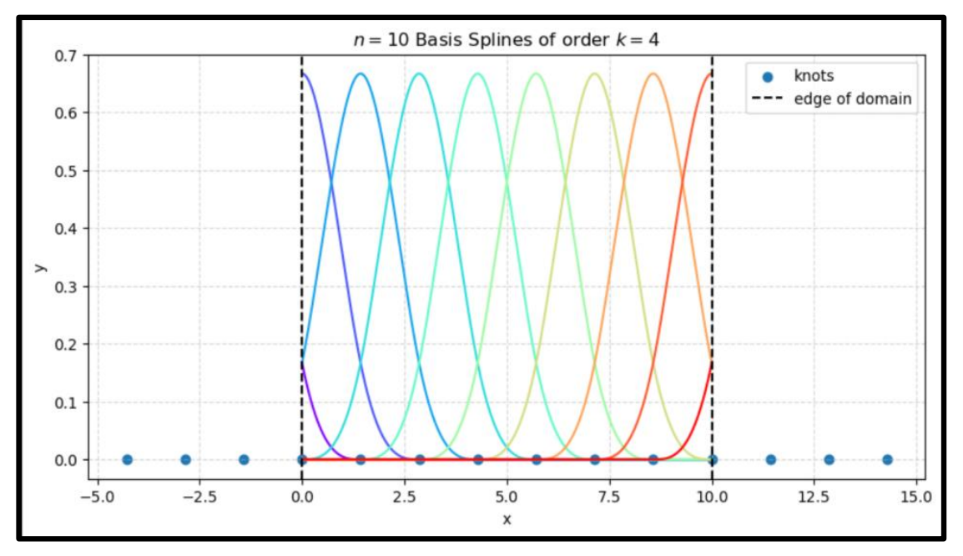
Now, a key goal of this Hierarchical Inference is to estimate the posterior distribution for our hyper-parameters.

In the Hierarchical case, we consider a marginalized likelihood that looks like so:

$$\mathcal{L}_{\rm tot}(\vec{d} \mid \Lambda) = \prod_{i=1}^{N} \frac{Z_{\emptyset}(d_i)}{n_i} \sum_{k=1}^{n_i} \frac{\pi(\theta_i^k \mid \Lambda)}{\pi(\theta_i^k \mid \emptyset)}.$$

With J indices, k posterior samples, and $\pi(\theta, z)$ as our prior; the posterior samples from *each* individual event in GWTC-3 are taken, and sampled from.

Subsequently, we model both the mass and spin distributions using B-splines. A Spline function is a piece-wise polynomial of order k stitched together from defined knots. They provide a very nice way to interpolate smooth functions from finite sampling parameters.



Results

Model	θ	Priors
Primary Mass Model Parameters		
B-Spline Primary	a	$smooth(\tau,\sigma,r,n)$
	τ_m	1, 5, 10
	n	20, 50, 80
Mass Ratio Model Parameters		
B-Spline Ratio	a	$smooth(\tau,\sigma,r,n)$
	τ	25
	n	50
Redshift Model Parameters		
B-Spline x Power	a	$normal(0,3)$
	τ	1
	n	50
Spin Magnitude Model Parameters		
B-Spline Magn.	a	$smooth(\tau,\sigma,r,n)$
	τ	1
	n	50
Spin Tilt Model Parameters		
B-Spline Tilt	a	$smooth(\tau,\sigma,r,n)$
	τ	25
	n	50

This analysis concerns itself with the underlying distribution of Black Hole Mass, Mass Ratio, Spin Magnitude, Spin Orientation, and Redshift across the observable universe. Figure 4 shows the primary mass distribution inferred with our B-Spline Model considering several different constraints. Our B-Spline model finds peaks at both 12 $M_\odot$ and 32 $M_\odot$ further agreeing with those reported in The LIGO collaboration et al. (2021b)[2].

