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THE EXISTENCE OF TRANSLATION COMPLEMENTS*

1. Introduction

Let $\mathscr{P}$ be a (possibly infinite) projective plane, and G a collineation group of $\mathscr{P}$. Suppose temporarily that $\mathscr{P}$ is a translation plane with respect to L, and G is the group of collineations fixing L. Then $G=TG_x$, where T is the translation group with respect to L, and x is an affine point. If, moreover, the characteristic of $\mathscr{P}$ is not 2, then $G_x=C_G(t)$ and $G=TC_G(t)$, where t is an involutory homology with center x.

In this note we will consider a sort of converse of these facts. Take a *finite* collineation group G generated by involutory homologies, where $\mathscr{P}$ is again any projective plane. We will assume that G has an involutory homology t whose behavior resembles that of the t in the preceding paragraph, and then deduce the existence of a factorization $G=TC_G(t)$ as above.

The precise statement is as follows. Let $0(G)$ denote the largest normal subgroup of G of odd order. Define $Z^*(G)\geq 0(G)$ by: $Z^*(G)/0(G)$ is the center of $G/0(G)$.

THEOREM. *Let G be a finite collineation group of a projective plane which is generated by $0(G)$ and involutory homologies. Suppose G has a Klein subgroup generated by homologies having different axes, but that $G/0(G)$ is not dihedral of order 4 or 8. Assume further that $Z^*(G)$ contains an involutory homology t. Then $G=TC_G(t)$, where $T\lhd G$ is a (possibly trivial) group of elations, all having the same center as t or the same axis as t.*

In particular, G fixes a point or a line. This is not at all obvious – and, in fact, easily implies the theorem. Another way of looking at the theorem is that $C_G(t)$ must contain most of G.

This result is similar in statement and proof to [4], Theorem A(iv). That result considered the situation where the hypothesis $t\in Z^*(G)$ is replaced by: $Z^*(G)$ contains no involutory homology. However, neither result implies the other.

The proof is relatively elementary. For example, it does not involve the solvability of groups of odd order: the only deep group-theoretic fact required is Glauberman's Z^*-theorem [2].

The proof is somewhat simplified if G contains no Baer involution and is

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solvable. The solvable case is, however, a very interesting and unexpected case, as can be seen from the second corollary.

COROLLARY 1. *Let H be a collineation group of a projective plane. Suppose that the involutory homologies in H generate a finite group G satisfying the conditions of the theorem. Then* $H = TC_H(t)$.

COROLLARY 2. *Let S be a (finite) 2-group of collineations of a projective plane containing two involutory homologies generating a group of order at least* 16. *If S normalizes a collineation group X of odd order, then SX fixes a point or a line (both, if X contains no nontrivial elations).*

2. Background

Let $\mathscr{P}$ be a projective plane. If $g \neq 1$ is a perspectivity, c_g and A_g will denote its center and axis. Let G be a finite collineation group of $\mathscr{P}$. If c is a point, and A is a line, then $G(c)$ (or $G(A)$) is the group of perspectivities in G with center c (or axis A); also, $G(c, A) = G(c) \cap G(A)$.

We will need the following facts.

(α) (See [4], (3.1), (3.2.) Let $\langle t, u \rangle \leq G$ be a Klein group with t and u homologies having different axes. Then tu is a homology; t is the only involutory homology in $G(c_t, A_t)$; each collineation fixing c_t and A_t centralizes t; G has no elementary abelian subgroup of order 8 generated by homologies; and G has no Klein group $\langle t', u' \rangle$ with $A_{t'} = A_{u'}$.

(β) ([4], (3.3).) If $S \leq G$ is a 2-group, and S contains an involutory homology, then so does $Z(S)$.

(γ) If a perspectivity fixes a subplane, then its center and axis are in that subplane.

(δ) ([1], p. 120.) Let t and u be involutory homologies in $G(L)$ with $c_t \neq c_u$. Then $tu \in G(L \cap c_t c_u, L)$.

(ε) Let t and u be involutory homologies with $c_t \neq c_u$ and $A_t \neq A_u$. Then all fixed points of tu are on $c_t c_u$, with the possible exception of $A_t \cap A_u$.