Likewise, the mass ratio sees a shallow slope from q ~ 0.4 to q ~ 0.9 with persistently large uncertainty through most of it's range (Figure 1)

Result 1

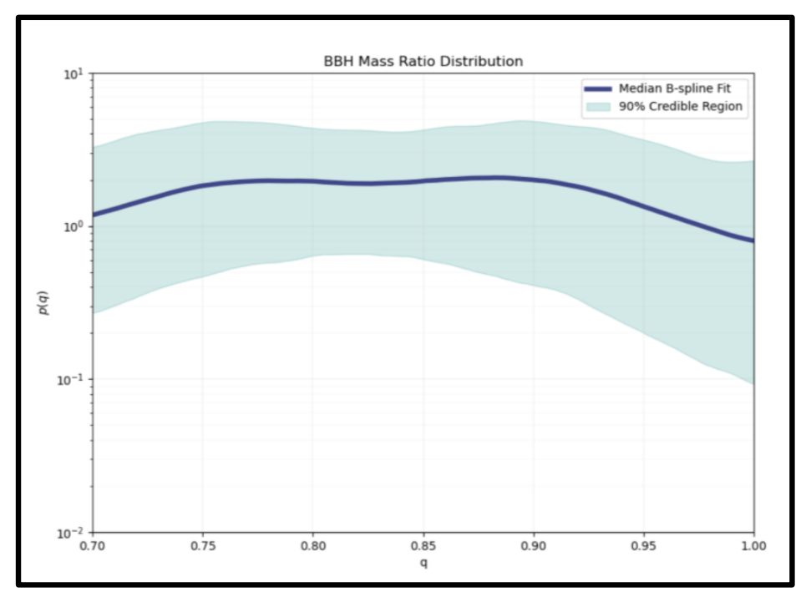


Figure 1: The Mass Ratio Distribution inferred with the B-Spline Model.

Result 2

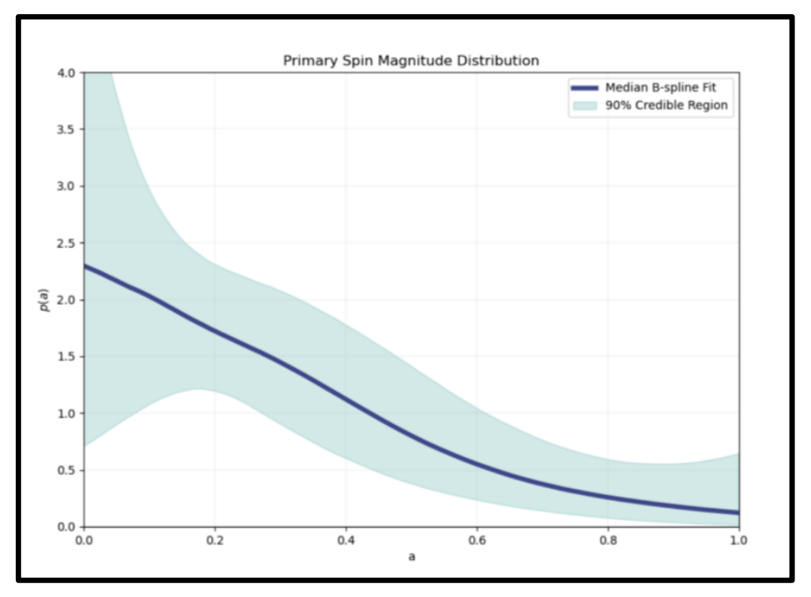


Figure 2: Primary spin magnitude distribution 25 knots linearly spaced from 0 to 1.

Result 3

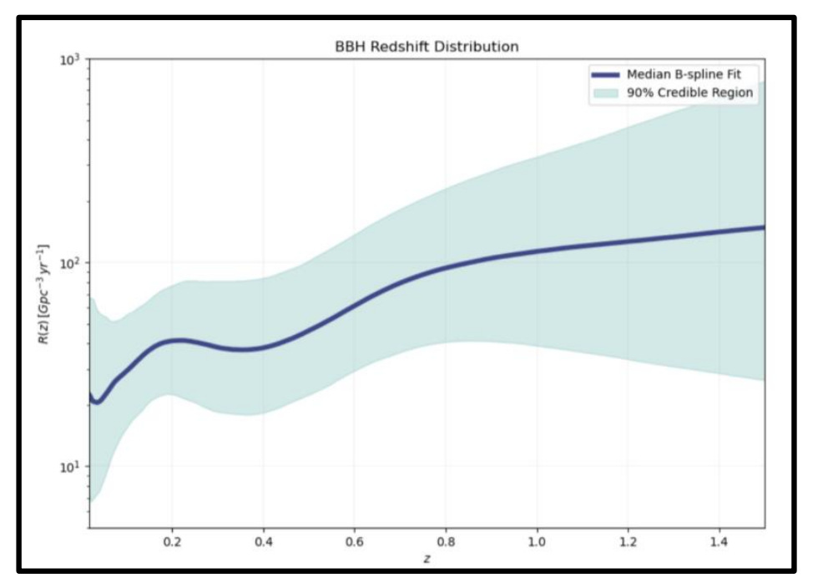


Figure 3: Merger Rate as a function of redshift.