(ζ) ([4], (3.5).) Let p be an odd prime, and let M be an elementary abelian p-subgroup of G inverted by the involutory homology t. Write $N = N_G(M)$. Then one of the following holds.

1. All centers of involutions in $\langle t \rangle M$ coincide, and N fixes c_t (or dually).
2. There are two involutions in $\langle t \rangle M$ having different centers and axes, and all centers of involutions in $\langle t \rangle M$ lie on a line fixed by N.
3. $p = 3$, and each $m \in M - \{1\}$ fixes exactly three points, which are noncollinear and are permuted transitively by M.

4. M fixes exactly three points x, y, z, which are non-collinear; $x^t = x$, $y^t = z$, and N induces S_3 on $\{x, y, z\}$.

(η) (Frattini argument.) If $H \trianglelefteq G$ and P is a Sylow subgroup of H, then $G = HN_G(P)$.

(θ) ([3], p. 180.) If P is a p-group (where $p > 2$) and t is an involution normalizing P, then $P = C_P(t)[P, t] = [P, t]C_P(t)$. Here, $C_P(t)$ and $[P, t] = \langle [g, t] \mid g \in P \rangle$ are $N_G(P) \cap C_G(t)$-invariant.

(ι) ([3], p. 240.) If P is a Sylow subgroup of G, and if $a, b \in Z(P)$ are conjugate in G, then they are conjugate in $N_G(P)$.

($\varkappa$) Let t be an involution acting on a group H of odd order. Suppose t normalizes a p-subgroup P of H. Then t normalizes a Sylow p-subgroup of H containing P. (*Proof.* Let $Q \geq P$ be a p-group maximal with respect to being normalized by t. Then $N_H(P)^t = N_H(P)$. Since $N_H(P)$ has an odd number of Sylow p-subgroups, t normalizes one of them. The maximality of P now implies that it is Sylow in $N_H(P)$, and hence in H.)

(λ) (Glauberman's Z^*-theorem [2].) Let t be an involution in G. Then, $t \in Z^*(G)$ if and only if $tt^g = t^g t$ and $g \in G$ imply $t^g = t$.

3. Proof of the theorem

Let G be as in the theorem. We will use the following notation:

$\langle t, u \rangle$ is a Klein group generated by homologies, where $t \in Z^*(G)$;

$S \geq \langle t, u \rangle$ is a Sylow 2-subgroup of G; and

$c = c_t$, $A = A_t$.

We begin with a general group-theoretic remark.

LEMMA 1. *Suppose $H \leq G$ and $G = 0(G)H$. Let H^* be the subgroup generated by the involutory homologies in H. Then $G/0(G) \approx H^*/0(H^*)$.*

Proof. Since $0(G)0(H^*) \trianglelefteq G$, we have $0(H^*) = 0(G) \cap H^*$. Each element of H is a product of involutory homologies $\bmod 0(G)$ and hence also $\bmod 0(H)$. Thus, $H = 0(H)H^*$, so $G = 0(G)0(H)H^* = 0(G)H^*$. This implies the lemma.

LEMMA 2. (i) *t is the unique involutory homology in $Z(S)$.*

(ii) *$A = A_t$ is the unique fixed line of S (and dually).*

Proof. (i) Deny! By (α), if $u \neq t$ is another involutory homology in $Z(S)$, then $S - \langle t, u \rangle$ contains no involutory homologies. Since S is Sylow and $N_G(S)$ fixes t, it also fixes u and tu by (ι). Then $u \in Z^*(G)$ by (λ). Thus, $G \trianglerighteq 0(G)\langle t, u \rangle$, so $G = 0(G)\langle t, u \rangle$, contrary to the hypotheses of the theorem.

(ii) Suppose S fixes $L \neq A_t$. Then $\langle t, u \rangle$ fixes L, so $L = A_u$ or A_{tu}. In either case, this contradicts (i).

LEMMA 3. *Let G be as in the theorem. Suppose G fixes a line L. Then L is the axis A of t, and $G=TC_G(t)$ with $T=[G, t]$ a group of elations having axis A.*

Proof. By Lemma 2, $L=A$. Let $g\in 0(G)$. Then $[g, t]=t^g t$ is an elation with axis A (by (α) and (δ)). Thus, $[0(G), t]$ consists of elations having axis A. By (θ), $G=[0(G), t]C_{0(G)}(t)\cdot C_G(t)=C_G(t)[0(G), t]$. This proves the lemma.