Result 4

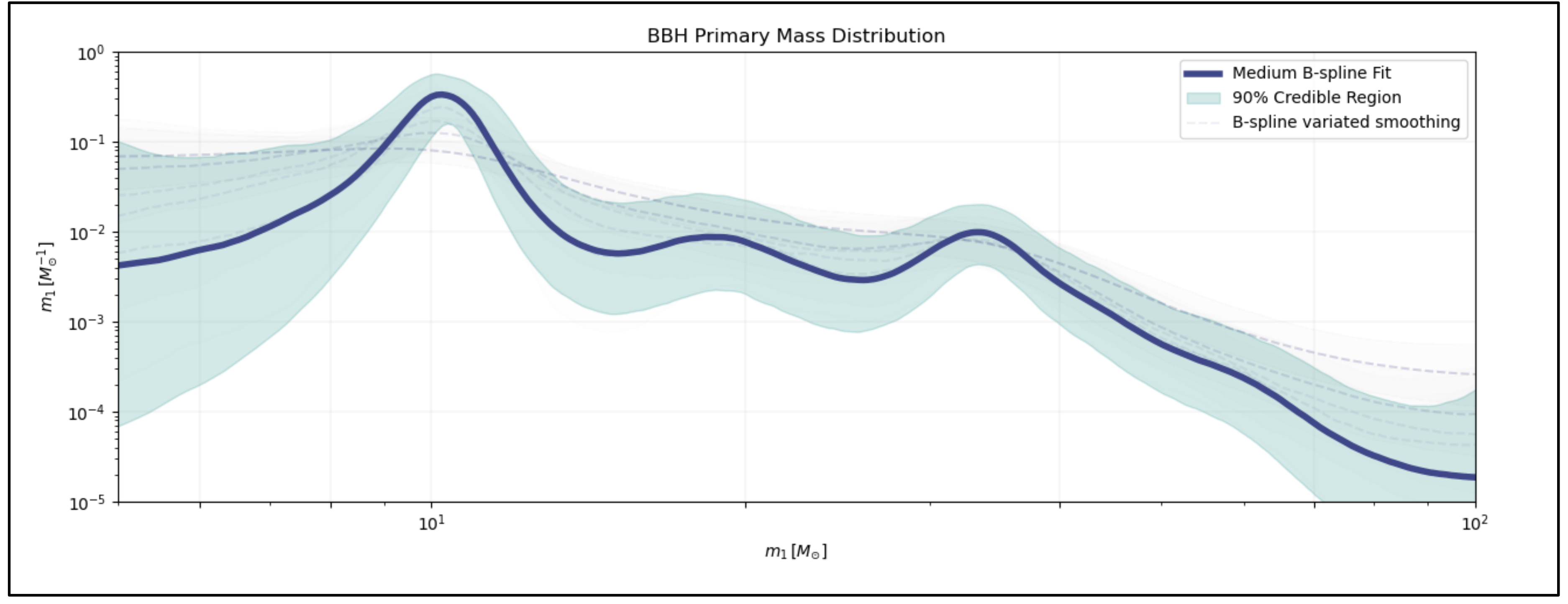


Figure 4: Marginal mass distribution for the primary Black Hole inferred with B-spline using 25 knots placed linearly from 5 $M_\odot$ to 100 $M_\odot$.

Indeed, Figure 2 is especially important as it pertains to spin. The B-Spline model shows results in disagreement with work done by Edelman and Farr (Edelman, Farr et al. 2023)[1], in which my clear peak is absent. However, it is found that 90% of BBH spins exist below a ~ 0.75 (which is in agreement). Furthermore, the RedShift (Figure 3) exhibits a large increase from z ~ 0.1 to z ~ 0.2, followed by decrease form z ~ 0.2 to z ~ 0.43. At large z we see high uncertainty (sparse data), but a continuation of power-like behavior.

Lastly, with discussion to the adjustments made in the smoothing priors for mass, Figure 4 demonstrates that changes to the current priors exhibit similar features, but less exaggerated peaks.

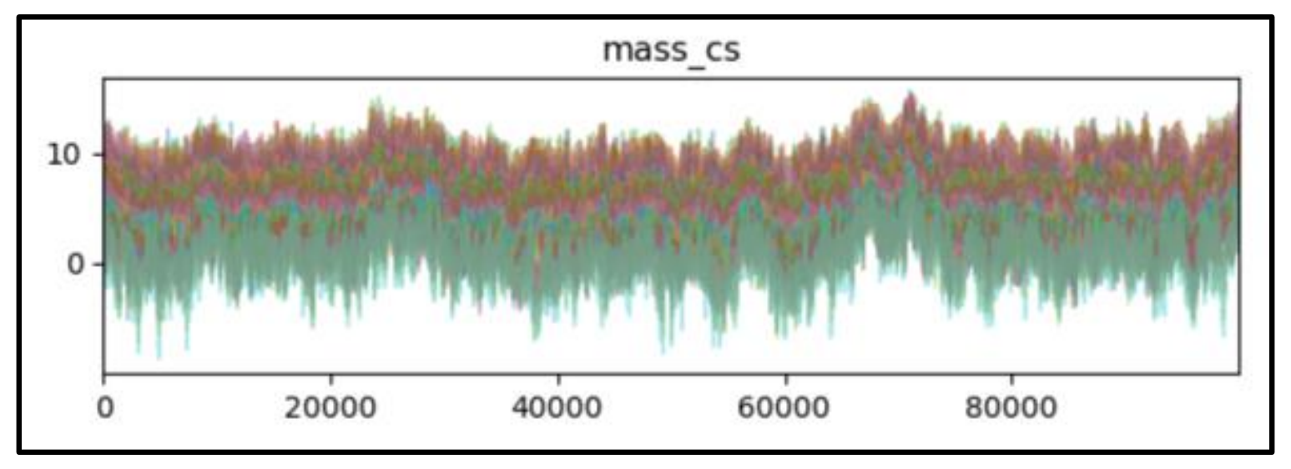


Figure 5: Trace plot (i) for sensitivity to tau (the smoothing prior used for the primary mass).