Let G and $\mathscr{P}$ yield a counterexample to the theorem with $|G|$ minimal. By Lemma 3, G moves all points and lines.

We will obtain a contradiction in a series of steps.

(I) *G has no normal subgroup H such that $u\in H$ and $t\notin H$.*

Proof. Suppose H exists. By (β) there is an involutory homology in $Z(S\cap H)$. This contradicts Lemma 2(ii).

(II) *The following situation cannot occur: $S=\langle u, v\rangle X\triangleright X$ with u and v involutory homologies, $|\langle uX, vX\rangle|=4$ or 8, and X planar.*

Proof. Suppose $S=\langle u, v\rangle X$ has these properties. We will show that $\langle u, v\rangle 0(C_G(t))/\langle t\rangle 0(C_G(t))\leq Z^*(C_G(t)/\langle t\rangle 0(C_G(t)))$. Once this is known, we will have $\langle u, v\rangle 0(C_G(t))\trianglelefteq C_G(t)$, so $\langle u, v\rangle 0(C_G(t))0(G)\trianglelefteq C_G(t)0(G)=G$. But G is generated by $0(G)$ and involutory homologies, and (as we will show) $\langle u, v\rangle$ contains all involutory homologies in S. Thus, $G=\langle u, v\rangle 0(G)$, and this contradicts one of the hypotheses of the theorem.

By (γ), all centers and axes of involutions in $\langle u, v\rangle$ are in the fixed point subplane $\mathscr{P}_0$ of X. By (α), $\langle u, v\rangle$ centralizes X. Also by (α), all involutions in $\langle u, v\rangle$ are homologies, so $\langle u, v\rangle\cap X=1$ and hence $|\langle u, v\rangle|=4$ or 8. Since t centralizes a Klein group in $\langle u, v\rangle$, by (α) t must be in $\langle u, v\rangle$.

Suppose $h\in S$ is an involutory homology. Then h induces an involutory homology of $\mathscr{P}_0$. Hence, h agrees on $\mathscr{P}_0$ with some involution $h'\in\langle u, v\rangle$. Now $hh'\in X\leq C_S(h')$. Thus, hh' is 1 or an involution. But $hh'\in X$, so $h=h'$. Consequently, $\langle u, v\rangle$ contains all involutory homologies in S.

In particular, if $u^g\in S$ with $g\in C_G(t)$, then $u^g\in\langle u, v\rangle$. If $u^g\in\langle t, u\rangle$ whenever $u^g\in S$, then $u\langle t\rangle 0(C_G(t))\in Z^*(C_G(t)/\langle t\rangle 0(C_G(t)))$ by (λ). Thus, the only situation left to consider is: $|\langle u, v\rangle|=8$, and $u^g\notin\langle t, u\rangle$ for some $g\in C_G(t)$.

Since $\langle u, v\rangle$ has exactly two Klein groups, $\langle t, u\rangle^g=\langle t, v\rangle$. Also, $\langle t, u\rangle \triangleleft\langle u, v\rangle$, so $N_G(\langle t, u\rangle)$ and $N_G(\langle t, v\rangle)$ contain S. Then $N_G(\langle t, u\rangle)^g =N_G(\langle t, v\rangle)$ implies that $S^g=S^{n^{-1}}$ for some $n\in N_G(\langle t, v\rangle)$. Then $gn\in N_G(S)$ and $\langle t, u\rangle^{gn}=\langle t, v\rangle$, so we may replace g by gn.

Since $\langle u, v\rangle$ contains all involutory homologies in S and g normalizes S, g also normalizes $\langle u, v\rangle$. Now $\langle t, v\rangle^g=\langle t, u\rangle$, so $\langle t, u\rangle^{g^2}=\langle t, u\rangle$. Thus, g has even order. Let $\langle g'\rangle$ be the Sylow 2-subgroup of $\langle g\rangle$. Then g' normalizes S, so $g' \in S$. However, $\langle t, u\rangle^{g'} = \langle t, v\rangle$, so this is impossible. This proves (II).

(III) *We may assume that $\mathscr{P}$ is the subplane of $\mathscr{P}$ generated by $c^{0(G)}$.*

Proof. First, suppose $c^{0(G)}$ does not generate a subplane $\mathscr{P}^*$ of $\mathscr{P}$. Since $c^{0(G)}$ is invariant under $G = C_G(t)0(G)$, it is not contained in a line. Similarly, $c^{0(G)}$ cannot have three or more collinear points. Thus, $c^{0(G)}$ is a triangle $\{c, d, e\}$. Note that t cannot fix d (if it did, t would normalize $G(d)$ and hence commute with some conjugate of itself having center d, which is impossible since $t \in Z^*(G)$). Thus, $G \rhd G_{cde}$ and $|G/G_{cde}| = 6$, where $t \notin G_{cde}$. However, $\langle t, u\rangle \cap G_{cde} \neq 1$, so this contradicts (I).

This provides us with $\mathscr{P}^*$. Clearly G acts on $\mathscr{P}^*$, inducing the collineation group $G^* = G/K$, where K is the pointwise stabilizer of $\mathscr{P}^*$. By (γ), each involutory homology in G is also an involutory homology of $\mathscr{P}^*$, and (by (α)) centralizes K. Also, G^* fixes no point or line of $\mathscr{P}^*$.

If $K = 1$, there is nothing to prove. Thus, suppose $K \neq 1$. Then $|G^*| < |G|$, so the minimality of $|G|$ implies that $G^*/0(G^*)$ is dihedral of order 4 or 8.

Consequently, $S = \langle u, v\rangle (S \cap K)$ with u and v involutory homologies. This contradicts (II).

(IV) *G has a nontrivial normal p-subgroup for some odd prime p.*

Proof. Deny! For each prime $p \mid |0(G)|$, there is a Sylow p-subgroup P of $0(G)$ normalized by S. (For, $0(G)$ has an odd number of Sylow p-subgroups.) By the Frattini argument, $G = 0(G) N_G(P) = 0(G)(N_G(P) \cap C_G(t))$. If P fixes $A = A_t$ for each p and P, then so does $0(G)$ and hence also $G = 0(G)\, C_G(t)$.

Thus, we can find p and P such that P moves A. Let H be the subgroup generated by the involutory homologies of $N_G(P)$. By Lemma 1, $G/0(G) \approx H/0(H)$. Also, $H \geq [P, t]$, where $P_1 = [P, t]$ moves A since $P = C_P(t)[P, t]$ does (see (θ)). If H moves each point, then $H = G$ by the minimality of G, so (IV) holds. Thus, H must fix a point – and hence must fix c by Lemma 2(ii). Consequently, $P_1 \leq G(c)$ by the dual of Lemma 3.

Dually, there must a prime q and a Sylow q-subgroup Q of $0(G)$ normalized by S such that Q fixes A and moves c. Then $Q_1 = [Q, t]$ also moves c, and $Q_1 \leq G(A)$.

Since S normalizes Q_1, while S fixes no point of A (by Lemma 2(ii)), $Q_1(x, A) \neq 1$ for at least two points $x \in A$. Hence, the group of elations with axis A is an elementary abelian q-group. Dually, the group of elations with center c is an elementary Abelian p-group. In particular, t inverts both P_1 and Q_1.

Let E be the group of all elations in $0(G) \cap G(A)$. By ($\varkappa$), E is contained in a Sylow p-subgroup of $0(G)$ normalized by t. Hence, we may assume that $E=P_1$. In particular, $C_G(t)$ normalizes P_1. Dually, we may assume that Q_1 consists of all elations in $G(c)$, and then $C_G(t)$ normalizes Q_1.

Let $r\neq p, q$ be a prime, and R a Sylow r-subgroup of $0(G)$ normalized by t. By what has already been proved, R must fix c and A, and hence centralize t by (α). Since each r-subgroup of $0(G)$ normalized by t is contained in a Sylow r-subgroup of $0(G)$ normalized by t (see ($\varkappa$)), it follows that t inverts no nontrivial r-element of $0(G)$.