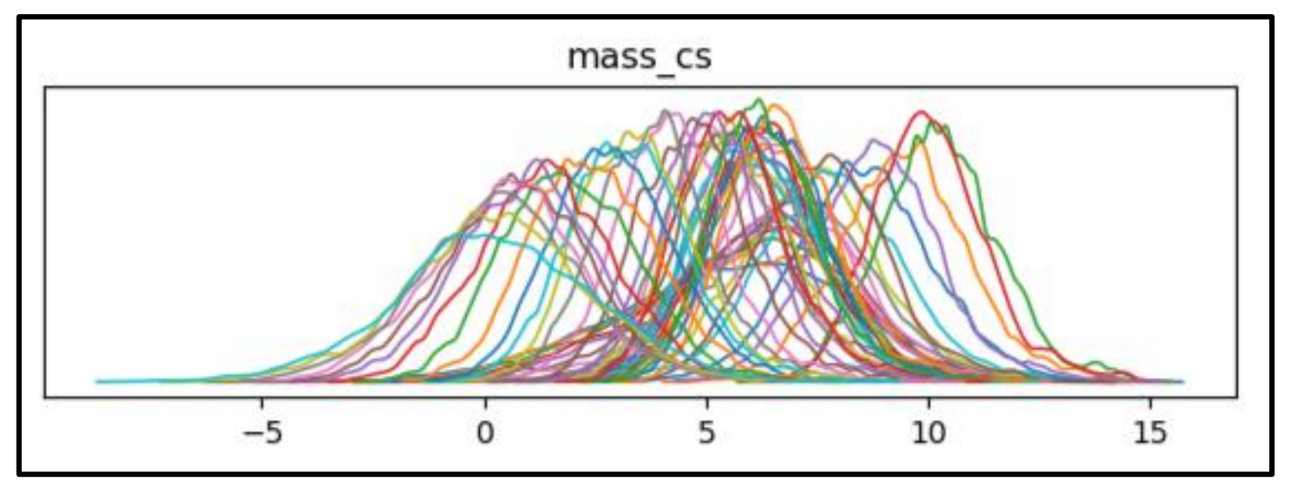


Figure 6: Trace plot (ii) for the smoothing prior used in the primary mass.

Conclusion

Indeed, it is now apparent that the presented analysis wonderfully characterizes the distributions for BBH in GWTC-3. Let's highlight a few key points:

- Non-parametric and data-driven models, speficially Basis Splines, can be used with great success in analysis for populations of Black holes.
- B-Splines highlight the danger in parametric models when data is scarce. For instance, in the low and high primary mass region, where detections are quite rare, the uncertainty for these model's is quite large; this isn't seen in parametric models.
- Changes to priors, specifically in the mass regime, is mostly immune to analytical changes in the fits, but is capable of hiding exaggerations in the peaks of our models.
- Binary Black Holes, within mergers, tend to exist in our universe at around $m1$ ~ 12 $M_\odot$ to $m2$ ~ 32 $M_\odot$

These are, of course, only a few notable conclusions among many. I'd like to briefly finish with the original motivation for this analysis: the likelihood of a certain Binary Black Hole Formation scenarios. By using our non-parametric model, we were able to find two most-common masses for Binary Black Hole Mergers. However, due to the still large uncertainties that exist in these model's, this analysis has no grounds to make any real alignments with theorized formation scenarios Godfrey and Farr (Godfrey, Farr et al. 2023)[3] .

It is here that I must make a hopeful assertion: with more detections, and thus greater growth of the catalog, the uncertainties will greatly reduce, and the analysis's ability to make assertions on BBH formation will greatly increase. The future in this analysis is with time and luckily time likes to proceed and Black Holes like to smash.

References

[1] Edelman, B., Farr, B. and Doctor, Z., 2023. Cover Your Basis: Comprehensive Data-driven Characterization of the Binary Black Hole Population. *The Astrophysical Journal*, *946*(1), p.16.
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[3] Abbott, Benjamin P., Richard Abbott, TDea Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams et al. "GWTC-1: a gravitational-wave transient catalog of compact binary mergers observed by LIGO and Virgo during the first and second observing runs." *Physical Review X* 9, no. 3 (2019): 031040.

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