We now wish to count the number of conjugates of t. Let $t'\in t^G-\{t\}$, and consider tt'. Since t inverts tt', necessarily $|tt'|=p^\alpha q^\beta$ for some α and β. If $\beta=0$, then tt' is contained in a Sylow p-subgroup of $0(G)$ normalized by t, so tt' is an elation in $0(G)\cap G(c)$, and hence $tt'\in P_1$. Similarly, if $\alpha=0$ then $tt'\in Q_1$. Finally, suppose $\alpha\geq 1$ and $\beta\geq 1$. Then $\langle tt'\rangle\cap P_1\neq 1$ and $\langle tt'\rangle\cap Q_1\neq 1$ so $\langle tt'\rangle\cap P_1$ fixes the axis A of $\langle tt'\rangle\cap Q_1$. Since $\langle tt'\rangle\cap P_1$ consists elations with center $c\notin A$, this is impossible.

Thus, $t^G=tP_1\cup tQ_1$, so $|G:C_G(t)|=|P_1|+|Q_1|-1$. We know that $C_G(t)$ normalizes P_1, and $C_G(t)\cap P_1=1$. Thus, $|P_1|=|P_1C_G(t):C_G(t)|$ divides $|P_1|+|Q_1|-1$, so $|P_1|$ divides $|Q_1|-1$. Similarly, $|Q_1|$ divides $|P_1|-1$. This is absurd.

(V) *G has a normal elementary Abelian subgroup not centralized by t.*

Proof. By (IV), G has a normal elementary Abelian subgroup $N\neq 1$. Here, $N\not\leq C_G(t)$. For, if $N\leq C_G(t)$ then $N\leq C_G(t^g)$ for all $g\in 0(G)$, and then N must fix each point in the generating set $c^{0(G)}$ of $\mathscr{P}$ (see (III)).

(VI) *$G=MC_G(t)$, where M is a minimal normal elementary Abelian p-subgroup of G inverted by t.*

Proof. Let M be a nontrivial elementary Abelian p-subgroup of G normalized by $C_G(t)$, not centralized by t, contained in a normal elementary Abelian p-subgroup of G, and minimal with respect to these properties; by (V), M exists. By (θ), $M=C_M(t)\times[M, t]$, where $[M, t]=\{m\in M\mid m^t=m^{-1}\}$ is a nontrivial $C_G(t)$-invariant subgroup. Thus, $M=[M, t]$ is inverted by t.

Suppose M moves A and c. Then so does $H=MC_G(t)$. By Lemma 2(ii), H moves every line. By Lemma 1, $H/0(H)\approx G/0(G)$. The minimality of $|G|$ now implies that $H=G$.

Suppose now that M fixes A. Then $M\langle t\rangle=\langle t^M\rangle\leq G(A)$. By ($\delta$), M consists of elations. Let $M\leq N\trianglelefteq G$ with N Abelian. Then N fixes A. Take any $g\in G$ such that $A^g\neq A$. Then $M^g\leq N\cap G(A^g)$ fixes A, so that $z=A\cap A^g$ is the center of all elements of M^g. Similarly, $M\leq G(z, A)$. Thus, $\langle M^g\mid g\in G\rangle\leq G(z)$, where $\langle M^g\mid g\in G\rangle$ is normal in G. It follows that G must fix z, and this contradicts Lemma 3.

(VII) *If* $t' \in tM - \{t\}$, *then* $A_t \neq A_{t'}$ *and* $c_t \neq c_{t'}$.

Proof. Suppose $A_t = A_{t'}$. By (δ), $tt' \in M$ is an elation with axis A_t. Hence, $C_G(tt') \geq M$ fixes A_t, so $G = MC_G(t)$ also does.

Completion of the Proof

By (I), $u \notin C_G(M)$ and $tu \notin C_G(M)$. By (θ), it follows that $C_M(u) \neq 1$ and $C_M(tu) \neq 1$ (since t inverts M). Let $1 \neq m \in C_M(u)$, and write $m = tt'$ with $t' \in t\langle m \rangle$. By (VII) and ($\varepsilon$), all fixed points of m are on $L = c_t c_{t'}$, except possibly for $x = A_t \cap A_{t'}$.

Since $N_G(M) = G$, Lemma 3 and (ζ) imply that either (a) each element of $M - \{1\}$ fixes exactly three points, which are non-collinear and are permuted transitively by M, or (b) M fixes exactly three points x, y, z, which are non-collinear, are not all fixed by t, but are such that G fixes $\{x, y, z\}$.

Here, (b) leads to the same contradiction as in the first part of the proof of (III). Thus, (a) must hold. In particular, m fixes just three points x, y, z, and M fixes $\{x, y, z\}$. Moreover, $y, z \in L$ and $x \notin L$.

Since u centralizes $\langle t, m \rangle = \langle t, t' \rangle$, u fixes $x, c_t, c_{t'}, A_t \cap L$, and $A_{t'} \cap L$. Thus, $L = A_u$ and $x = c_u$. It follows from (α) that $M_x = C_M(u)$, where $|M : M_x| = 3$.

Similarly, there is a point x^* such that $M_{x^*} = C_M(tu)$ and $|M : M_{x^*}| = 3$. But $C_M(u) \cap C_M(tu) = 1$ since t inverts M. Thus, $|M| = 9$.

It follows that $G/C_G(M)$ is isomorphic to a subgroup of $GL(2, 3)$, where $C_G(M) = M(C_G(t) \cap C_G(M))$. Here $C_G(t) \cap C_G(M)$ fixes each point of $c^M = c^G$ (since $G = C_G(t)M$). Thus, $C_G(t) \cap C_G(M) = 1$ by (III), so $C_G(M) = M$. Hence, G/M is isomorphic to a subgroup of $GL(2, 3)$.

Since M is a minimal normal subgroup of G, G/M acts irreducibly on M. Also, G/M is generated by its involutions, and has order > 8, so we must have $G/M \approx GL(2, 3)$. Let $g \in C_G(t)$ have order 3 and be normalized by $\langle t, u \rangle$. Then $g^u = g^{-1}$. We may assume that g centralizes $\langle m \rangle = C_M(u)$ (as otherwise, g would centralize $C_M(tu)$). Then $\langle g, t, u \rangle$ fixes the set $\{x, y, z\}$ of fixed points of m, where $x = c_u$ and $yz = A_u$. Clearly, t fixes x. But t cannot fix y and z, as $m \notin C_G(t)$ (see (α)). Thus, $y^t = z$. Also, g cannot fix x (as otherwise g would fix c_u and A_u, contrary to (α)). It follows that $\langle t, g \rangle$ induces a subgroup of S_3 on $\{x, y, z\}$ with t and g acting nontrivially and commuting. This is ridiculous.

Proof of Corollary 1. By Lemma 2(i), $\langle t \rangle 0(G) \lhd H$, so $H = 0(G) C_H(t) = TC_{0(G)}(t) C_H(t) = TC_H(t)$.

Proof of Corollary 2. Set $H = SX$, so $X = 0(H)$. If S has involutory homologies having different axes, then Corollary 1 applies.

Suppose commuting involutory homologies always have the same center and axis. By (β), $Z(S)$ contains an involutory homology t. We will show that X fixes c_t and A_t. Suppose $g \in X$ moves c_t, and consider S^g. By hypothesis, there is an involutory homology $u \in S^g$ not conjugate to t^g in S^g, and hence in H. Then $\langle t, u\rangle$ has an involution v commuting with t and u. Clearly, v is a homology. Thus, $c_t^g = c_u = c_v = c_t$, which is not the case.

COROLLARY 3. *Let S be a 2-group of collineations of a projective plane. Assume that S is dihedral of order* 16, *and that one of its dihedral subgroups of order* 8 *is generated by two involutory homologies having different axes. If S normalizes a collineation group X of odd order, then SX fixes a point or a line (both, if X contains no nontrivial elations).*

Even when S contains Baer involutions, Corollary 3 can be proved in exactly the same manner as the theorem.

4. Concluding remarks

The proof of the theorem used all of S primarily to deal with the case where t inverts some element of order 3. It is straightforward to further modify the proof in order to deal with this case when $S = \langle t\rangle$, if one is willing to accept weaker conclusions (such as G having an orbit of length 1, 3 or 9).

BIBLIOGRAPHY

1. Dembowski, P., *Finite Geometries*, Springer, Berlin-Heidelberg-New York, 1968.
2. Glauberman, G., 'Central Elements in Core-Free Groups', *J. Algebra* 4 (1966), 403–420.
3. Gorenstein, D., *Finite Groups*, Harper, New York 1968.
4. Kantor W.M., 'On the Structure of Collineation Groups of Finite Projective Planes' (to appear).

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